Simulation of Multipoint Ultrasonic Flowmeter

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Abstract

The paper can be seen as a feasibility study of a multipoint ultrasonic flow meter which is capable of non-intrusive measurement of flow rate in cases of 2D or 3D flows with the capability of displaying the velocity field. The flow meter is a vector tomograph by its principle. We show that by taking proper transversal interaction measurements the problem can be broken down to three separate scalar field reconstruction problems which is a well described problem even for cases of incomplete data sets. Compared to other vector tomographic methods found in the literature we introduce an additional regularizer which finds a solution satisfying the continuity equation. The proposed method is tested on two flow fields: 2D jet flow and a 3D passenger car wake. The latter test showed that the proposed evaluation scheme is capable of a successful reconstruction of the topology of the wake including its trailing vortices. This has an important practical impact concerning the application of such a device as it offers the applied research an extremely powerful tool which is capable of experimentally visualizing a complicated 3D flow pattern within minutes.

Key words: tomography, 3D vector field, flow meter, ultrasonic measurement

1. Introduction

Vector tomography has been introduced by Norton [1] in his famous article. Norton suggested an analytical solution to the reconstruction problem of vector fields in two and later three dimension. The analytical solution is rather impractical as it needs a large number of measurements oriented in parallel beams in various orientations. Soon many numerical solutions to the problem started to appear, some of which are covered by [2], [3], [4]. The downside of the numerical solutions is that they are seriously ill-posed and the solution based only on the ultrasonic measurements is not unique. To get a stable solution another information needs to be brought to the solution. A favorite choice in scalar tomography is the minimization of Total Variation of the image, e.g. minimization of the sum of gradients, such as described in [5]. We will adopt this approach and add another regularizer which referrers such solutions which satisfy the continuity equation.

2. Ultrasonic flow measurement

2.1. Fluid interaction

Ultrasonic waves passing through a moving fluid are being influenced by the velocity of the fluid. Let's adopt a ray model of the path the waves take on the shortest route from the transmitter to the receiver. We can then define two kinds of interfere, longitudal and transversal which influence the time of flight of the signal and the angle under which it arrives at the receiver respectively. The interaction is well covered by Jovanovich [6], for the longitude interaction we can write:

$$\tau = \int_\Gamma \frac{1}{u_g} ds = \int_\Gamma \frac{1}{(cn + v)s} ds$$

Where τ is the time of fligh of the signal, u_g is the longitudal part of the ray velocity, c is the speed of sound. For flow velocity v considerably smaller compared to the speed of sound this relationship can be linearized and a line integral along the ray path can be expressed:

$$(\tau_0 - \tau)c_0^2 = \int_\Gamma (\Delta cn + v)ds$$

$$l_\Gamma = \int_\Gamma (\Delta cn + v)ds$$

Here we empathize the linear relationship between local flow velocity and measured time of flight. This is extremely important for a later formulation of the tomography problem. Similarly we can construct an integral for the transversal interaction:

$$t_\Gamma = \int_\Gamma (\Delta cn + v)ds_\perp$$

Again the integral is linear with the transverse velocity.

2.2. Helmholtz decomposition

Each vector field can be decomposed into its solenoidal (divergence free) and irrotational (curl free) components, we call this the Helmholtz decomposition. It was first shown by Norton [1] and later by others that only solenoidal part of vector fields can be reconstructed from longitudal interaction (time of flight) measurements while both kinds of interaction are necessary to obtain the irrotational part.

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2.3. Measurement setup

Single transmitter/receiver pairs are sufficient for longitudal interaction measurement. The transversal interaction is far more difficult to obtain, a simple and effective approach of acoustic tripoles was suggested by [6] and is demonstrated in Figure 1 for a two dimensional case. The difference in the time of arrival of an acoustic signal at the receivers R0, R1 and R2 can be used to calculate the angle of arrival and therefore the magnitude of the transversal interaction. The principle can easily be expanded into three dimensions where a minimum of 5 receivers would be necessary.
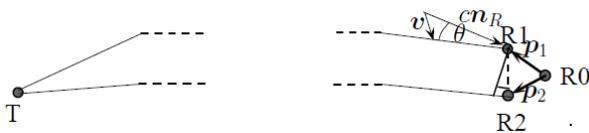
Figure 1: Acoustic triplet for transversal interaction measurement

2.4. Ultrasonic vector tomograph

Tomographic reconstruction of vector fields needs only integral values of transversal and longitudal influence of the field. The Multipoint Ultrasonic Flowmeter has several ultrasonic transceivers spaced on the outside of the measured domain, see Figure 2. At one measurement step one of the transceivers acts as a transmitter of an ultrasonic pulse and the other transceivers receive that signal. This step is repeated for all the other transceivers until $n(n-1)$ signals are obtained where n is the number of transceivers.
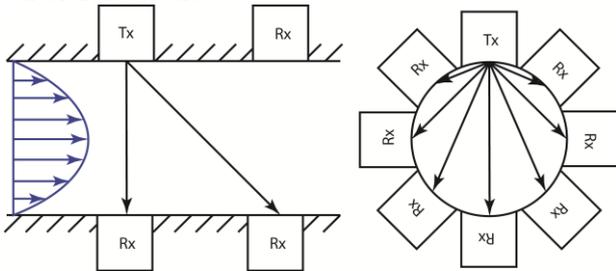
Figure 2: Principle of the Multipoint Ultrasonic Flowmeter

Although demonstrated in three dimension the same principle can be applied on a two dimensional problem. For each ray in a 3D space we obtain one longitudal interaction value and two transversal interaction values. These can be seen as cylindrical coordinates of a measured vector and can be transformed into Cartesian coordinates of a global coordinate system. By doing so for all the rays we can separate the x,y and z components of velocity and the whole system can be solved as three independent scalar tomography problems. Additional dimensional constrains will be introduced later on when we regularize the system by the continuity equation.

3. Inverse problems

3.1. General formulation

A simple matrix description can be constructed for the reconstruction of a general scalar field [3]. Consider a single ray passing through the control area which has been discretized by mesh, Figure 3. The symbol a_{ij} stands for the length of the jth ray passing through the ith mesh cell.
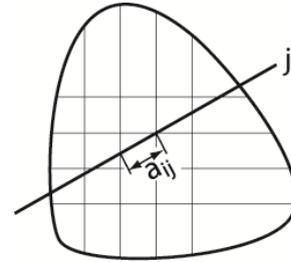
Figure 3: Principle for the construction of the system matrix

We can then construct a system matrix as:
$$b = A \cdot x$$
Where x stands for a vector of local ray velocity as described in section 2.1, b is a vector of measurements. The immediately apparent solution of a matrix inverse is not applicable in tomography problems since the system is underdetermined. Let's define an approximate solution to the problem which minimizes the *L2* norm of errors of our solution:

$$\min \frac{1}{2} \|Ax - b\|_2^2$$

3.2. Regularization

The under determination of the solution to the inverse problem describing tomography is given by the fact that we do not have enough information to reconstruct the values of all cells when we use a limited number or projections (limited number of Tx/Rx pairs). The classical tomography such as the popular medical computerized tomograph (CT) use analytical solution through the Radon transform. One needs to have an large number of projection lines organized in parallel streams to be able to reconstruct the field analytically. Since this limitation is quite impractical for the multipoint flowmeter we will need to accept that there is currently not enough information for us to reconstruct the field. We need to add some additional information to regularize the system. A favorite regularizer in the field of tomography is the Total Variation (TV) regularizer, which has been covered by several authors; we will adopt the solution given by [5]. The TV term computes the total gradient of the field. By minimizing the TV we are seeking for a solution which satisfies both the initial problem of ultrasonic measurement and contains as little gradients as possible. Our problem is now formulated as a minimization of the expression:

$$\min \frac{1}{2}\|Ax - b\|_2^2 + \lambda TV(x)$$

Where the $TV(x)$ operator stands for the total variation given by

$$\sum_i l_i |x_{m(i)} - x_{n(i)}| = \|Lx\|_1$$

Where $m(i)$ and $n(i)$ denote two elements which share the ith edge and l_i is the length of the edge. The Total Variation can also be approximated by a matrix operator D, in which case we obtain:

$$\min \frac{1}{2}\|Ax - b\|_2^2 + \lambda\|Dx\|_1$$

Subject to minimization by the augmented Lagrangian method, after linearization and thresholding given by [5] we obtain the algorithm:

$$x^{k+1} = x^k - \delta(A^T(Ax^k - b) + D^T(\alpha(Dx^k - y^k) - p^k))$$

$$y^{k+1} = \begin{cases} Dx^{k+1} - \frac{p^k}{\alpha} - \frac{\lambda}{\alpha} & if\ \alpha Dx^{k+1} - p^k \geq \lambda \\ 0 & if\ |\alpha Dx^{k+1} - p^k| \leq \lambda \\ Dx^{k+1} - \frac{p^k}{\alpha} + \frac{\lambda}{\alpha} & if\ \alpha Dx^{k+1} - p^k \leq -\lambda \end{cases}$$

$$p^{k+1} = p^k + \alpha(y^{k+1} - Dx^{k+1})$$

Where $y = Dx$ is the TV term, $\alpha > 0$ and $\delta < 1/\|A^TA + \alpha D^TD\|_2$

3.3. Continuity regularizer

The given algorithm is applicable to any scalar or separated vector field. For our special case of multipoint flowmeter we can adopt another constrain to the vector field – the continuity equation, the continuous form for 3D flow is:

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} = 0$$

The continuity equation can be properly discretized and written in the matrix form. The problem is then expanded by the minimization of the $L1$ norm of the magnitude by which the continuity equation is not satisfied:

$$\min \frac{1}{2}\|Ax - b\|_2^2 + \lambda_{TV}\|Dx\|_1 + \lambda_c\|Cx\|_1$$

The given algorithm can be expanded analytically with the TV operator.

4. Simulation

The given algorithm was programmed in Matlab and tested by simulation. The simulation computed the virtual experimental data first and then reconstructed the velocity field based only on that data. Flow fields are given for comparison with the etalon values.

4.1. 2D jet flow

The first test case is a two dimensional flow field of a jet. The field was generated by an analytical solution to the problem [8]:

$$v_x = \frac{\dot{f}(\eta)}{3x^{1/3}}$$

$$v_y = -\frac{v^{\frac{1}{2}}}{3x^{\frac{2}{3}}}(f - 2\dot{f}\eta)$$

$$f(\eta) = 2a \tanh(a\eta)$$

$$\dot{f}(\eta) = 2a^2 \operatorname{sech}^2(a\eta)$$
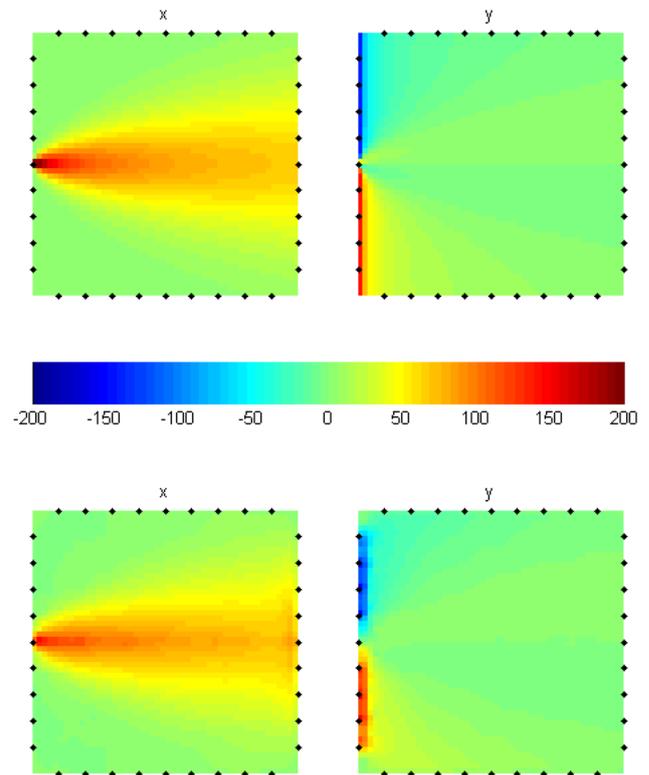
Figure 4: Simulated reconstruction of a 2D jet flow

Figure 4 gives the x a y components of velocity on the upper left an upper right picture respectively. The two lower pictures give the resulting reconstructed field. Black dots denote the simulated ultrasonic transceivers. The reconstruction was done according to the algorithm given by section 3.2 with the TV regulazer only, $\lambda = 3, a = 0.1$, 1000 iterations. One can notice two major disagreements between the fields – first one in the

region of the right edge of the region where the *x* velocity component was introduced a jump which is not present in the input picture. The second region is the left upper and left lower corner of the *y* component where the incoming flow ends before the edge of the region. Both of these errors clearly violate the continuity equation since mass is appearing or disappearing in the middle of the control area. Let's have a look at the same simulation with continuity constrain involved, see Figure 5. The error in the former region has completely disappeared while the error in the latter region is still present. One can notice an increased *x* velocity component in the latter region as the continuity constrain is getting the missing mass from outside of the simulated region. We have demonstrated the advantage of the additional continuity constrain.
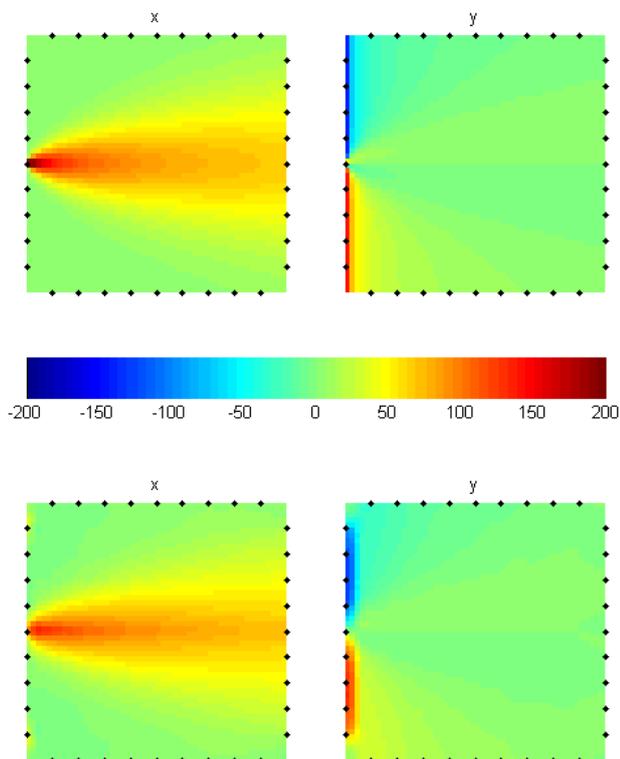
Figure 5: Simulated reconstruction of a 2D jet flow with the continuity constrain

4.2. 3D passenger car wake

For the second test case a wake of a passenger car was chosen as an example of a complicated three dimensional flow pattern. The simulated velocity field was taken from a CFD simulation of a sedan passenger car, total of 192 ultrasonic transceivers were placed around the domain. Results are displayed in Figure 6, left column of images is the input velocity field while the right column represents the simulated results. One can notice artifacts near the free stream but the topology of the wake has been simulated successfully. This is further demonstrated by an ISO surface of constant velocity magnitude on

the very low of the picture. Please note that the view angle has been changed for the ISO surface so that the two trailing vortices are visible.
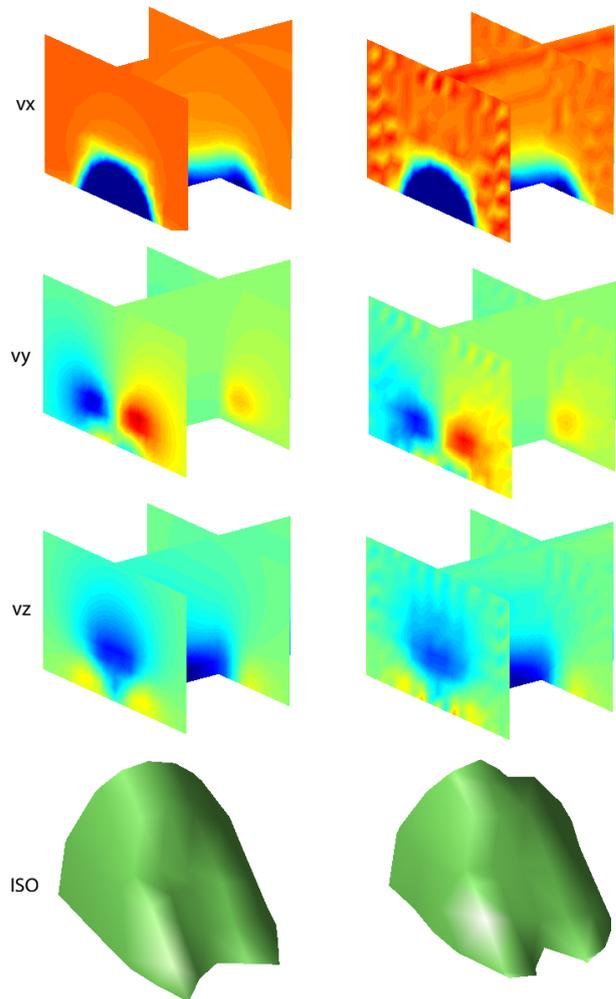
Figure 6: Simulated reconstruction of a wake of a passenger car

5. Conclusion

The Multipoint Ultrasonic Flow meter has been proven to be capable of reconstructing a 3D flow pattern as complicated as a wake of a passenger car. We have showed that by taking appropriate measurements the transversal interactions of the acoustic signals can be used to break down the vector tomography problem into two or three (depending on the dimensionality) separate scalar reconstruction problems. Standard tools for image reconstruction were used to find an optimal solution constrained by Total Variation, which is a standard and favorite regularization tool, together with the equation of continuity. Further work will include more simulations on other flow fields; parallelization will be needed to introduce finer computational grids. The flow meter will also be verified experimentally.

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