The investigation of acoustical signal induced by vibrations of human vocal folds

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Abstract

This paper is focused on the numerical solution of coupled mechanical-acoustical problem. The goal is to investigate the influence of sound generated by vocal fold vibration and compared this contribution with acoustic results generated by the aerodynamic mechanism. Here, the results of earlier simulation of fluid-structure interaction (FSI) are used to obtain vocal fold vibration and the backward coupling from acoustic field to mechanical is neglected. Then the coupled problem reduces purely to solution of wave equation with prescribed interface acceleration as boundary conditions. The FSI problem as well as acoustic problem is solved by the finite element method based solver, which is developed in-house. For simulation of open-boundary problem the so called perfectly matched layer technique is used.

Keywords: acoustics, vibro-acoustics, human phonation, perfectly matched layer, finite element method

1. Introduction

The human voice production is very interesting phenomenon, which is not fully understand yet. General theory assumes, see [1], that the basic acoustical signal is produced by vocal folds excited by the fluid flow. This signal propagates then through vocal tract and it is finally articulated in mouth into form of human speech. Here, we will considered only the mechanism of the basic acoustical signal formation. There are described three basic physical principles of sound production.

The first principle represents the pulsating jet through glottis, the narrowest part of channel. The modulation of fluid flow is caused by opening and closing gap between both vocal folds. This phenomenon plays main role in acoustic signal production, see [2] or [3]. The second principle is connected with the turbulence downstream of glottis. The separation of flow appears periodically near the orifice formed by vocal folds and it creates quite complex vortex structures in flow. These structures can be associated to the broadband acoustic spectrum. The third principle is the vibro-acoustic sound generation. The vibration of vocal fold interface represents additional acoustic source proportional to interface acceleration in normal direction, i.e. it has different origin than first two aerodynamic sound sources. This sound source should have lower magnitude nevertheless it will be studied in detail in this work.

2. Mathematical model

Let restrict us for the purpose of simplicity to two dimensional problem. In following paragraphs are described governing equations of acoustics, coupled vibro-acoustic problem and finally our modelling approach.

2.1. Acoustics

The acoustic wave propagation in homogeneous media represented by domain Ω^a is described by the partial differential equation in the form

$$\left(\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x_i^2}\right) p^a = 0, \quad x \in \Omega^a,\ t \in (0, \mathrm{T}) \quad (1)$$

where $p^a(x,t)$ is the sought acoustic pressure and c_0 is speed of sound. Problem (1) is closed with zero initial conditions and the following combinations of boundary conditions are considered:

$$\begin{aligned}
&\text{a)} \quad p^a(\mathbf{x}, t) = 0, && \mathbf{x} \in \Gamma_{\mathrm{Dir}},\ t \in (0, \mathrm{T}) \\
&\text{b)} \quad \frac{\partial p^a(\mathbf{x}, t)}{\partial \mathbf{n}} = 0, && \mathbf{x} \in \Gamma_{\mathrm{Wall}}, t \in (0, \mathrm{T}) \qquad (2)\\
&\text{c)} \quad \frac{\partial p^a(\mathbf{x}, t)}{\partial \mathbf{n}} = -\rho_0 \frac{\partial v_n}{\partial t}, && \mathbf{x} \in \Gamma_{\mathrm{W_0}},\ t \in (0, \mathrm{T}) \\
&\text{d)} \quad \text{PML approach.}
\end{aligned}$$

The boundary condition (2 a)) is called sound soft because it stands for farfield boundary with no reflection. The boundary condition (2 b)) is called sound hard because it expresses presence of walls with perfect (full) reflection of acoustic waves. The wall has outer unit normal $\mathbf{n}$. This boundary condition can be further modified to (2 c)) for the case, when the reflecting boundary is moving, vibrating. Then the right hand term represents acoustic emission from surface in the form of time derivative of wall normal velocity, see e.g. [4]. The Perfectly matched layer (PML) approach is described below.

2.1.1. PML

In order to simulate the open-boundary problem in bounded domains the PML technique or Absorbing boundary condition approach are usually used, see [4]. The PML approach consists of a few additional layers of elements along the normal direction of boundaries, which represents interface with open space. Inside these layers the sound waves are effectively damped to zero. The most important property is that there is no reflection at the interface between propagation region and PML, therefore the name perfectly matched.

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For the formulation and numerical implementation we refer to [5].

2.2. Vibro-acoustics

The vibro-acoustic problem is coupled problem, where the vibrating body emits acoustic waves into surrounding acoustic medium and in reverse its deformation is influenced by the impinging acoustic waves.

2.2.1. Elastic body

The deformation $\mathbf{u}(\mathbf{x},t) = (u_1, u_2)$ of the elastic body Ω^s is governed by equations

$$\rho^s \frac{\partial^2 u_i}{\partial t^2} - \frac{\partial \tau^s_{ij}(\mathbf{u})}{\partial x_j} = f_i{}^s \quad \text{in } \Omega^s \times (0, \mathrm{T}), \tag{3}$$

where ρ^s denotes the structure density, the vector $\mathbf{f}^s = (f_1^s, f_2^s)$ is the volume density of an acting force and the tensor τ^s_{ij} is the Cauchy stress tensor. The Cauchy stress tensor can be expressed by the generalized Hook law in the form, see e.g. [6]

$$\tau^s_{ij} = \lambda^s(\operatorname{div}\mathbf{u})\,\delta_{ij} + 2\mu^s e^s_{ij}, \tag{4}$$

where λ^s, μ^s are Lamé coefficients, the tensor $\mathbb{I} = (\delta_{ij})$ is Kronecker's delta and tensor $\mathbf{e}^s = (e^s_{jk})$ is the strain tensor of small displacements.

The formulation of elastic problem (3) is completed by the zero initial conditions and following boundary conditions

$$\begin{aligned} &\text{a)} \quad \mathbf{u}(x,t) = \mathbf{0}, && x \in \Gamma^s_{\mathrm{Dir}},\ t \in (0,\mathrm{T}), \quad (5)\\ &\text{b)} \quad \tau^s_{ij}(x,t)\, n^s_j(x) = q^s_i(x,t), && x \in \Gamma_{\mathrm{W}_0},\ t \in (0,\mathrm{T}), \end{aligned}$$

where the $\Gamma_{\mathrm{W}_0}, \Gamma^s_{\mathrm{Dir}}$ are mutually disjoint parts of the boundary $\partial\Omega^s = \Gamma_{\mathrm{W}_0} \cup \Gamma^s_{\mathrm{Dir}}$ and $n^s_j(x)$ are components of the unit outer normal to Γ_{W_0}. The condition (5 b)) is explained below.

2.2.2. Vibro-acoustic coupling

The both subproblems are coupled via common interface. From the nature of problem the continuity of velocities and stresses should be preserved across interface in the normal direction. From the first condition follows

$$\left(\frac{\partial \mathbf{u}}{\partial t} - \mathbf{v}^a\right) \cdot \mathbf{n} = 0 \tag{6}$$

and together with linearized momentum conservation of acoustics in the form $\frac{\partial \mathbf{v^a}}{\partial t} = -\frac{1}{\rho_0}\nabla p^a$ and notation $v_n = \frac{\partial u}{\partial t} \cdot \mathbf{n}$ it gets exactly the condition (2 c), i.e. the boundary condition prescribed to acoustic problem.

The second condition reads

$$\tau^s_{ij}\, n^s_j = p^a n^s_j =: q^s_i(x,t), \tag{7}$$

so it determines term q^s_i from condition (2 c), i.e. the boundary condition for elastic problem.

2.3. Simplification of coupled problem

The vibro-acoustic problem have to be solved as coupled problem, if the intensity of acoustic field is high and it changes the vibration of elastic body. It is usually satisfied in a simulation of flapping flags or loudspeakers. But focus of this paper is paid to the sound produced by vocal folds vibration, where the low intensity of acoustic field compared to the energy of elastic vibration can be assumed. Therefore the mutual coupling becomes just the forward coupling from known deformation of interface to acoustic part of solution, i.e. the problem simplifies to Eq. (1) together with conditions (2). The results of previously performed FSI simulation, see [7], are used to get normal component of velocity at interface Γ_{W_0}.

2.4. Acoustic domain

The acoustic domain is chosen according to the domain studied in [8]. Shortly, it is composed of domain with vocal folds adjusted for CFD computation, vocal tract model of vowel [u:] based on article [9], farfield domain and finally enclosed by PML domain. The scheme of acoustic domain is depicted in Figure 1.

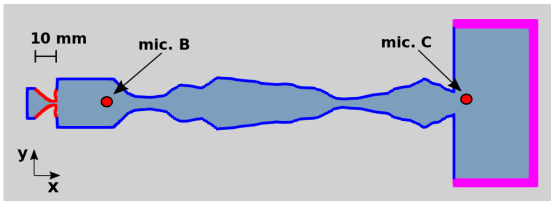

Figure 1. *Scheme of acoustic domain with boundaries. The blue boundary is Γ_{Wall}, the red boundary represents the interface of vocal folds Γ_{W_0} and the pink part is five layers of PML elements. For exact dimensions see [8]. Further the position of two microphones ($B = [x = 0.05\,\mathrm{m}, y = 0\,\mathrm{m}]$ and $C = [x = 0.25\,\mathrm{m}, y = 0\,\mathrm{m}]$) are illustrated.*

3. Numerical model

Firstly, the derivation of weak formulation is provided. Equation (1) is multiplied by test function η and integrated over the whole acoustic domain Ω^a, which leads to

$$\left(\frac{1}{c_0^2}\frac{\partial^2 p^a}{\partial t^2}, \eta\right)_{\Omega^a} - (\Delta p^a, \eta)_{\Omega^a} = 0, \tag{8}$$

where $(\cdot,\cdot)_\Omega$ denotes scalar product in Lebesque spaces $L^2(\Omega)$. The application of the Green theorem together with boundary condition (2) gives us

$$\left(\frac{1}{c_0^2}\frac{\partial^2 p^a}{\partial t^2}, \eta\right)_{\Omega^a} + (\nabla p^a, \nabla\eta)_{\Omega^a} = \left(\rho_0 \frac{\partial v_n}{\partial t}, \eta\right)_{\Gamma_{\mathrm{W}_0}}. \tag{9}$$

Finally, the restriction of test functions to finite element space and seeking approximative solution in form $p^a_h(t,x) = \sum_{j=1}^{N_h}\gamma_j(t)\eta_j(x) \in V_h \subset W^{1,2}(\Omega^a)$, where $W^{1,2}(\Omega)$ is the Sobolev space and N_h is the dimension of V_h, yields

$$\mathbb{M}^a\ddot{\gamma} + \mathbb{K}^a\gamma = \mathbf{b}^a(t), \tag{10}$$

where elements of matrices $\mathbb{M}^a = (m^a_{ij}), \mathbb{K}^a = (k^a_{ij})$ are given as

$$m^a_{ij} = \left(\frac{1}{c_0^2} \eta_i, \eta_j \right)_{\Omega^a}, \quad k^a_{ij} = \left(\frac{\partial \eta_i}{\partial x_j}, \frac{\partial \eta_j}{\partial x_j} \right)_{\Omega^a}, \quad (11)$$

and the components of vector $\mathbf{b}^a(t) = (b^a_i)$ are

$$b^a_i = \left(\rho_0 \frac{\partial v_n}{\partial t}, \eta_i \right)_{\Gamma_{W_0}}. \quad (12)$$

System (10) is numerically discretized in time by the Newmark method with uniform time step $\Delta t = \frac{T}{N}, N >> 1$.

4. Numerical results

First paragraph of this section describes the results of FSI simulation followed by the second paragraph dedicated to sound propagation induced by vibrating vocal folds. The obtained results are analyzed and compared with results obtained by aeroacoustic analogies, i.e. with the sound induced by fluid flow around vocal folds.

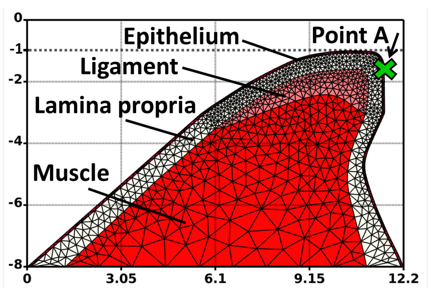

Figure 2. *The computational mesh of vocal tract model with marked different layers of materials and with dimensions shown in mm. The point A with coordinates* [11.57mm, −1.50mm] *is shown.*

4.1. FSI problem

Figure 2 shows the vocal fold (VF) model shape based on the article [10] composed of four parts with different material parameters, see [7]. The fluid parameters were chosen as for air with 293K. The FSI simulation was driven by pressure difference between inlet and outlet $\Delta p = 1600$ Pa. The time step in simulation was selected as $2.5 \cdot 10^{-5}$ s.

After short transition time the stable vibration of VF have developed. Figure 3 illustrates typical behaviour of the flow induced vibration of VF. The Fourier transform shows the excitation of first two eigenmodes of VF, see 4. This is in good correspondence with results of article [10].

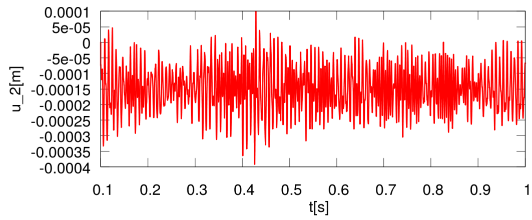

Figure 3. *The time evolution of displacement of point A in y-direction.*

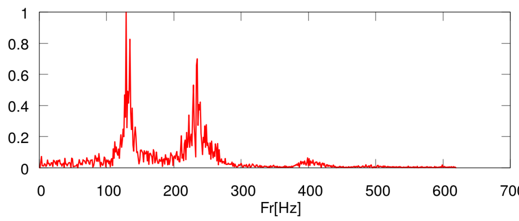

Figure 4. *The (normalized) Fourier transform of y-component of point A displacement.*

4.2. Vibro-acoustics

From computed VF deformation the interface acceleration was computed and then used for the evaluation of boundary condition (2 c). Then the wave equation (1) was solved and the acoustic pressure signal was stored at two microphone positions, see Fig. 5. First microphone was located in the end of CFD domain, the second one in the farfield region, i.e. in front of the model mouth.

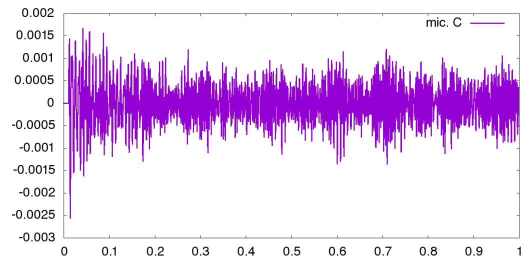

Figure 5. *Acoustic pressure monitored in point C in the farfield region. Dimension of x axis is* [*s*]*, y axis is* [*Pa*]*.*

The Fourier transform of signal from both microphones are shown in Figures 6 and 7. The frequency spectrum of acoustic pressure at point B shows that the most dominant frequencies answer the dominant frequencies of VF motion, where additionally the frequency of 930 Hz is present. The Fourier transform in point C, i.e. point in the end of vocal tract model, exhibits the relative strengthening of frequencies 282, 930 and 2550 Hz. These frequencies approximately correspond with the peak of transfer function of this vocal tract model, see [8], i.e. this behaviour is expected. Nevertheless it can be stated that the sound pressure level of sound induced by VF vibration is a few dB more silent than in the case of flow induced sound, see Fig. 8, so the investigated frequencies do not play too significant role in comparison with airborne sound. The vibrationborne sound rather helps to colour voice timbre than to form formants, i.e. its characteristic frequencies.

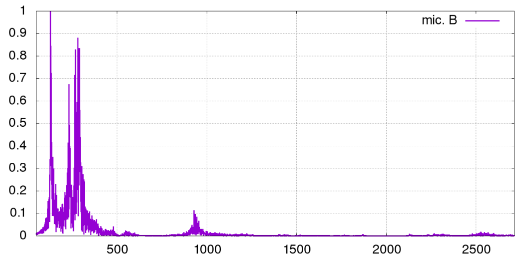

Figure 6. *The normalised Fourier transform of the acoustic pressure from microphone B.*

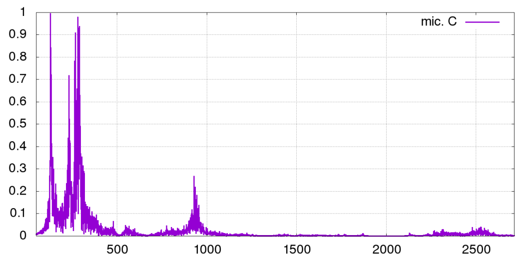

Figure 7. *The normalised Fourier transform of the acoustic pressure from microphone C.*

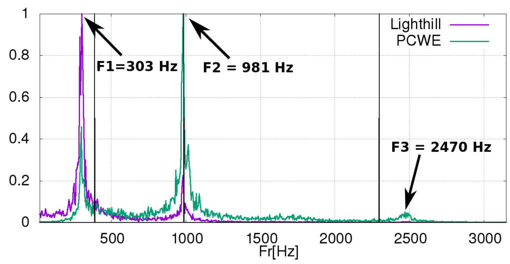

Figure 8. *The normalised frequency spectra of the acoustic pressure obtained by aeroacoustic analogies – Lighthill and PCWE analogy. There are highlighted dominant frequencies of simulation with arrows and values. The black vertical lines mark the frequencies* 389 Hz, 987 Hz *and* 2299 Hz *taken from article [9]. Results preprinted from article [8].*

5. Conclusion

The coupled problem of sound emission produced by vibrating elastic body was mathematically described. For the purpose of investigation of sound produced by vocal folds vibration the problem was simplified to solution of wave equation, where the sound excitation is given by the known normal acceleration of interface coming from FSI simulation. This approach can be easily used also for many other technical applications.

The obtained acoustic pressure and its frequency spectra mainly copy the dominant frequencies of vocal fold motion. For the microphone in front of mouth the amplification of frequencies corresponding to the peak frequencies of vocal tract model can be observed as it was expected. The resulting sound pressure level is few dB lower than in the case of flow induced sound, so the studied frequencies rather creates the voice timbre than the voice formants.

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