**Direct solution of so called 3D method of data reduction**

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**Abstract**

The paper deals with new derivation and solution of balance equations. These equations are used to process experimental data of 3-dimensional flow fields of compressible fluids. Strict balance of mass and momentum flux and energy of local measured values of 3-D flow fields with assumption that the gas is ideal and flow is adiabatic, is conducted. Results of this balance are integral parameters of the whole flow field. These are later used for evaluation of kinetic energy loss and further aerodynamic investigation.

*Keywords: compressible flow, balance equations, three-dimensional flow field, experimental data evaluation*

## 1. Introduction

Rapid development of measuring equipment in fluid mechanics had brought its own, unique problems. One of those is large amount of data that needs to be processed. Large amount of data cannot serve for evaluation of any investigated experiment. Two-dimensional example is showed in Figure 1.

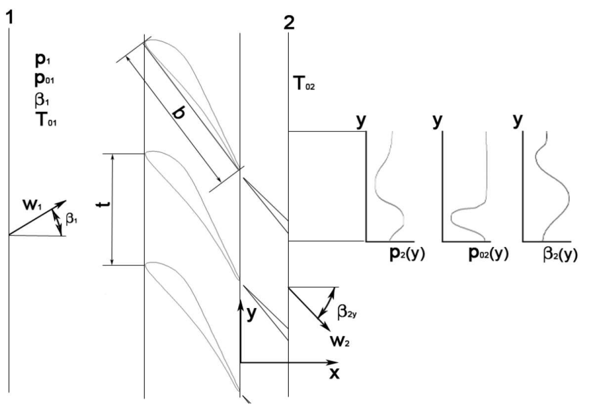


*Figure 1 - 2-D experimental results illustration*

There is distribution of values of static and total pressure and angle of stagnation flow. It’s obvious, that the courses of the values differ and therefore can’t be related easily to quality of the flow field. This is especially valid point in three-dimensional flow, because in any measured point, there will be additional angle (yaw) and there will be much more data. Hence individual points of data have no evaluation value at all.

Therefore, the balance equations were to be utilized. General idea behind those equations is that the pressure and flow direction will be established in each measurement point. Therefore mass flow of whole area can be obtained as well as momentum flux. The equations will then give one value of speed and direction to the whole area that would hypothetically set in in the infinite distance from the measuring plane. It can be perceived as kind of physical average. Having assigned one value of speed and its direction to the whole investigated area gives researcher an option to evaluate quality of flow field.

## 2. Physical background

Situation that method of data reduction will be most commonly used with combination of aerial reading of parameters downstream the investigated object, as seen on *Figure 2*.

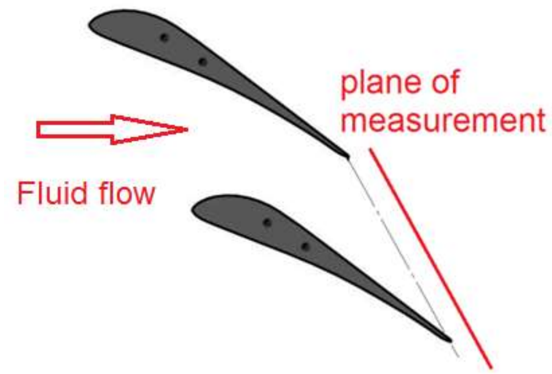


*Figure 2 - Illustration of experiment configuration.*

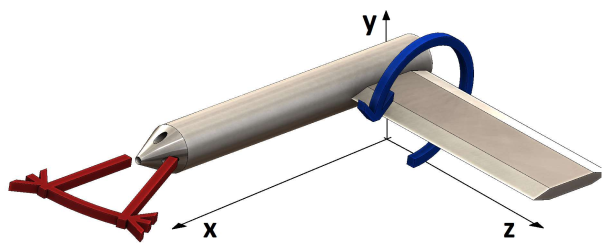


*Figure 3 - measured angles explained*

The measurements require a probe capable of measuring the direction of stagnation flow. In case this method was developed for 5-hole conical probe. In the *Figure 3* the probe used is depicted. Rotation around the

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axis that's holding profile (represented by blue arrow) is denoted $\beta$ and figures as pitch; active PID regulator balances the probe in the direction of the flow. This rotation occurs in $x-z$ plane. Deviation in plane perpendicular to rotation – yaw, denoted $\gamma$ and represented by red arrow is evaluated solely by calibration of the probe and different pressures that are measured; see [1].

The equations balance mass flux, and momentum flux, which is also dependent on mass flux. The following formulation of mass flux is stated:

$$\dot{m} = \iint_A \rho_{2(y,z)} \cdot v_{2(y,z)} \cdot \cos(\beta_{2(y,z)}) \cdot \cdot \cos(\gamma_{2(y,z)}) \cdot dy \cdot dz \tag{1}$$

Equation (1) contains several parameters that are not directly measured. Density $\rho$ and velocity $v$ cannot be obtained in experiment. However static pressure in front of measured objects is measured and assuming that flowing media is an ideal gas and that the process is adiabatic, equation can be modified.

Equation of state of ideal gas:

$$\rho_2 = \frac{p_2}{r \cdot T_2} \tag{2}$$

Equation of energy:

$$\frac{T_2}{T_{02}} = 1 - \frac{\kappa+1}{\kappa-1} \cdot M_2^{*2} \tag{3}$$

Where $\kappa$ is Poisson's ratio, $M^*$ is dimensionless speed, defined by following equation:

$$M_2^* = \sqrt{\frac{\kappa+1}{\kappa-1} \cdot \left[1 - \left(\frac{p_2}{p_0}\right)^{\frac{\kappa-1}{\kappa}}\right]} \tag{4}$$

Velocity was substituted by following form:

$$v_2 = \sqrt{\frac{2\kappa \cdot r}{\kappa-1} \cdot T_0 \cdot \left[1 - \left(\frac{p_2}{p_{02}}\right)^{\frac{\kappa-1}{\kappa}}\right]} \tag{5}$$

Implementing energy equation into equation of state of ideal gas gives us substitution for density, while presuming adiabatic process, therefore $T_0 = T_{01} = T_{02}$:

$$\rho_2 = \frac{p_2}{r \cdot T_{02} \cdot \left(\frac{p_2}{p_0}\right)^{\frac{\kappa-1}{\kappa}}} \tag{6}$$

These substitutions allow the argument of mass flow integral equation to be stated as following:

$$\frac{A \cdot \sqrt{r \cdot T_0}}{r \cdot T_{02}} \cdot \left(\frac{p_2}{p_{02}}\right)^{\frac{1}{\kappa}} \cdot p_{02} \cdot \cdot \sqrt{\frac{2\kappa}{\kappa-1} \cdot \left[1 - \left(\frac{p_2}{p_{02}}\right)^{\frac{\kappa-1}{\kappa}}\right]} \cdot cos(\beta_2) \cdot cos(\gamma_2) \tag{7}$$

The equation of mass flux density as stated in [1] was used for simplification of equation (7)

$$q(M_2^*) = = \sqrt{\frac{2}{\kappa-1}\left(\frac{\kappa+1}{2}\right)^{\frac{\kappa+1}{\kappa-1}}\left[1 - \left(\frac{p}{p_0}\right)\right]^{\frac{\kappa-1}{\kappa}}} \left(\frac{p}{p_0}\right)^{\frac{1}{\kappa}} = \left(\frac{\kappa+1}{2}\right)^{\frac{1}{\kappa-1}} M_2^* \cdot \left(1 - \frac{\kappa-1}{\kappa+1} M_2^{*2}\right)^{\frac{1}{\kappa-1}} \tag{8}$$

The integral parameter of mass flux can then be stated in simplified form. All the unknowns in the equation have indexes $(y,z)$ which means that they were obtained during measurement, when the probe was traversed trough its path and covered area of measurement. Note that the parameter also contains constants, so numeric value does not represent actual mass flux.

$$I_m = \iint_A p_{02_{(y,z)}} \cdot q\left(M^*_{2_{(y,z)}}\right) \cdot cos(\beta_{2(y,z)}) \cdot \cdot cos(\gamma_{2(y,z)}) \cdot dy \cdot dz \tag{9}$$

The mass flux equation in form seen above is related only to pressures and stagnation flow angles, static pressure and temperature, which are measured during the experiment.

Values of other quantities would be obtained in very similar manner, using different dimensionless functions, as ratio of pressures [1]:

$$\pi(M_2^*) = \frac{p}{p_0} = \left(1 - \frac{\kappa-1}{\kappa+1} \cdot M_2^{*2}\right)^{\frac{\kappa}{\kappa-1}} \tag{10}$$

And function characterizing kinetic pressure [1]:

$$\omega(M_2^*) = \left(1 - \frac{\kappa-1}{\kappa+1} M_2^{*2}\right)^{\frac{1}{\kappa-1}} \cdot \frac{\kappa}{\kappa+1} \cdot M_2^{*2} = \frac{\kappa}{\kappa-1} \cdot \left(\frac{p}{p_0}\right)^{\frac{1}{\kappa}} \cdot \left(1 - \left(\frac{p}{p_0}\right)^{\frac{\kappa-1}{\kappa}}\right) \tag{11}$$

The momentum flux parameter in direction of x axis is then written as follows:

$$I_x = \iint_A p_{02(y,z)} \cdot \left[\pi\left(M^*_{2(y,z)}\right) + +2 \cdot \omega\left(M^*_{2_{(y,z)}}\right) \cdot cos^2(\beta_{2(y,z)}) \cdot cos^2(\gamma_{2(y,z)})\right] dy \cdot dz \tag{12}$$

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and momentum flux parameters in direction of axis y and z respectively:

$$I_y = \iint_A 2 \cdot p_{02_{(y,z)}} \cdot \omega\left(M^*_{2(y,z)}\right) \cdot cos\left(\beta_{2(y,z)}\right) \cdot sin\left(\beta_{2(y,z)}\right) \cdot cos\left(\gamma_{2(y,z)}\right) \cdot dy \cdot dz \quad (13)$$

$$I_z = \iint_A 2 \cdot p_{02_{(y,z)}} \cdot \omega\left(M^*_{2(y,z)}\right) \cdot cos\left(\beta_{2(y,z)}\right) \cdot sin\left(\gamma_{2(y,z)}\right) \cdot cos^2\left(\gamma_{2(y,z)}\right) \cdot dy \cdot dz \quad (14)$$

## 3. Reference parameters

Concept of reference parameters is based on assumption, that were the measurements conducted very far from the investigated profile, the parameters would stabilize over the investigated plane and have specific value that's valid for whole experiment. Calculation of these parameters is then only finding values of investigated quantities that comply with all the measured data. In order to do that system of equations must be solved.

$$A \cdot p_{02} \cdot q(M_2^*) \cdot \cos(\beta_2) \cdot \cos(\gamma_2) = I_m \quad (15)$$

$$A \cdot p_{02} \cdot (\pi(M_2^*) + 2 \cdot \omega(M_2^*) \cdot \cos^2(\beta_2) \cdot \cos^2(\gamma_2)) = I_x \quad (16)$$

$$2 \cdot A \cdot p_{02} \cdot \omega(M_2^*) \cdot cos(\beta_2) \cdot sin(\gamma_2) \cdot cos(\gamma_2) = I_y \quad (17)$$

$$2 \cdot A \cdot p_{02} \cdot \omega(M_2^*) \cdot cos(\beta_2) \cdot sin(\beta_2) \cdot cos^2(\gamma_2) = I_z \quad (18)$$

### 2.1. Direct solution and results

Key to direct solution was not to use any other than mathematical substitutions. The equations had to be rearranged and solution obtained. Equations (17) and (15) were divided to eliminate angle $\gamma$. Equations (16) and (18) were also divided, for elimination of $p_{02}$ and finally equations (15) and (18) were divided for elimination of $\beta$ from the system. Thus $M_2^*$ could be obtained, and it was stated as follows:

$$\frac{I_x}{I_z} = \frac{\kappa + 1 - M_2^{*2} \cdot \left\{\kappa + 1 - 2 \cdot \kappa \cdot \left[1 - \left(\frac{I_z}{I_m}\right)^2 \cdot \frac{C}{A}\right] \cdot (1 - B)\right\}}{2 \cdot M_2^{*2} \cdot \kappa \cdot \sqrt{1 - \left(\frac{I_z}{I_m}\right)^2 \cdot \frac{C}{A} \cdot (1 - B) \cdot \left(\frac{I_z}{I_m}\right)^2 \cdot \frac{C}{\sqrt{A}}}} \quad (19)$$

For:

$$A = M_2^{*2} \cdot \kappa^2 - \left(\frac{I_y}{I_m}\right)^2 \cdot \left(\frac{\kappa + 1}{2}\right)^{\frac{2 \cdot \kappa}{\kappa - 1}}$$

$$B = 1 - \left(\frac{I_y}{I_m}\right)^2 \cdot \left(\frac{\kappa + 1}{2}\right)^{\frac{2 \cdot \kappa}{\kappa - 1}} \cdot \frac{1}{M_2^{*2} \cdot \kappa^2}$$

$$C = \left(\frac{\kappa + 1}{2}\right)^{\frac{2 \cdot \kappa}{\kappa - 1}}$$

The equation (19) is composed only of unknown dimensionless speed and constants, thus presenting a solution. Rearranging of the said equation into explicit form proved to be difficult. The symbolical (Matlab) solution exists, however is far too complex to be analyzed yet.

### 2.2. Known other solutions

Method of data reduction with similar or identical input has already been solved using different approaches. One of those is using of substitution for velocity in direction $v_y$ and $v_z$, $v_{yz} = \sqrt{v_y^2 + v_z^2}$; see [2] for further information. Quite similar is approach with measurement of wall angles instead of one wall and one dihedral angle was used to investigate radial cascade in cylindrical coordinates, see [3], and in Cartesian coordinates, see [4]. Another form of approach, relying on substitution of directional angles, is being utilized at IT CAS for evaluation of measured cascades.

All proposed solutions share same characteristic, which is indirect approach. With this approach it is difficult to implement measurement uncertainty into equations. It is also difficult to perform theoretical analysis of the formulae and of expected results. Method described in [4] allows analysis to certain extent, but theoretical investigation of uncertainty is rendered next to impossible.

## 4. Conclusion

The system of balance equations has been build and solved providing another tool for investigation 3-D flow fields of compressible fluid. Analysis of the results is yet to be made as well as uncertainty analysis. The solution proved to be difficult, but this was expected, as all prior authors used substitutions and other simplifications. Reference parameters of the flow past investigated body can be calculated. In near future, the analysis of the equation will be made. Evaluation of nature of the results will be conducted.


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## List of symbols

| | |
|---|---|
| $A$ | area of measurement ($m^2$) |
| $\beta$ | pitch angle (deg.) |
| $\gamma$ | yaw angle (deg.) |
| $I_m$ | mass flux integral parameter ($kg \cdot m \cdot s^{-2}$) |
| $I_x$ | momentum flux in direction of x axis integral parameter ($kg \cdot m \cdot s^{-2}$) |
| $I_y$ | momentum flux in direction of y axis integral parameter ($kg \cdot m \cdot s^{-2}$) |
| $I_z$ | momentum flux in direction of z axis integral parameter ($kg \cdot m \cdot s^{-2}$) |
| $\kappa$ | Poisson's ratio (1) |
| $M_2^*$ | dimensionless speed (1) |
| $p_2$ | static pressure in measured area (Pa) |
| $p_{02}$ | total pressure in measured area (Pa) |
| $\pi$ | pressure ratio (1) |
| $q$ | mass flux density (1) |
| $r$ | specific gas constant ($J/kg \cdot K$) |
| $\rho$ | density ($kg/m^3$) |
| $T_0$ | static temperature (K) |
| $T_{01}$ | total temperature in front of experiment (K) |
| $T_{02}$ | total temperature in measuring area (K) |
| $\omega$ | kinetic pressure function (1) |

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