# Comparison of triangular meshes by using shape functions

Nikola Pajerová*

[1] ČVUT v Praze, Fakulta strojní, Ústav technické matematiky, Karlovo nám. 13, 121 35 Praha 2, Česká republika

**Abstract**

The problem of scanned surfaces reconstruction is often being solved in computer graphic. This article deals with similarity measure of given triangular meshes in STL format are obtained from several scanning of one calibration artefact. While scanning the object, the noise or another inaccuracy can appear. To compare these meshes several types of shape functions are used, that are applied onto individual meshes, and then the results are compared in graphs.

*Keywords:* surface reconstruction; shape function; meshes comparison

## 1. Introduction

In this paper, the shape functions and shape distributions [1-5] are used to determine similarity between five meshes. These meshes are in STL format, it means they are given as triangular meshes with face normals, in text form they are given as lists of vertex coordinates (in special order to save the sequence of vertices in each face) with normal vectors for each face, which are oriented outwards of the object. Meshes were obtained by optical scanning of ball-bar standard (formed by two spheres and cylinder), Fig. 1, which is used for calibration of optical scanners.

The aim of this paper is to find the similarity between meshes by using different shape functions and shape distributions.

***Fig. 1**. Ball-bar standard*

### 1.1. Triangular meshes processing

#### 1.1.1. Removal of inappropriate points

Since the five meshes obtained by optical scanning of the same ball-bar were in different position and had inappropriate points, the pre-processing had to be done. Inappropriate points had to be removed, because they could distort the comparison, or result. So that the meshes were moved into the same position, where the centre of line segment with endpoints in sphere centres (i.e. $S_1$ and $S_2$ in Fig. 2) was moved to origin and this line segment was situated on $x$-axis. Then the inappropriate points were removed by using parallel planes (see Fig. 2). The least square method was used to create a mesh of the nominal CAD model of the object in the suitable position (depicted in lower part of Fig. 2).

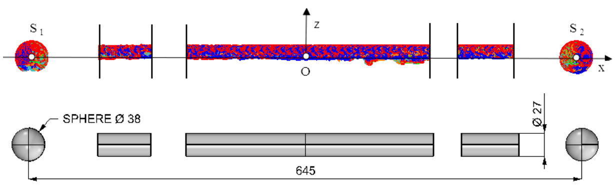


***Fig. 2**. Modified ball-bar meshes and nominal model*

#### 1.1.2. Application of shape functions

Various types of shape functions were used to compare these meshes. Shape functions described in [1] were used, where the D1 and D4 functions were modified. Function D2 was used in the same form. Modification of D1 function measures the distance of each point of the mesh from the origin (details in [2]). The D4 function modification measures the cube root of tetrahedron volume, where one vertex is in the centroid of the ball-bar mesh and one tetrahedron face is the mesh triangle.

The random sampling, described in [1], is in this paper represented by mesh vertices. The oriented distances are also taken into consideration – for this position of meshes the sign of $x$-coordinates of special points creates the orientations of each distance or volume (special points are for D1 the $x$-coordinates of the vertices, for D2 they are $x$-coordinates of centres of line segments, for D3 the $x$-coordinates of triangles centroids and for D4 the $x$-coordinates of tetrahedron centroids). So that the formula for oriented shape function D1 is:

$$f_i = \operatorname{sign}(x_i)\sqrt{{x_i}^2 + {y_i}^2 + {z_i}^2}, i = 0,1,\dots,n \quad (1)$$

where $V_i = (x_i, y_i, z_i)$ are vertices of the mesh. Then, formula for oriented D2 function is:

$$\bar{f_i} = \operatorname{sign}(x_i)|\boldsymbol{a}_i|, i = 0,1,\dots,k \quad (2)$$

where $S_i = (x_i, y_i, z_i)$ are centres of line segments and $\boldsymbol{a}_i$ are vectors between two mesh vertices. At last, the formula for oriented D4 function is:

$$\tilde{f_i} = \operatorname{sign}(x_i)(\boldsymbol{u} \times \boldsymbol{v}) \cdot \boldsymbol{w}, i = 0,1,\dots,l \quad (3)$$

* Kontakt na autora: Nikola.Pajerova@fs.cvut.cz

where $C_i = (x_i, y_i, z_i)$ are centroids of tetrahedrons and vectors $\boldsymbol{u}$, $\boldsymbol{v}$ and $\boldsymbol{w}$ are given by vertices of mesh triangle and centroid of the mesh.

For comparison, the shape functions without the orientation (it means formulas without $\text{sign}(x_i)$) are also included. But the oriented functions are more important, because they show rough shape of each mesh in the final histogram (it is clearly visible from histogram for D1 function – see Fig. 3 and compare it with Fig. 2).

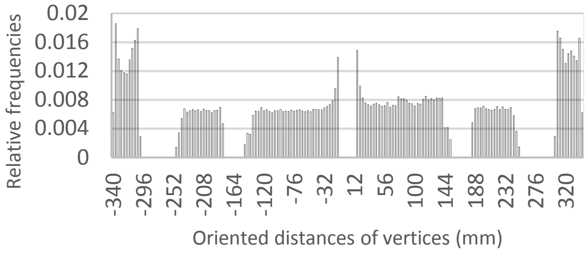


***Fig. 3**. Histogram for D1 function*

### 1.2. Shape distributions

After applying the shape functions, frequency histograms were constructed. Frequency histogram shows how many values (for example distances or volumes) fall into each of the fixed sized bins.

But since each mesh has different number of points, the frequencies had to be divided by the number of mesh points (i. e. must be normalized). This was done for each mesh and for each shape function.

To compare histograms, the frequencies were represented by piecewise linear functions with equally spaced vertices. This is so called shape distribution [1, 3].

The similarity measure of each two histograms (or equally two meshes) is represented by the Minkowski $L_1$ norm (presented in [1]):

$$D(f, g) = \sum_{i=1}^{k} |f_i - g_i|, \tag{4}$$

where $f_i$ and $g_i$ are the function values from formula (1), (2) or (3). This function measures "area" between two functions, or more precisely it sums the absolute value of differences between two frequencies in same bin over all bins. It is better than using integral for this case, because firstly graphs are only connections of discrete points from histograms, secondly this function shows the differences of two meshes more accurately for each bin. So that, the lower the number is, more similar the corresponding meshes are.

## 2. Comparison of results

### 2.1. Results for D1 function

Shape distributions for D1 function are depicted in Fig. 4 and Tab. 1. It is visible from Fig. 4, that the meshes 1 and 5 have the worst similarity, because its course is different from the course of nominal mesh. These similarities are preciously obvious from table of Minkowski norms for two meshes, Tab. 1. There are compared only meshes 1 to 5 to each other. This table shows the best similarity of meshes 3 and 4 (because the number of these two is the lowest) and the worst similarity between meshes 1 and 5 (it is the highest number from the table).

***Table 1.** Minkowski $L_1$ norm for shape distribution D1.*

| $L_1$ | M1 | M2 | M3 | M4 | M5 |
|---|---|---|---|---|---|
| M1 | 0.0000 | 0.0930 | 0.1462 | 0.1468 | 0.1525 |
| M2 | 0.0930 | 0.0000 | 0.0870 | 0.0832 | 0.0928 |
| M3 | 0.1462 | 0.0870 | 0.0000 | 0.0497 | 0.0593 |
| M4 | 0.1468 | 0.0832 | 0.0497 | 0.0000 | 0.0689 |
| M5 | 0.1525 | 0.0928 | 0.0593 | 0.0689 | 0.0000 |

### 2.2. Results for D2 function

It is obvious from Tab. 2 and Fig. 5 and 6, that the best similarity is (when using D2 function) between meshes 1 and 2. Vice versa, the worst similarity for this shape function is between meshes 2 and 4. You can compare graphs for directed (or oriented) and undirected measures (Fig. 5 and 6). Tab. 3 and Fig. 6 show the shape distributions for oriented distances of two random points of the mesh. Here the meshes 2 and 4 are similar and meshes 1 and 5 are least similar.

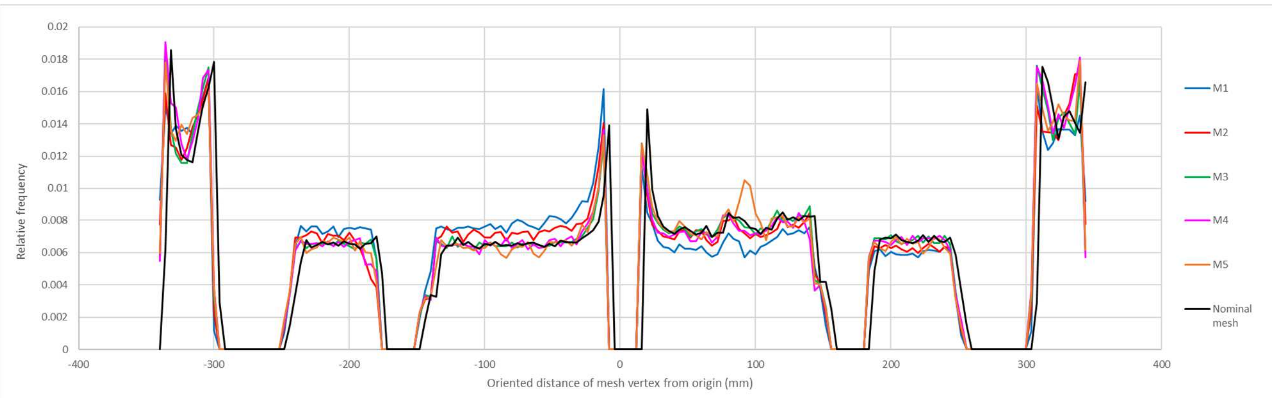


***Fig. 4**. Frequency histogram for D1 function (shape distributions)*

***Table 2.*** *Minkowski $L_1$ norm for shape distribution D2.*

| $L_1$ | M1 | M2 | M3 | M4 | M5 |
|---|---|---|---|---|---|
| M1 | 0.000 | 0.074 | 0.125 | 0.122 | 0.109 |
| M2 | 0.074 | 0.000 | 0.087 | 0.115 | 0.094 |
| M3 | 0.125 | 0.087 | 0.000 | 0.079 | 0.126 |
| M4 | 0.122 | 0.115 | 0.079 | 0.000 | 0.154 |
| M5 | 0.109 | 0.094 | 0.126 | 0.154 | 0.000 |

***Table 3.*** *Minkowski $L_1$ norm for oriented shape distribution D2.*

| $L_1$ | M1 | M2 | M3 | M4 | M5 |
|---|---|---|---|---|---|
| M1 | 0.0000 | 0.0127 | 0.0180 | 0.0292 | 0.0184 |
| M2 | 0.0127 | 0.0000 | 0.0159 | 0.0255 | 0.0190 |
| M3 | 0.0180 | 0.0159 | 0.0000 | 0.0165 | 0.0166 |
| M4 | 0.0292 | 0.0254 | 0.0165 | 0.0000 | 0.0219 |
| M5 | 0.0184 | 0.0190 | 0.0166 | 0.0219 | 0.0000 |

***Table 4.*** *Minkowski $L_1$ norm for shape distribution D4.*

| $L_1$ | M1 | M2 | M3 | M4 | M5 |
|---|---|---|---|---|---|
| M1 | 0.000 | 0.074 | 0.125 | 0.122 | 0.109 |
| M2 | 0.074 | 0.000 | 0.087 | 0.115 | 0.094 |
| M3 | 0.125 | 0.087 | 0.000 | 0.079 | 0.126 |
| M4 | 0.122 | 0.115 | 0.079 | 0.000 | 0.154 |
| M5 | 0.109 | 0.094 | 0.126 | 0.154 | 0.000 |

***Table 5.*** *Minkowski $L_1$ norm for oriented shape distribution D4.*

| $L_1$ | M1 | M2 | M3 | M4 | M5 |
|---|---|---|---|---|---|
| M1 | 0.000 | 0.088 | 0.111 | 0.122 | 0.115 |
| M2 | 0.088 | 0.000 | 0.083 | 0.138 | 0.107 |
| M3 | 0.111 | 0.083 | 0.000 | 0.101 | 0.125 |
| M4 | 0.122 | 0.138 | 0.101 | 0.000 | 0.157 |
| M5 | 0.115 | 0.107 | 0.125 | 0.157 | 0.000 |

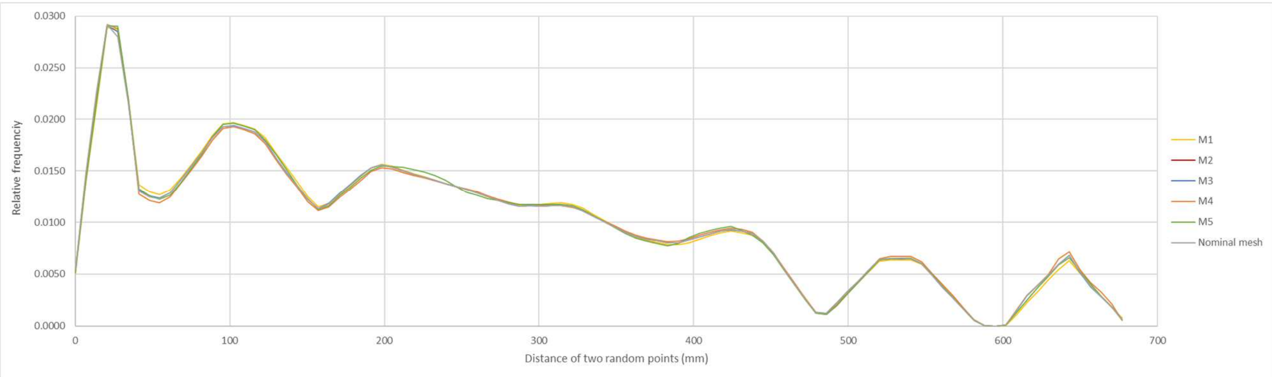


***Fig. 5.*** *Frequency histogram for D2 function*

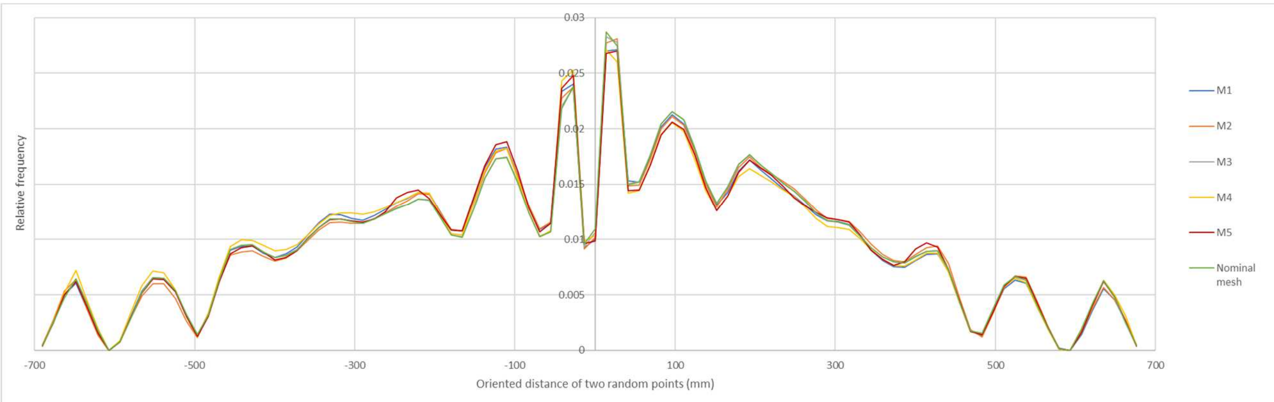


***Fig. 6.*** *Frequency histogram for oriented D2 function*

## 2.3. Results for D4 function

D4 function measuring the volumes of tetrahedrons gives these results for frequency histograms and Minkowski norms: for undirected measure the most similar meshes are 1 and 2, least similar are meshes 3 and 5 (see Tab. 4 and Fig. 7); for oriented measure are meshes 2 and 3 most similar and meshes 4 and 5 least similar (see Tab. 5 and Fig. 8).

## 2.4. Result from shape distributions

According to the results from Minkowski norms, from D1 function the best similarity is between meshes 3 and 4, from D2 function it is between meshes 1 and 2 (or 1 and 5, respectively) and from D4 function it is between meshes 1 and 2 (or 4 and 5, respectively). We can see that each function shows a bit distinct result, but the same similarity measure is for function D2 and D4 (for undirected measure) and they indicate, that the most similar meshes are meshes 1 and 2.

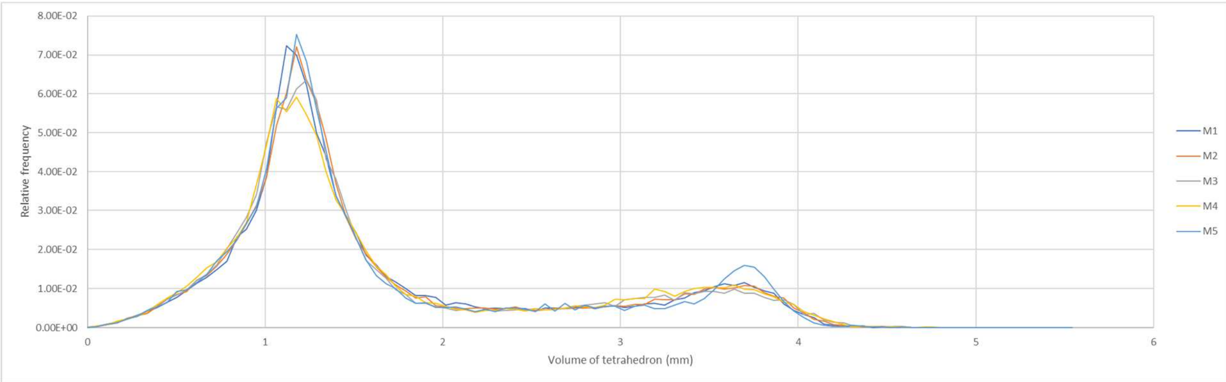


***Fig. 7***. *Frequency histogram for D4 function*

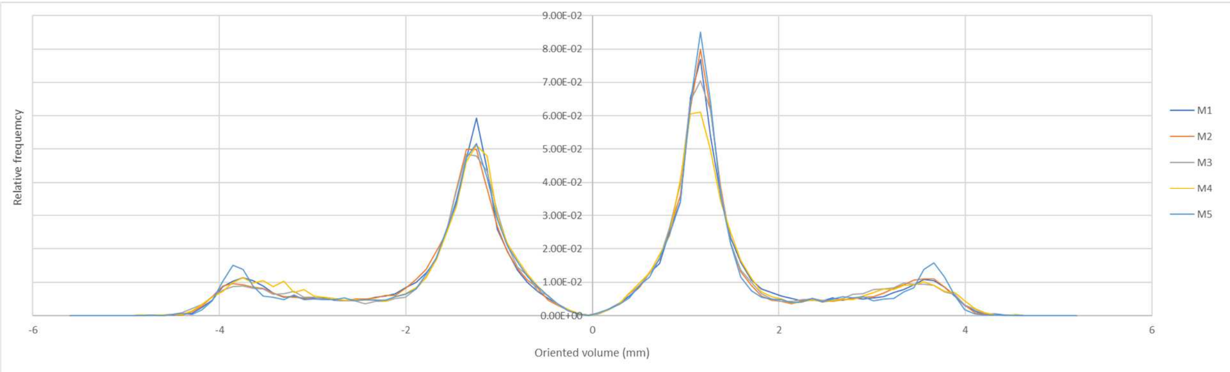


***Fig. 8***. *Frequency histogram for oriented D4 function*

We can also compare the arithmetic means of Minkowski norms for each mesh in each function. As depicted in Tab. 6, we can say that from measures for functions D2, D2-oriented and D4-oriented are the highest numbers in the column for mesh 4. When comparing functions D1 and D2 the lowest number is for mesh 3. The lowest number means the most suitable similarity (i.e. this mesh is the most similar to each other). Finally, we could say the most suitable mesh for processing can be the mesh 3, the worst can be the mesh 4.

***Table 5.*** *Minkowski $L_1$ norm for oriented shape distribution D4.*

| Arithm. means | Mesh 1 | Mesh 2 | Mesh 3 | Mesh 4 | Mesh 5 |
|---|---|---|---|---|---|
| D1 | 0.134 | 0.089 | 0.085 | 0.087 | 0.093 |
| D2 | 0.019602 | 0.019608 | 0.0168 | 0.0232 | 0.0190 |
| D2 oriented | 0.03118 | 0.03706 | 0.03709 | 0.0434 | 0.0333 |
| D4 | 0.108 | 0.093 | 0.104 | 0.117 | 0.121 |
| D4 oriented | 0.109 | 0.104 | 0.105 | 0.129 | 0.126 |

## 3. Conclusion

The shape functions can show differences between meshes. Shape distributions show how big these differences are and so we can compare them. The shape functions were used here as in [1], some of them with modifications. Because results of this study vary, the further processing have to be done (such as detailed comparison with another function or modification of these functions). This process can suggest good approach to find relations among meshes of same object obtained by optical scanning, but need continuation.

## List of symbols

$\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}$ vectors between two points
$|\boldsymbol{a}|$ Euclidean norm of vector $\boldsymbol{a}$
$C_i$ centroid of the object with coordinates $x_i, y_i, z_i$
$f, g$ two shape functions
$S_i$ $i$-th centre of line segment with coordinates $x_i, y_i, z_i$
$V_i$ $i$-th vertex of the mesh with coordinates $x_i, y_i, z_i$

## References

[1] OSADA. R., FANKHOUSER. T., CHAZELLE. B., DOBKIN. D.. *Matching 3D Models with Shape Distributions*. In *Proceedings International Conference on Shape Modeling and Applications*. Genova. 2001. ISBN 0-7695-0853-7

[2] PAJEROVÁ. N., LINEKOVÁ I.. *Application of shape distributions to compare triangular meshes*. In *17th conference on applied mathematics - Aplimat 2018*. Bratislava. 2018. pages 795-802 (*in printing*)

[3] OSADA. R., FANKHOUSER. T., CHAZELLE. B., DOBKIN. D.. *Shape distributions*. ACM Transactions on Graphics. Vol. 21. No. 4. 2002. pages 807-832

[4] YU I.C., LAPADAT. D., SIEGER. L., REGLI. W.C.. *Using shape distributions to compare solid models*. In *Proceedings of the seventh ACM symposium on Solid modeling and applications* (SMA '02). ACM. New York. NY. USA. 273-280

[5] MONTEVERDE L.C., RUIZ C.R., HUANG Z.. *A Shape Distribution for Comparing 3D Models*. In: *Advances in Multimedia Modeling*. MMM 2007. Lecture Notes in Computer Science. Vol. 4351. Springer. Berlin. Heidelberg