Shifting method of relay feedback identification with use of PLC

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Abstract

This paper shows the use of the Shifting method for relay feedback identification. An identification program using this method was created for the programmable logic controller Tecomat Foxtrot. The program finds two points of the Nyquist frequency characteristics from a single relay feedback test and calculates model parameters of the identified system. For this purpose it is assumed that the system is describable by the second order time delayed model. The code was written in the structured text language according to the standard IEC 61131-3. The program was tested on two simulated systems and on a laboratory controlled plant called "Air Aggregate". The obtained results show a good applicability of the proposed identification method for identification of time invariant aperiodic systems.

Key-words: Shifting method, relay feedbeck identification, frequency characteristics, second order plus time delay model

1. Introduction

Relay feedback identification is relatively old but still developing set of methods of system identification. At the beginning was the work of Tsypkin [1]. After him Åström and Hägglund used relay feedback identification for automatic estimation of controller parameters [2]. Relay feedback identification is a widely used method. Lots of papers and books about relay feedback identification were published, for example by Q. G. Wang, T. H. Lee and C. Lin [3] or T. Liu and F. Gao [4]. Work of S. H. Shen, J. S. Wu and C. C. Yo introduces a way how to determine static gain from relay feedback test with asymetric relay [5]. In this event we can find two points of frequency characteristics from a single relay feedback test. The shifting method, which is based on biased relay, is a method of relay feedback identification of time invariant processes [6], [7] and [8]. This method shows that it is posible to find three points of frequency characteristics from a single relay feedback test. We verify the correct working of this method with use of programable logical controller (PLC). I wrote for PLC Tecomat Foxtrot script of shifting method in structured text. Structured text is a programing language according to IEC 61131-3. Therefore, the written program can be used on other PLCs. The written program identifies controlled plant automaticaly only with knowledge of plant input value in an operating point and its limit values. Identify model can be used for regulator parameters estimation [9].

2. Shifting method

Shifting method was developed for identification of time invariant single input single output systems. For relay feedback identification based on Shifting method biased relay in circuit is used, which is shown in Fig. 1.

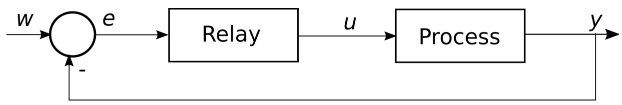

Fig. 1. *Controlled plant under relay feedback test.*

This method is able to find three points of frequency characteristics from a single masurement. Biased relay is needed calculation of point of frequency characteristics $G_{ident}(0)$ with frequency $\omega = 0$. The first frequency characteristics point gives us value of static gain of the controlled plant. The point could be calculated from integrals over $k \cdot T_p$, where k is the positive integer and T_p is the period of relay oscilations, as

$$G_{ident}(0) = G_{ident}(j\omega_0) = K = \frac{\int_t^{t+k\cdot T_p} y(\tau) d\tau}{\int_t^{t+k\cdot T_p} u(\tau) d\tau}, \tag{1}$$

where y is the controlled plant output, u is the plant input and t is the stable oscilation reach time. The second point of Nyquist frequency characteristics could be estimated from

$$G_{ident}(j\omega_1) = \frac{\int_t^{t+k\cdot T_p} y(\tau) e^{-j\omega_1\tau} d\tau}{\int_t^{t+k\cdot T_p} u(\tau) e^{-j\omega_1\tau} d\tau}, \tag{2}$$

where ω_1 is the frequency of stable oscillation during the relay feedback test. Following equation holds for ω_1.

$$\omega_1 = \frac{2\pi}{T_p}. \tag{3}$$

The third point of Nyquist characteristics has a frequency

$$\omega_2 = 2\omega_1. \tag{4}$$

Controlled plant input $\overline{u}$ and output $\overline{y}$ with the frequency of oscillation ω_2 could be calculated from the plant input u and plant output y with the frequency

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of oscilation ω_1 as

$$\overline{u} = u(t) + u(t - \frac{T_p}{2}), \quad (5)$$

$$\overline{y} = y(t) + y(t - \frac{T_p}{2}), \quad (6)$$

where t is the time of stable oscillation reach. The third point of Nyquist frequency characteristics is

$$G_{ident}(j\omega_2) = \frac{\int_t^{t+k \cdot T_p} \overline{y}(\tau) e^{-j\omega_2 \tau} d\tau}{\int_t^{t+k \cdot T_p} \overline{u}(\tau) e^{-j\omega_2 \tau} d\tau}. \quad (7)$$

Now, we have three points of frequency characteristics. One of them is on a positive part of the Nyquist characteristics real axis. The remaining two points are mostly in the 2nd and the 3rd quadrant. In this situation, we need some other information about frequency characteristics in the 4th quadrant. The described problem could be solved by adding a transport delay to the output of the relay as in Fig. 2. When we add transport delay, points of frequency characteristics $G(j\omega_1)$ and $G(j\omega_2)$ are shifted towards the 4th quadrant as $G_{delayed}(j\omega_1)$ and $G_{delayed}(j\omega_2)$, see Fig. 3.

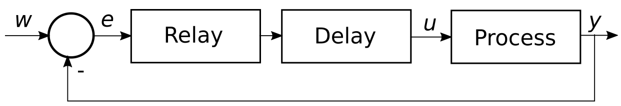

Fig. 2. *Controlled plant under relay feedback test with added transport delay.*

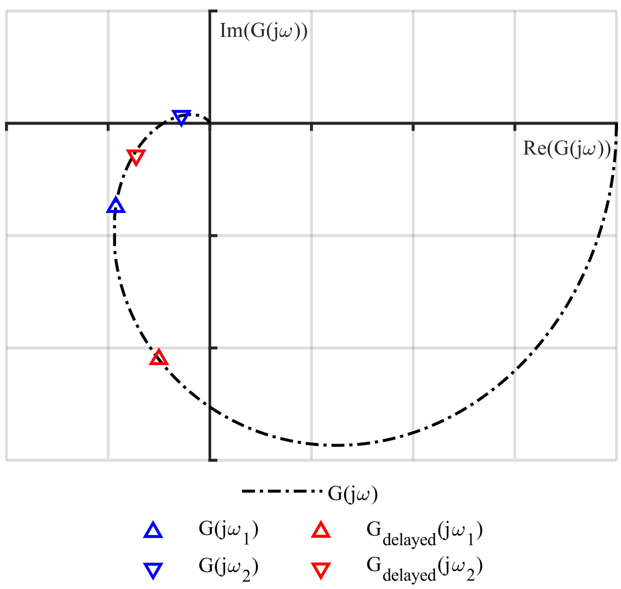

Fig. 3. *Shifting of points of Nyquist frequency characteristics from identification with added delay.*

3. Model estimation

We assumed model of controlled plant with transfer function

$$M(s) = \frac{K e^{-T_d s}}{a_2 s^2 + a_1 s + 1} \quad (8)$$

where s is Laplace transform argument, K is the static gain and a_2 and a_1 are time constants. Model parameters K, a_2, a_1 and T_d could be calculated as

$$K = G_{ident}(j\omega_0) = M(j\omega_0) \quad (9)$$

$$a_2 = \frac{1}{2\omega_1^2} \sqrt{\frac{1}{3} \left(\frac{K^2}{|G_{ident}(j\omega_2)|^2} - \frac{4K^2}{|G_{ident}(j\omega_1)|^2} + 3 \right)} \quad (10)$$

$$a_1 = \frac{1}{\omega_1} \sqrt{\frac{K^2}{|G_{ident}(j\omega_1)|^2} - (1 - a_2 \omega_1^2)^2} \quad (11)$$

$$T_d = \frac{1}{2} \left(\sum_{l=1}^{2} \frac{arg\left(\frac{1}{a_2 (j\omega_l)^2 + a_1 j\omega_l + 1}\right) - arg(G_{ident}(j\omega_l))}{\omega_l} \right) \quad (12)$$

These equations were published in [8].

4. Examples

The identification program was tested on three systems. Two of them were simulated systems. The other one was real system.

4.1. Simulated controlled plant

4.1.1. Third order simulated controlled plant

The simulated stable nonoscillatory controlled plant had transfer function

$$G(s) = \frac{2}{(1s+1)(0.5s+1)(0.9s+1)}. \quad (13)$$

The setpoint was given by the value of the action variable $u_0 = 5 \ V$. The first point of the frequency characteristics $G_{ident}(j\omega_0)$ was estimated from system static characteristics. Two other points of the frequency characteristics were found from the Shifting relay feedback identification method. A quarter of the period of stable oscilation of a system without transport delay T_0 was used as an added transport delay during the relay feedback test. This value of transport delay shifted the points of frequency characteristics of the simulated plant out of the 2nd quadrant.

The relay feedback test was repeated 100 times. Results from repeated identification are the almost identical points of the frequency characteristics. Similarity of results from repeated identification is evidence of good repeatibility of the identification process. Mostly found points frequencies are $\omega_1 = 0.88123 \ rad \cdot s^{-1}$ and $\omega_2 = 1.76246 \ rad \cdot s^{-1}$. Points with this value of frequency were estimated in 75 cases. The most different results with the same frequencies are in Tab.1. The identification program identified controlled plant sucessfully also in other 25 cases.

Table 1. *Most different results of the third order simulated system.*

	1st measurement	2nd measurement
$G_{ident}(j\omega_0)$	2.00001	2.00001
$G_{ident}(j\omega_1)$	$-0.19950 - 1.08527j$	$-0.17609 - 1.10618j$
$G_{ident}(j\omega_2)$	$-0.36701 - 0.19709j$	$-0.37845 - 0.21245j$

Result differences are caused by the nonconstant length of PLC cycle. The PLC cycle length depends on complexity of the part of a program, which is working in actual PLC cycle. The found points of the frequency characteristics were used to calculate model parameters, Tab.2.

Table 2. *Model parameters of the third order simulated system.*

	1st measurement	2nd measurement
K	2.00001	2.00001
a_1	2.05635	2.02604
a_2	1.33557	1.25973
T_d	0.19440	0.20285

A comparison of the Nyquist frequency characteristics of the system and the models is in the Fig. 4. The identified models have almost the same Nyquist characteristics as the identified simulated controlled plant. At higher frequencies, the models and simulated system are a little different. Bode amplitude plots of both models are almost the same, see Fig. 5. They differ from the characteristics of the simulated controlled plant only at higher frequencies. The curves confirm that the static gain determination is correct.

The simulated controlled plant did not have a transport delay. The specified model belongs to lower order. The transport delay replaces the higher order of the identified system. Therefore, the phase frequency characteristics can not be the same at higher frequencies, see Fig. 6. The identified models are oscillating but the step functions are similar, see Fig. 7.

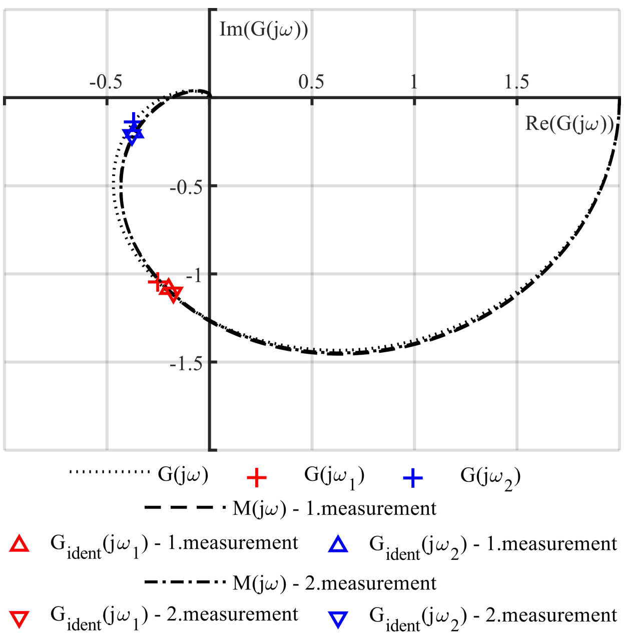

Fig. 4. *Example 1 - Nyquist frequency characteristics of simulated system and its models.*

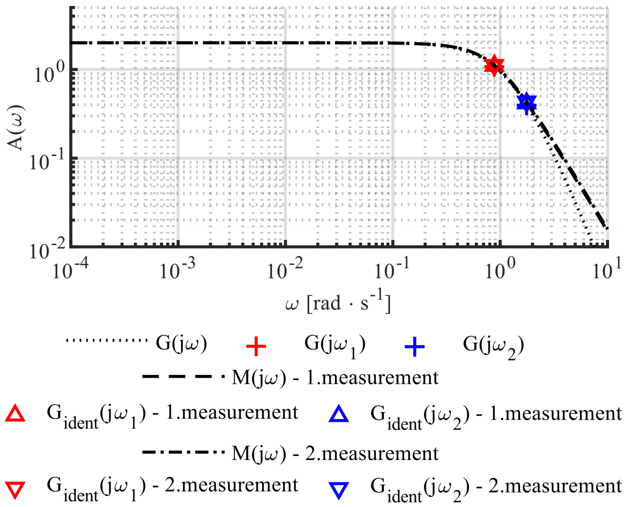

Fig. 5. *Example 1 - Bode amplitude plot of simulated system and its models.*

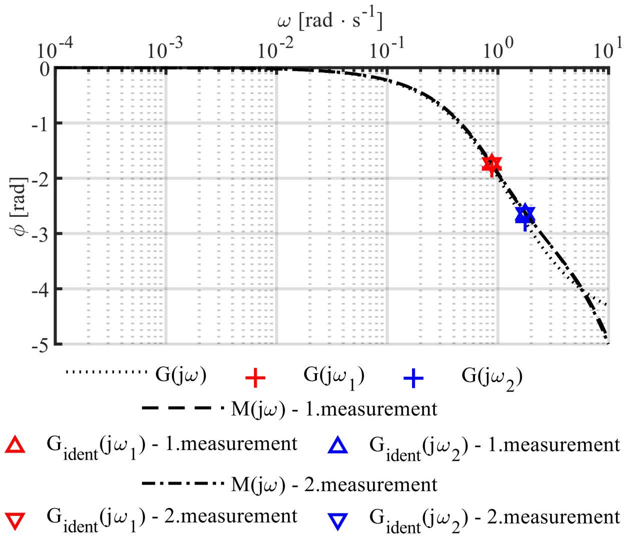

Fig. 6. *Example 1 - Bode phase plot of simulated system and its models.*

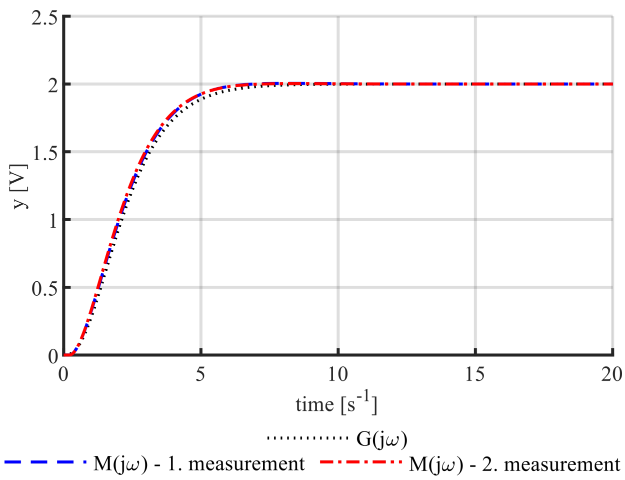

Fig. 7. *Example 1 - Step functions of simulated system and its models.*

4.1.2. Second order plus time delay simulated controlled plant

The simulated stable nonoscillatory controlled plant had transfer function

$$G(s) = \frac{2e^{1s}}{(1s+1)(1.5s+1)}. \tag{14}$$

The setpoint was given by value of the action variable $u_0 = 3\ V$. The first point of frequency characteristics $G_{ident}(j\omega_0)$ was estimated from static characteristics. Two other points of the frequency characteristics were found from the Shifting relay feedback identification method. A quarter of the period of stable oscillation of system without transport delay T_0 was used as an added transport delay during relay feedback test. This value of transport delay shifted the points of frequency characteristics of simulated plant closer to the 4th quadrant. The third point of controlled plant with frequency ω_2 is still in the 2nd quadrant, see Fig. 8. The relay feedback test was repeated 100 times. Results from repeated identification are almost identical points of frequency characteristics. Mostly found points frequencies are $\omega_1 = 0.60415\ rad \cdot s^{-1}$ and $\omega_2 = 1.20830\ rad \cdot s^{-1}$. Points with this value of frequency were found in 74 cases. The most different results with same frequencies are in Tab.1. The identification program identified controlled plant successfully also in other 26 cases.

Table 3. *Most different results of the second order plus time delay simulated system.*

	1st measurement	2nd measurement
$G_{ident}(j\omega_0)$	2.00001	2.00001
$G_{ident}(j\omega_1)$	$-0.22373 - 1.33223j$	$-0.30259 - 1.26257j$
$G_{ident}(j\omega_2)$	$-0.67736 - 0.14155j$	$-0.63720 - 0.09432j$

Result differences are caused by the nonconstant length of PLC cycle. The found points of the frequency characteristics were used to calculate model parameters, Tab. 4.

Table 4. *Model parameters of the second order plus time delay simulated system.*

	1st measurement	2nd measurement
K	2.00001	2.00001
a_1	2.28445	2.41858
a_2	1.27178	1.40337
T_d	0.88438	0.90791

The Nyquist frequency characteristics depicts the points of frequency characteristics, which were found during identification, and points of models, which have the same frequency as points from identification (Fig. 8). The Nyquist frequency characteristics of models and controlled plant look almost the same, but it is evident that the points do not have absolutely the same phase.

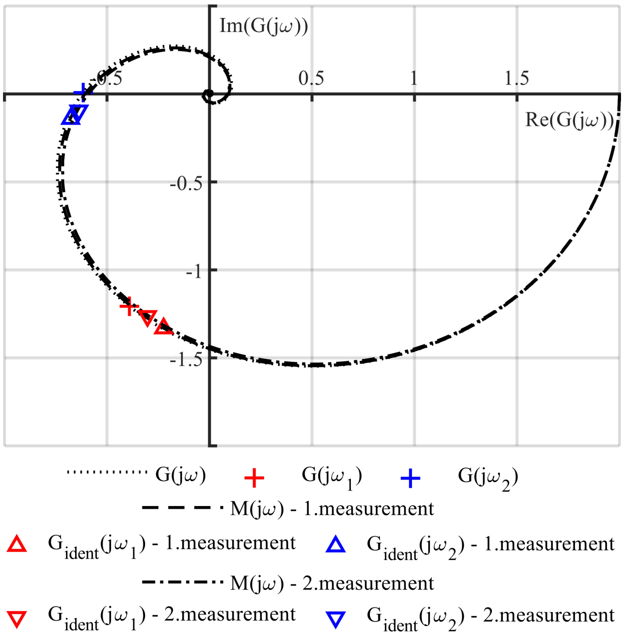

Fig. 8. *Example 2 - Nyquist frequency characteristics of simulated system and its models.*

The differences in phases are not too significant, therefore Shifting method relay feedback identification found good models. We should use greater value of added transport delay for better results of identification.

The following results are from identification of the same simulated plant 14 with use of a greater transport delay. When we used a third of period T_0 as added time delay, the identification program estimated better points of frequency characteristics (Tab. 5). Mostly found points frequencies are $\omega_1 = 0.54259\ rad \cdot s^{-1}$ and $\omega_2 = 1.08518\ rad \cdot s^{-1}$. Parameters of second order plus time delay model are in Tab. 6.

Table 5. *Most diferent results of the second order plus time delay simulated system with use of a greater value of added delay.*

	1st measurement	2nd measurement
$G_{ident}(j\omega_0)$	2.00001	2.00001
$G_{ident}(j\omega_1)$	$-0.20361 - 1.34668j$	$-0.20245 - 1.34738j$
$G_{ident}(j\omega_2)$	$-0.67808 - 0.15965j$	$-0.67145 - 0.16035j$

Table 6. *Model parameters of the second order plus time delay simulated system with use of a greater value of added delay.*

	1st measurement	2nd measurement
K	2.00001	2.00001
a_1	2.52182	2.53051
a_2	1.58631	1.63339
T_d	0.95625	0.93680

Nyquist frequency characteristics of estimated models are almost the same as controlled plant frequency characteristics (Fig. 9). Bode frequency characteristics of models are also the same as frequency characteristics of controlled plant, see Fig. 10 and Fig. 11.

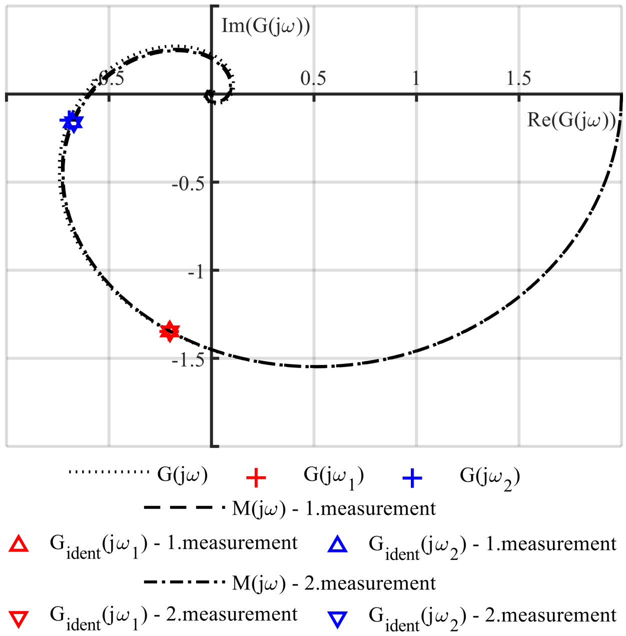

Fig. 9. *Example 2 - Nyquist frequency characteristics of simulated system and its models with use of a greater value of added delay.*

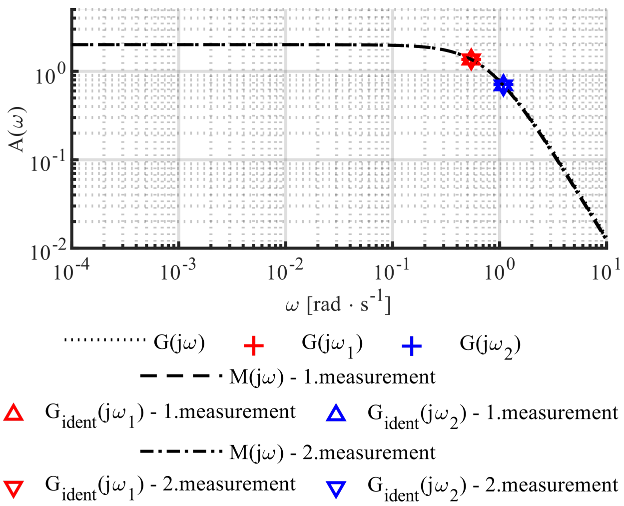

Fig. 10. *Example 2 - Bode amplitude plot of simulated system and its models with use of a greater value of added delay.*

4.2. Real laboratory controlled plant

The real identified controlled plant is a laboratory system called "Air Aggregate". The system is sheet-metal tunnel, which consists of a fan, a flow rate meter, a bulb and a thermometer, (Fig. 12). We can control the air flow with use of a fan and flow rate meter or bulb temperature with use of a bulb and thermomether.

4.2.1. Air Aggregate - fan control

The first version of the laboratory system "Air Aggregate" use is based on the fan and the flow rate meter. In this case, the controlled plant input u_{Ex3} is the voltage on the fan. The controlled variable y_{Ex3} is the voltage on the flow rate meter. Both are unified signals in a range of $0 - 10\ V$.

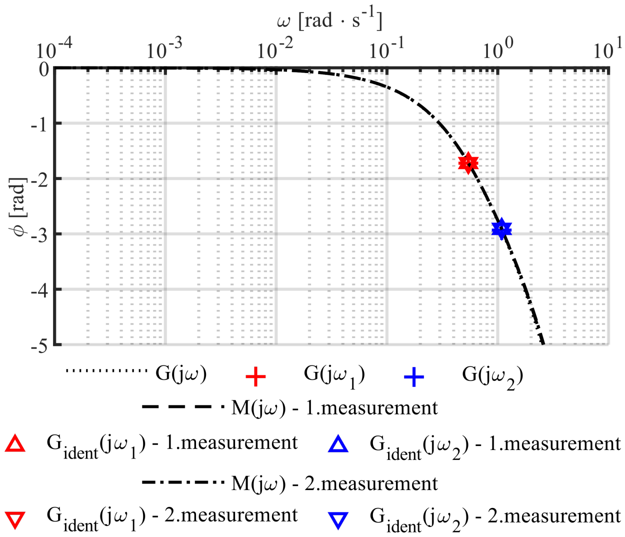

Fig. 11. *Example 2 - Bode phase plot of simulated system and its models with use of a greater value of added delay.*

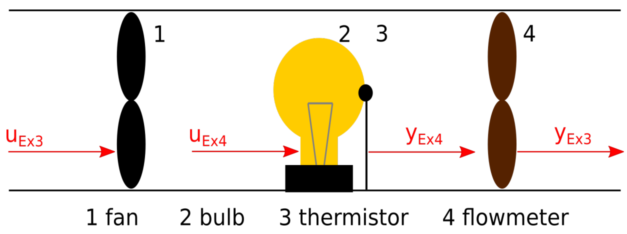

Fig. 12. *The laboratory model "Air Agreggate".*

The system was identified at the setpoint given by the value of the action variable $u_0 = 2.3\ V$. The added transport delay was chosen to be half of the period T_0. This value of transport delay shifts points of "Air Aggregate - fan control" frequency characteristics out of the 2nd quadrant. The estimated points of frequency characteristics are in Tab.7.

Table 7. *Results of the "Air Agreggate - fan control" identification.*

ω_1	0.2057
ω_2	0.4114
$G_{ident}(j\omega_0)$	1.64209
$G_{ident}(j\omega_1)$	$-0.15101 - 0.59708j$
$G_{ident}(j\omega_2)$	$-0.17173 - 0.26230j$

The found points of the frequency characteristics were used to calculate model parameters, which are in Tab. 8.

Table 8. *Model parameters of the "Air Agreggate - fan control".*

K	1.64209
a_1	12.6377
a_2	9.64180
T_d	1.70382

The controlled plant is stable as well as the identified model. Points of Nyquist frequency characteristics of the controlled plant and identified model Nyquist frequency characteristics were compared to verify the accuracy of the estimated model (Fig. 13).

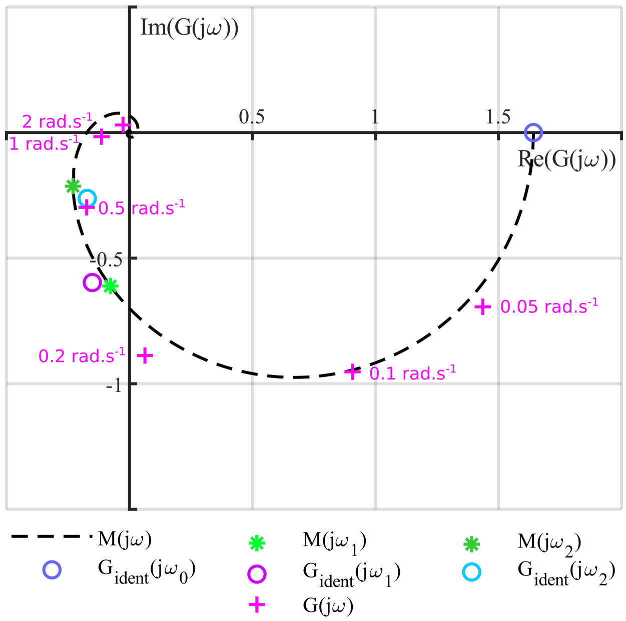

Fig. 13. *Example 3 - Nyquist frequency characteristics of the "Air Agreggate - fan control" system and its model.*

Bode frequency characteristics were also compared, see Fig. 14 and Fig. 15.

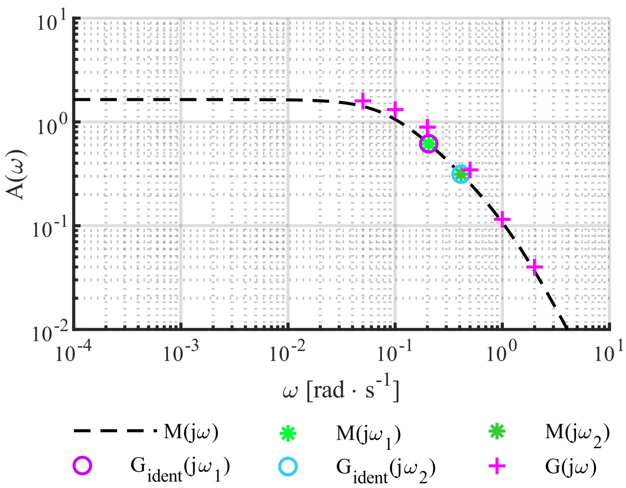

Fig. 14. *Example 3 - Bode amplitude plot of the "Air Agreggate - fan control" system and its model.*

In the amplitude characteristic are shown corresponding points from identification and the points from the controlled plant mathematical model of the same frequency. The phase characteristics are slightly different. The controlled plant is a real system, which can be affected by many different influences. That is why we consider the model as being sufficiently accurate.

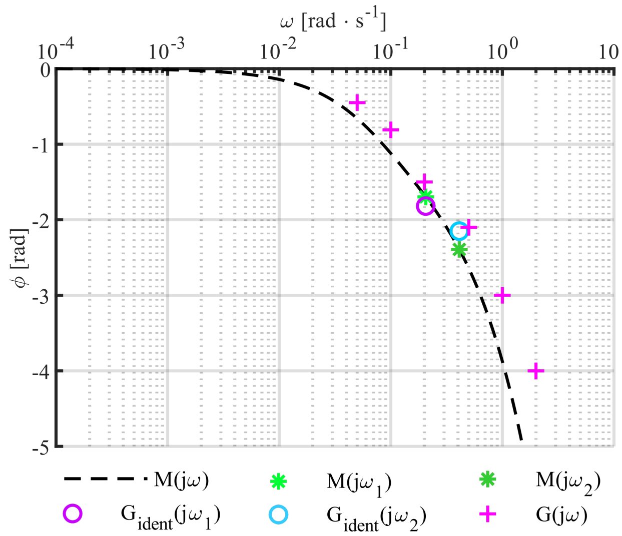

Fig. 15. *Example 3 - Bode phase plot of the "Air Agreggate - fan control" system and its model.*

4.2.2. Air Aggregate - bulb control

The same model as in the previous example was used for tests on the second real system. In the second model configuration, the manipulated variable u_{Ex4} is the voltage on the bulb and the controlled variable y_{Ex4} is the thermistor voltage. Both are unified signals of the range of $0 - 10 \quad V$.

The system was identified at the setpoint given by the value of the manipulated variable $u_0 = 3\ V$. The added transport delay was chosen to be half of the period T_0. This value of transport delay shifts points of "Air Aggregate - bulb control" frequency characteristics out of 2nd quadrant. Estimated points of frequency characteristics are in Tab 9.

Table 9. *Results of the "Air Agreggate - bulb control" identification.*

ω_1	0.1277
ω_2	0.2554
$G_{ident}(j\omega_0)$	2.58692
$G_{ident}(j\omega_1)$	$-0.09644 - 0.27691j$
$G_{ident}(j\omega_2)$	$-0.10321 - 0.06673j$

The found points of the frequency characteristics were used to calculate model parameters, Tab.10.

Table 10. *Model parameters of the "Air Agreggate - bulb control".*

K	2.58692
a_1	66.59007
a_2	205.46207
T_d	1.18257

The determined model of the system is stable, that corresponds to the observed behavior of the system. We plotted the frequency characteristics of the estimated model and points of "Air Agreggate - bulb control" frequency characteristics. The Nyquist frequency characteristics of the estimated model is close to the points of Nyquist frequency characteristics of controlled plant (Fig. 16). The difference in the position of the points on the Nyquist frequency characteristics is due to a small difference in the points in the phase plot, see Fig. 17. The points of controlled plant Bode amplitude plot are almost on the model amplitude characteristics. Therefore, it is possible to say that the Bode amplitude characteristics of the model and the controlled plant are identical. Therefore we can consider the model to be sufficiently accurate.

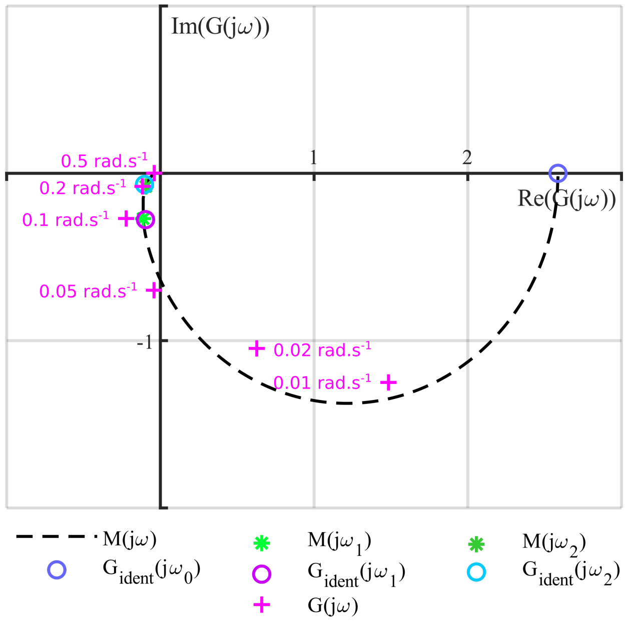

Fig. 16. *Example 4 - Nyquist frequency characteristics of the "Air Agreggate - bulb control" system and its model.*

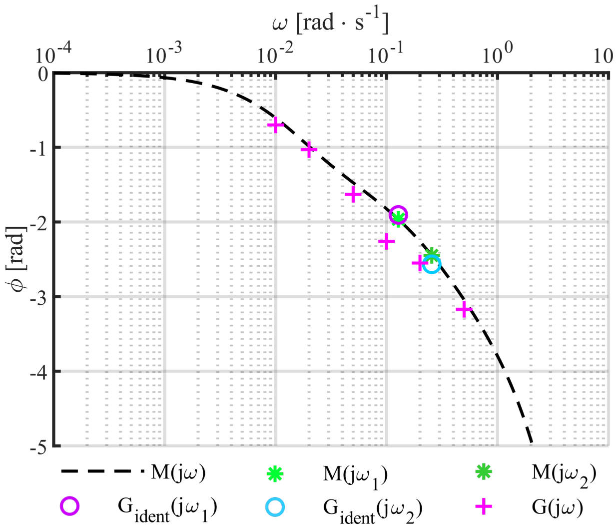

Fig. 17. *Example 4 - Bode phase characteristics of the "Air Agreggate - bulb control" system and its model.*

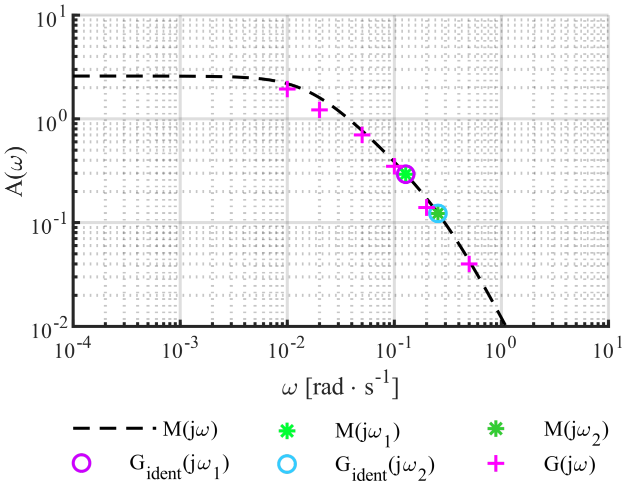

Fig. 18. *Example 4 - Bode amplitude characteristics of the "Air Agreggate - bulb control" system and its model.*

5. Conclusions

All used controlled plants were succesfully identified. The accuracy of estimated models was shown on comparison of frequency characteristics of estimated models and controlled plants. Repeated identification of simulated plants gives almost the same results. The identification program found in one case for the nonoscilatory system oscilatory model. Nevertheless step characteristics of model and controlled plant are almost the same. Therefore the controlled plant was identified sucessfully. The Shifting method is a well functioning relay feedback identification method. Written identification program works correctly and could be used for repeated identification of time invariant systems.

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Nomenclature

a_1	time constant of second order plus time delay model (s)
a_2	time constant of second order plus time delay model (s)
$G_{ident}(j\omega_0)$	point of frequency characteristics estimated by Shifting identification method (−)
$G_{ident}(j\omega_1)$	point of frequency characteristics estimated by Shifting identification method (−)
$G_{ident}(j\omega_2)$	point of frequency characteristics estimated by Shifting identification method (−)
$G_{delayed}(j\omega_1)$	point of frequency characteristics estimated by Shifting identification method with added delay (−)
$G_{delayed}(j\omega_2)$	point of frequency characteristics estimated by Shifting identification method with added delay (−)
$G(j\omega_1)$	point of controlled plant frequency characteristics (−)

$G(j\omega_2)$	point of controlled plant frequency characteristics $(-)$
$G(s)$	controlled plant transfer function $(-)$
j	imaginary unit $(-)$
k	non-zero integer $(-)$
K	static gain $(-)$
$M(s)$	identified model transfer function $(-)$
s	Laplace operator $(-)$
t	time of reach of stable oscillation (s)
T_0	period of system stable oscillation under relay feedback test without added transport delay (s)
T_d	transport delay of second order plus time delay model (s)
T_p	period of system stable oscillation under relay feedback test with added transport delay (s)
u	system input (V)
$\overline{u}$	system input when system oscilating with period $T_p/2$ (V)
u_{Ex3}	system input of Air Aggregate - fan control (V)
u_{Ex4}	system input of Air Aggregate - bulb control (V)
y	system output (V)
$\overline{y}$	system output when system oscilating with period $T_p/2$ (V)
y_{Ex3}	system output of Air Aggregate - fan control (V)
y_{Ex4}	system output of Air Aggregate - bulb control (V)
ω	frequency $(\mathrm{rad \cdot s^{-1}})$
ω_0	frequency of frequency characteristics point $(\mathrm{rad \cdot s^{-1}})$
ω_1	frequency of frequency characteristics point $(\mathrm{rad \cdot s^{-1}})$
ω_2	frequency of frequency characteristics point $(\mathrm{rad \cdot s^{-1}})$

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