Numerical verification of a method for determining stiffness parameters in mass-spring models for large displacements and deformations

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Abstract

The aim of the study was to verify a method of determination of stiffness parameters in mass-spring models in the range of large deformations. The mass-spring method is based on a mesh of elements. It consists of masses connected by ideal springs. By assigning the spring stiffness coefficient according to (Lloyd, 2007) the model can be given material properties. In this paper the method presented in (Lloyd, 2007) was verified for large deflections. A model of a deformable body was created in a program written in Python. Several loading conditions were considered. Nodal displacements were determined using Newton-Rhapson's method. The results were compared with FEM models - for small deformations and with geometric non-linearity. Simulations confirmed that in both cases mass-spring obtain similar results as FEM. The mass-spring method may be applied in large deflections modelling, such as in surgical simulations

Keywords: numerical modelling; mass-spring; MSM; FEM; fast simulation; surgical simulation

1.Introduction

Currently the need for numerical modelling of deformable structures has been observed in biomechanics. Such models are very important because they can be used in surgical simulations. It imposes a number of requirements on them. Soft tissues undergo high deformations, which is a limitation for many currently existing methods. Also, the time needed to get the solution should be as short as possible to allow real-time simulations. Some of the available modelling methods aren't strictly based on physical laws, but there is a growing number of methods using mechanical engineering calculation procedures. They achieve high accuracy of results, but in the case of modelling large deformations the calculation time is significantly longer.

Selected numeric modelling methods:

I. Methods based on material continuum

- Finite element method (FEM)

II. Heuristic methods

- Terzopoulos method [1]
- Mass-spring system (MSM)
- Hyperelastic mass-links (HEML) [2]

III. Hybrid methods

- Boundary element method (BEM)+ mass-spring system [3]
- Mass-tensor[2]

Table1.*Comparison of properties of selected modelling methods.*

	FEM	Terzopoulos	MSM	HEML	BEM + MSM	Mass-tensor
computational speed	+/-	+	+	+	+/-	+/-
material properties	+	-	+/-	+	+	+
results accuracy	+	-	+/-	-	+	+
large deflection	+	-	+	+/-	-	-

In case of large deflections, crucial in surgery simulation, many methods are difficult to apply. For instance, FEM is very expensive computationally and requires a lot of time to find a solution. The heuristic methods are effective in visualizing deformations, but their results may differ from reality.

One of the most popular approaches in heuristic modelling is the MSM. The method uses a mesh of springs connecting masses. As the literature indicates, it is a fast numerical modelling method of low computational complexity. One of the key problems in MSM simulations lies in assigning the material properties to the springs. In [4], a popular method for determining the

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spring stiffness coefficient based on the Young's modulus is presented. The Authors only briefly mention its accuracy under large displacements and deformations, which are crucial for surgical simulations. Therefore, the aim of the study was to verify the method presented in [4] under large deformations and displacements. Using this method, a deformable body model was created and its response to the set load was checked. Due to the high accuracy of the FEM, it was used as a reference method. Particular attention was paid to the analysis of large deformations.

2. Method

Firstly, custom software for simulating static MSM models was created in Python. Then, several models of deformable bodies were tested under static loading conditions. The software can be divided into the following subprograms: preprocessor, solver and postprocessor.

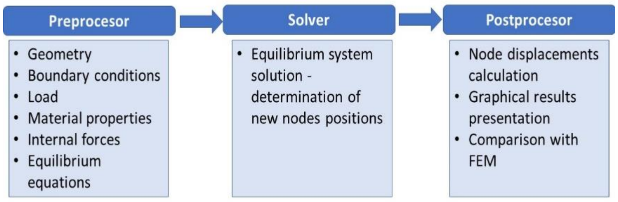

Figure1. *Implementation of the method into a program written in Python*

2.1. Preprocessor

In the first stage the geometry, the material properties, the boundary conditions and the external load were selected using the preprocessor. As mentioned before, Young modulus was assigned to the model using the method presented in [4]. The study focused on a two-dimensional model divided into square elements, as seen Fig. 2.

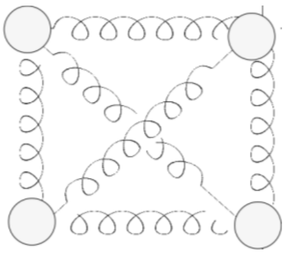

Figure 2. *Single 2D square element used in mass-spring model*

In this case, the stiffness coefficients of the springs can be computed with (1) and (2), as per [4]:

$$k_{edge} = \frac{5}{16} tE \tag{1}$$

$$k_{diagonal} = \frac{7}{16} tE \tag{2}$$

where:

k_{edge} − stiffness coefficient of edge spring$(\frac{N}{m})$,

$k_{diagonal}$ − stiffness coefficient of diagonal spring $(\frac{N}{m})$,

E −Young's modulus (Pa),

t −element's thickness (m).

The square elements contained: nodes (point masses), springs forming the side, and of the diagonal springs. The diagonal springs allowed the element resist shearing loads. Some nodes were shared by more than one element.

Static cases were considered, hence the principle of calculating the displacements in the model was based on the assumption that each node was in equilibrium under its external load. Therefore, the external forces were balanced by the forces generated in the springs. Their spring forces were calculated according to the formula (3)

$$\boldsymbol{F}_{int}^{i} = k_{ij} \Delta l \frac{\boldsymbol{b_s} - \boldsymbol{a_s}}{\|\boldsymbol{b_s} - \boldsymbol{a_s}\|}, \tag{3}$$

where:

$\boldsymbol{F}_{int}^{i}$ −spring force acting on the node (N),

k_{ij} − stiffness coefficient of spring connecting two nodes $(\frac{N}{m})$,

Δl – the increase of the spring element's length (m),

b_s – vector connecting origin with end of spring,

a_s – vector connecting origin with beginning of spring.

At the preprocessor stage, equilibrium equations were written for each node and then collected into a system of equations for the whole model.

2.2. Solver

In the solver, the previously prepared equation system based on (3) was solved using Newton-Rhapson's method. The positions of the nodes of the deformed body were determined.

2.3. Postprocessor

Based upon obtained results, the values of the deformations were found by the program. The deformed model was displayed graphically and the software compared the results with FEM.

2.4. Program application

A model of a rectangular deformable body was created in the software. Static external load of small and large magnitudes was applied in several ways. In each case, the displacements of the selected node were examined. A similar model was made using FEM in Ansys Workbench. The ANSYS models were solved in two ways, using linear analysis and taking into account geometric non-linearity

3. Results and discussion

In order to verify the method presented in [4], a rectangular model of deformable body was made using mass-spring method. Its geometry, after being divided into elements, is shown in Fig. 3. The node which displacements were analyzed was marked with a red dot.

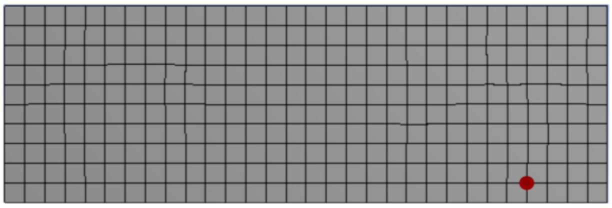

Figure 3*. Undeformed model divided into elements. Red dot is a node which displacements are taken into account when comparing MSM and FEM*

External forces were applied in two ways: tensile test and bending test. Similar models were made in Ansys Workbench, a program using FEM in engineering simulations. The deformed finite element mesh was exported to stl format and then uploaded to the Python software for comparison. The diagrams below show the FEM mesh in black and the mass-springs in blue. The values of the displacements were shown in the force dependence diagrams.

3.1. Stretching

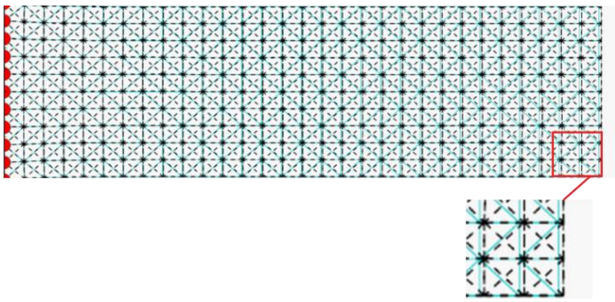

Figure 4*. Deformed by stretching finite element mesh using linear analysis (black) together with the deformed spring mesh (blue).*

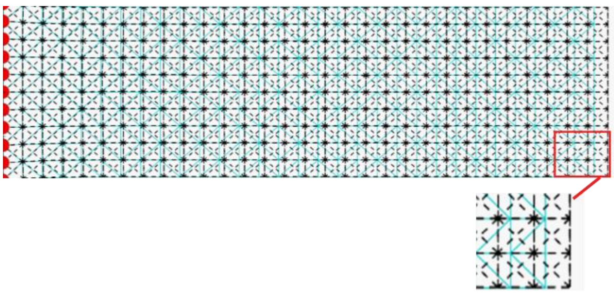

Figure 5*. Deformed by stretching finite element mesh using non-linear analysis (black) together with the deformed spring mesh (blue).*

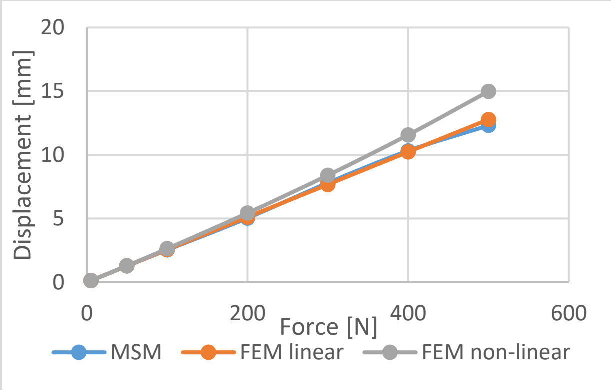

Figure 6*. Plot of analysed node displacement versus stretching force*

3.2. Bending

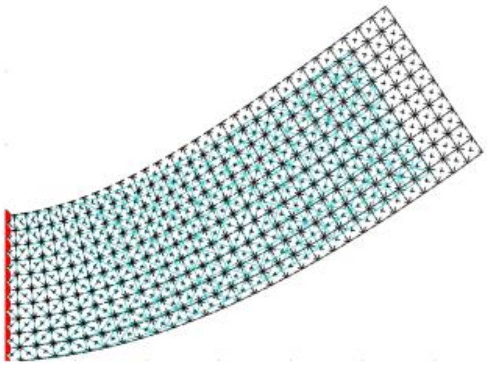

Figure 7. *Deformed by bending finite element mesh using linear analysis (black) together with the deformed spring mesh (blue).*

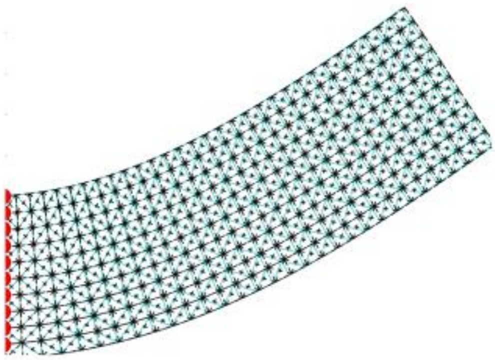

Figure 8. *Deformed by bending finite element mesh using non-linear analysis (black) together with the deformed spring mesh (blue).*

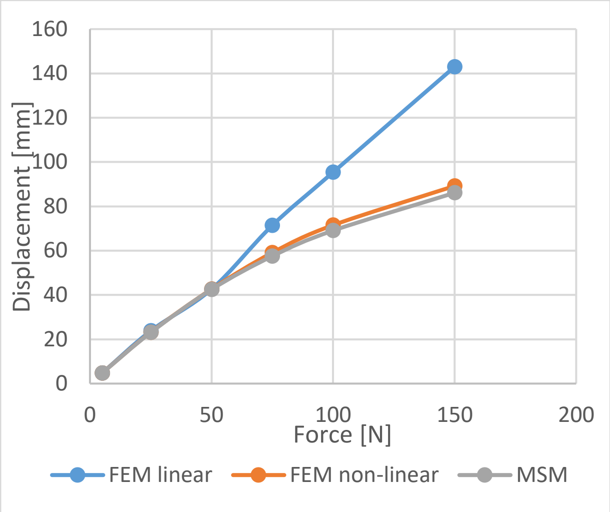

Figure 9. *Plot of analysed node displacement versus bending force.*

3.3. Discussion

On the basis of the obtained results it can be concluded that, in the case of small values of external loads the displacements obtained with MSM were comparable with both, linear and geometric non-linear FEM. For larger load magnitudes, when the model was stretched, the results were consistent with the linear FEM. When the model was bending, results were comparable with the non-linear FEM. Depending on the type of load, the MSM gives results consistent with FEM either linear or with geometric non-linearity. That is why it is worth to verify the results obtained for a specific type of load before using the mass-spring system method.

4. Conclusion

In this paper a verification of mass-spring system modelling method was presented. Using the example of a rectangular model, the method was compared with the FEM. Similar structures were exposed to the same loads; the results obtained, being displacements of nodes, indicate a high potential of this method in modelling both small and large deformations.

MSM can be used in real-time surgical simulations due to its low computational cost and relatively good accuracy of results. Soft tissues undergo large deformations, so in this case other methods of numerical modelling may turn out to be too complicated in terms of calculations - longer time needed to obtain the result.

Symbols

k_{edge} − stiffness coefficient of edge spring$(\frac{N}{m})$
$k_{diagonal}$ − stiffness coefficient of diagonal spring $(\frac{N}{m})$
E −Young's modulus (Pa)
t −element's thickness (m)
$\boldsymbol{F}^i_{int}$ −spring force acting on the node (N)
k_{ij} − stiffness coefficient of spring connecting nodes P_i and P_j $(\frac{N}{m})$
Δl – the increase of the spring element's length (m)
b_s – vector connecting origin with end of spring
a_s – vector connecting origin with beginning of spring

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