Analysis of Composite Beam Bending

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Abstrakt

Tato práce se týká modelování výpočtu ohybu kompozitních nosníků pomocí MKP modelů. Práce porovnává výsledky získané z MKP modelů jak mezi sebou, tak s analytickými metodami výpočtu ohybu kompozitních nosníků. Hodnoceny jsou tři základní modely MKP: skořepina, objemová skořepina a klasický objemový model. Pro porovnání s analytickými výpočty jsou vybrány metody ABD matic a klasická laminátová teorie s respektováním Bernoulliho teorie ohybu. Geometrie modelu nosníku je vybrána tak, aby vyhovovala předpokladům všech výše zmíněných metod výpočtu. Výsledky lze přímo porovnat mezi sebou. Jsou analyzovány změny průhybu nosníku při změně úhlu natočení vláken v jednotlivých vrstvách kompozitního materiálu.

Klíčová slova: *composite; beam; bending; FEM*

1. Introduction

The work is done to facilitate the design of composite beams. We compare the known methods of bending analysis of composite beams for the different composition of the composite material. The model of the beam, based on the thesis [3], has been improved to suit better all the methods of calculation. This work deals with larger size of inner diameter of the fixed beam and includes more options of the calculations.

The results of the deflection of the fixed beam loaded by a single force are presented. The deflection is calculated by the FEM method. The conventional shell, continuous shell and the volume model are used. The analytical models of the classical laminate theory and the calculation using ABD matrices are used for comparison of the results.

The geometry of the beam is chosen to meet the expectations of all above mentioned methods of calculation so the results can be easily compared. The deflection is analyzed for the changing angle of the fibres of the layers in the composite material.

2. Methods

The FEM calculations are realised in Abaqus and for the analytical calculations the MATLAB is used.

The geometry is the same for all models. The fixed beam is loaded by a single force (Fig.1.).

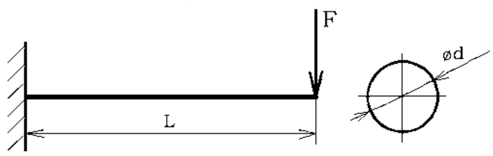

Figure 1: The geometry of the models

The composite layup is composed from three layers [90, α, -α]. The angle α is variable from 0° to 90°. The thickness of each layer is 1mm.

The material constants and the geometry are entered as parameters of the model or in the form of ABD matrices.

1.1 Conventional Shell

The geometry of the model is represented by a shell. The thickness of the beam is specified as a parameter of the model or it is included in ABD matrices which are used for calculations. So there are two ways for calculations. As seen in the results, the differences caused by the using both methods of setting the parameters of the model are minimal. The common elements S4R are used in both cases.

1.2 Continuum Shell

The continuum shell is modelled as a solid body. The real thickness of the beam is specified as the parameter of the model and by the composite layup. The elements of the model are distributed through the whole thickness of the solid body. The difference between the continuum and conventional shell is evident from the Fig. 2.

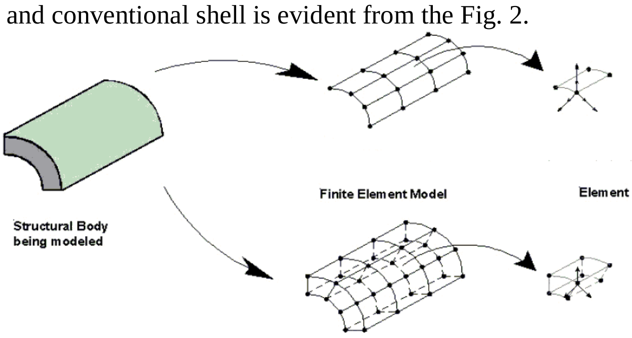

Figure 2: The difference between conventional and continuum shell

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The advantage of the continuum shell elements is that they can be stacked. According [1] they yield the exact elastic solution. The meshing is more difficult with continuum shell because the thickness of the shell has to be meshed. The SC8R elements are used for this model.

1.3 Volume Model

For the calculation using the volume model the full 3-D geometry is specified. Each ply is created separately as a separate solid body with his own material specification. The element type C3D8R (three-dimensional hexahedral element) has been used for meshing the tube. These elements are linear, reduced-integration elements.

1.4 Classical Laminate Theory

The main task in this way of the calculation is determination of the bending stiffness. The Hooke's law adapted for the laminate theory is used for its specification. The assumptions for the calculation are the plane strain and application of the Bernoulli's midline beam theory.
Bernoulli's beam theory assumes that a normal of the beam before deformation remains normal to the midline after deformation. This is illustrated in Figure 3.

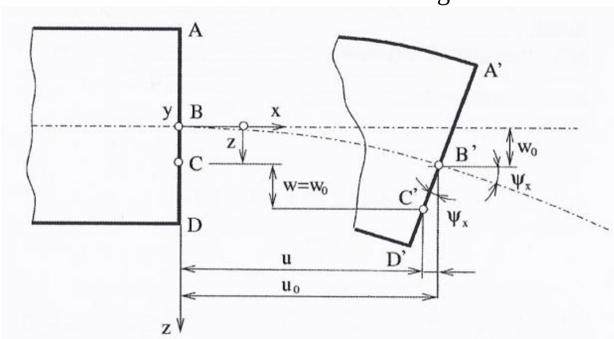

Figure 3: Bernoulli's assumption [2]

In the Hooke's law

$$\boldsymbol{\sigma} = \boldsymbol{S} \cdot \boldsymbol{\varepsilon} \rightarrow \boldsymbol{\varepsilon} = \boldsymbol{C} \cdot \boldsymbol{\sigma} \tag{1}$$

the matrix of compliance is

$$\boldsymbol{C} = \boldsymbol{S}^{-1} = \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_{xy}}{E_y} & 0 \\ -\frac{\nu_{yx}}{E_y} & \frac{1}{E_y} & 0 \\ 0 & 0 & \frac{1}{G_{xy}} \end{bmatrix} . \tag{2}$$

The orthotropic material is considered.
The load is presumed in the direction of the x-axis. The modulus of elasticity E_x from the compliance matrix is used for calculation of the bending stiffness.
The deflection of the beams is calculated according to the equation:

$$w''(x) = -\frac{M_o(x)}{\sum_k E_{x_k} \cdot J_{y_k}(x)} \tag{3}$$

From this equality it is evident that the main problem is the correct determination of the bending stiffness (product $E_{x_k} \cdot J_{y_k}(x)$) of composite material. This is solved by the sum of properties of single layers in composite. To calculate the bending stiffness a program in MATLAB was created.

1.5 Calculation Using the ABD Matrices

For the determination of the ABD matrices the knowledge of the composite layup, thickness of the layers and the material constants are needed. The main problem is still determination of the bending stiffness of the whole material and a related equivalent modulus of elasticity.
In this case the equivalent modulus of elasticity of the whole material is obtained from the elements of the tensile stiffness matrix **A** from equation of the ABD matrices of Kirchhoff beam theory [3]

$$\begin{bmatrix} \mathbf{N} \\ \cdots \\ \mathbf{M} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \vdots & \mathbf{B} \\ \cdots & \vdots & \cdots \\ \mathbf{B} & \vdots & \mathbf{D} \end{bmatrix} \begin{bmatrix} \boldsymbol{\varepsilon}^{\circ}{}_{\mathbf{m}} \\ \cdots \\ \mathbf{k} \end{bmatrix} , \tag{4}$$

where **A** is the extensional stiffness matrix, **B** is the bending-extension coupling stiffness matrix and **D** is the bending stiffness matrix. The equivalent Young's modulus E_{eq} is determined.

$$E_{eq} = \left(A_{11} - [A_{12} \quad A_{13}] \cdot \begin{bmatrix} A_{22} & A_{23} \\ A_{32} & A_{33} \end{bmatrix}^{-1} \cdot \begin{bmatrix} A_{21} \\ A_{31} \end{bmatrix} \right) \cdot \frac{1}{t} , \tag{5}$$

where A_{ij} are the elements of the tensile stiffness matrix and t is the thickness of the composite material. Than the equation (3) is used to obtain the deflection as in previous case.

3. Results

As the results the graphs of dependences of deflection on the angle of the fiber direction are obtained. In the Fig.4 the great difference between the calculation by the conventional shell and the other methods at larger angles is evident. It corresponds with the general assumptions for the calculations using shells. This problem disappeared when the more thin-walled profile is chosen as it is seen in the Fig.5.
The difference between calculation by conventional shell using ABD matrices and using the material constants and the thickness are very small.
For the calculation by the classical laminate theory the compliance matrix is used, so the results obtained from this method are on the safety side. The biggest differences in comparison of all used methods are at the centre of the spectrum of used angles.

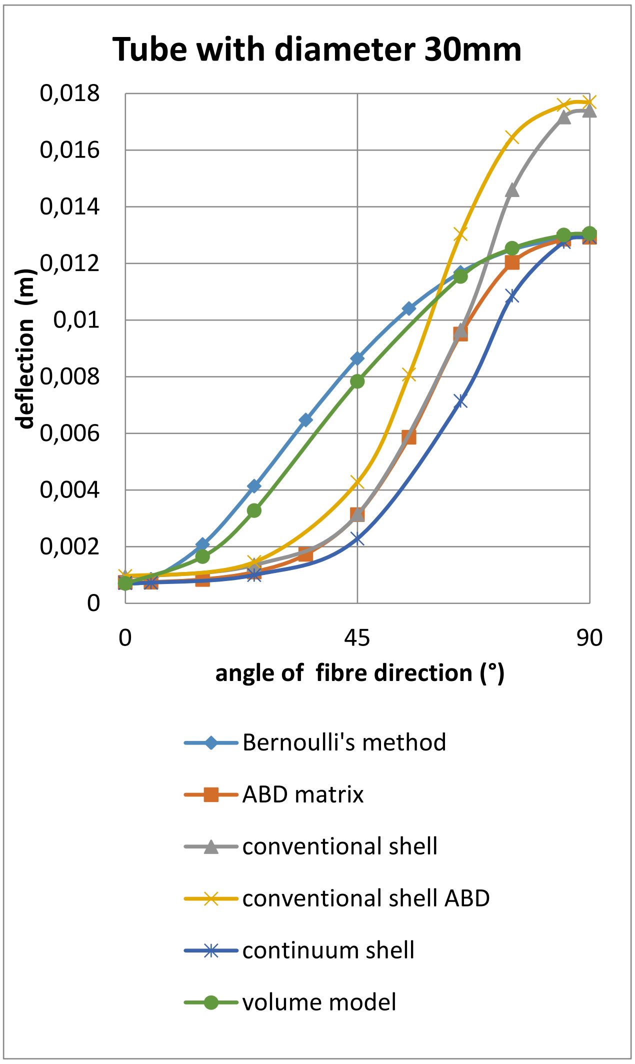

Figure 4: Dependence of the deflection on the fiber direction for the tube with inner diameter 30mm

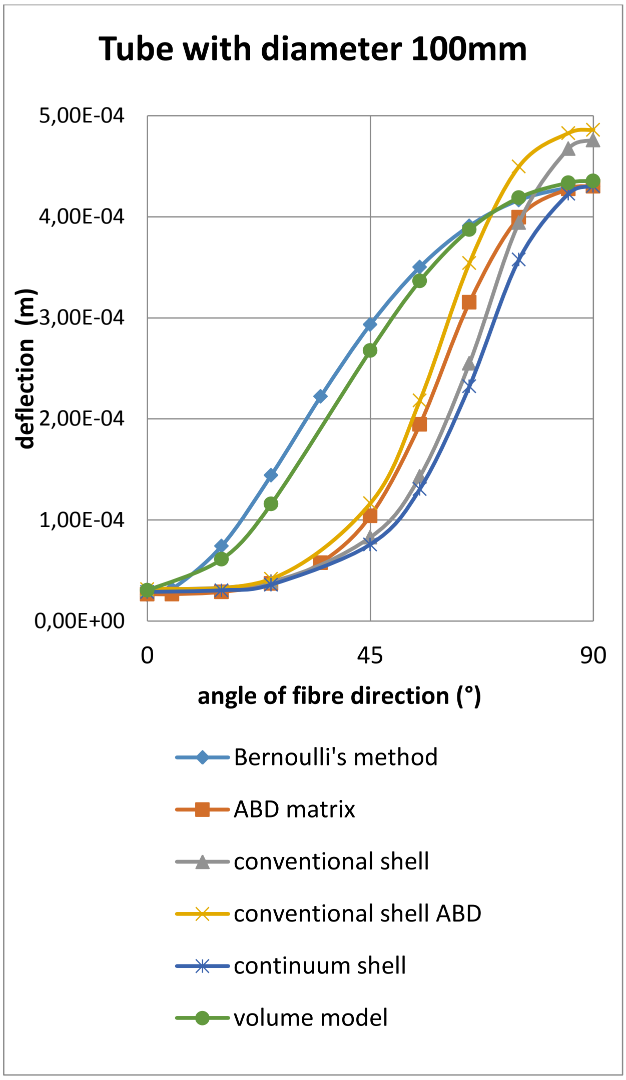

Figure 5: Dependence of the deflection on the fiber direction for the tube with inner diameter 100mm

4. Conclusion

The examined methods show conformity when the small angles of the fiber are used. The great differences are between the results for the intermediate fiber directions.
The calculations using conventional shell show the deviation at larger angles that disappears when the thin-walled profiles are used. The calculations using the conventional shell and continuum shell gives the similar results excluding the results at larger angles. Both FEM methods using shells are approaching the analytical method that uses ABD matrices.
The volume model gives the results comparable with the classical laminate theory. This two methods show the larger values in comparison with the other methods, but they are on the safety side of the calculation.
The continuation of this research requires the comparison of all used methods with the experimental data obtained from the three-point bending test.

Acknowledgment

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List of Symbols

	name	unit
$\boldsymbol{A}$	extensional stiffness matrix	$N.m^{-1}$
A_{ij}	element of extensional stiffness matrix	$N.m^{-1}$
$\boldsymbol{B}$	bending-extension coupling stiffness matrix	$N.m^{-1}$
$\boldsymbol{C}$	compliance matrix	Pa^{-1}
$\boldsymbol{D}$	bending stiffness matrix	N
E_{eq}	equivalent modulus of elasticity	Pa
E_x	modulus of elasticity in direction of the x-axis	Pa
E_y	modulus of elasticity in direction of the y-axis	Pa
G_{xy}	shear modulus in the plane xy	Pa
$i, j,$	general indices	
J_y	moment of inertia in direction y	m^4
$\boldsymbol{k}$	vector of curvature of the midplane of the	

	name	unit
	laminate	
M_O	bending moment	N.m
$\boldsymbol{S}$	stifness matrix	Pa
t	thickness	m
w	deflection	m
x, y	directions of the axes	
$\boldsymbol{\varepsilon}^{\circ}{}_{\boldsymbol{m}}$	strain of the midplane	
$\nu_{\boldsymbol{xy}'}$	Poisson’s ratio in main directions	

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