SOLUTION TRAVELING SALSMAN PROBLEM USING HEURISTIC ALGORITHMS

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Abstract

Okružní dopravní problém, někdy též nazýván jako úloha obchodního cestujícího, je kombinačním problémem hledajícím nejkratší trasu spojující množinu bodů s požadavkem, že trasa končí v bodě, ve kterém začala. Délka trasy je dána volbou pořadí při spojování jednotlivých bodů. Hledá se postup, který by našel nejkratší trasu, aniž by bylo třeba změřit všechny trasy, které existují. Dodnes nebyl nalezen takový algoritmus, který by toto umožnil s počtem kroků, který v závislosti na počtu bodů stoupá pouze lineárně nebo jako mocninná funkce. Tento příspěvek popisuje heuristický přístup k řešení tohoto problému, kde jsou uplatněny některé zákonitosti týkající se hodnot pravděpodobností výskytu jednotlivých spojnic bodů v nejkratší trase. Implementace těchto zákonitostí dokáže urychlit hledání řešení. Schopnost rychlého řešení tohoto problému má uplatnění v optimalizaci logistických tras, v plánování pořadí zpracování zakázek, naprogramování trajektorií pohybu obráběcích nástrojů apod.

Key words: *Algorithms, heuristic methods, models, Traveling Salesman Problem, optimal solution, probability, statistics.*

1. INTRODUCTION

Orbital traffic problem or traveling salesman problem consists in finding the shortest route connecting a given set of points, travelers must visit each point exactly once and return to the starting point, as shown for example ([3], p. 22). It is ironic that this seemingly simple task belongs to the mathematical problems of the millennium. Still is not found algorithm solving this problem without the number of steps in the algorithm increased depending on the number of seats a maximum polynomial as described ([1], pp. 20-23). In practice this problem is widespread in various fields and the ability of its solution has an impact on process efficiency hence the cost. Body may represent, for example: city or workplace when planning logistics routes, destinations to be drilled in the production of printed circuit boards in the optimization of sliding times. Points can also symbolize the contract at finding an optimal sequence to minimize the times for adjustment facility between these contracts etc.

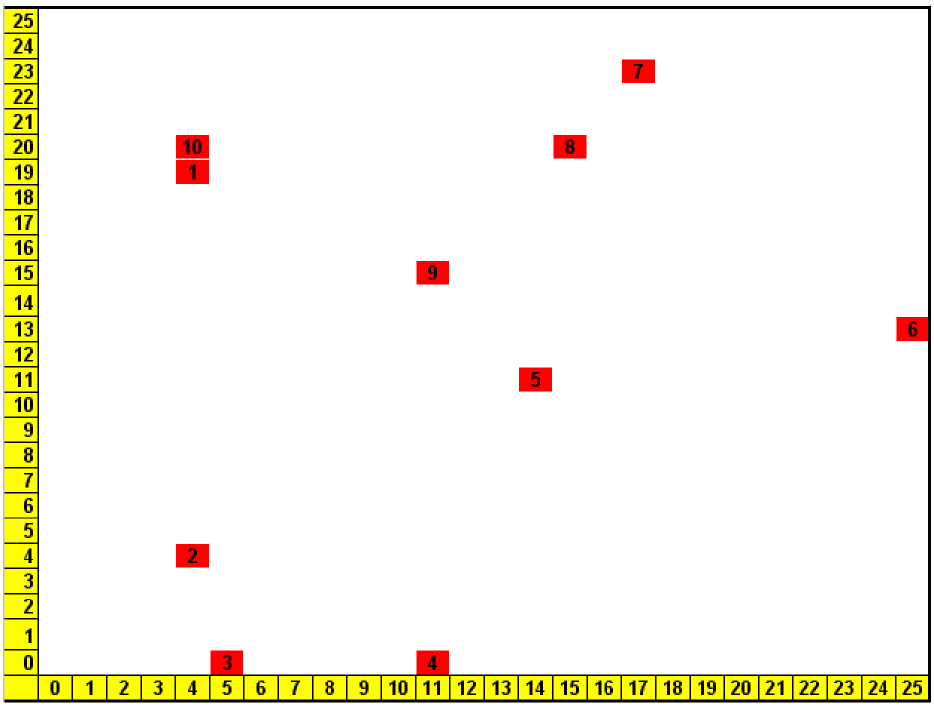


Fig. 1: *The order determines the optimal route point numbers*

Exact search for optimal solutions is relatively difficult problem, whether we choose any way. You may use mathematical notation situation and follow-up using a binary linear programming as described ([7], p. 208-209). To illustrate: for ten points we are looking for extreme bound 99 mostly binary variables.

This paper describes the model using a different method of solution based on heuristic approach, which can be described as a random search of feasible solutions as shown in ([9], p. 126), from which it is using other algorithms selected the best solution according to the criteria lowest length of the route. To achieve an optimal solution faster where speed considering proportional to the number of steps are incorporated into the program elements selectable solutions which are not promising. The functionality of this element was tested and confirmed, there is a significant acceleration of course finding the optimal route.

2. HEURISTIC METHODS

2.1 SELECTION CRITERIA IMPROBABLE SOLUTIONS

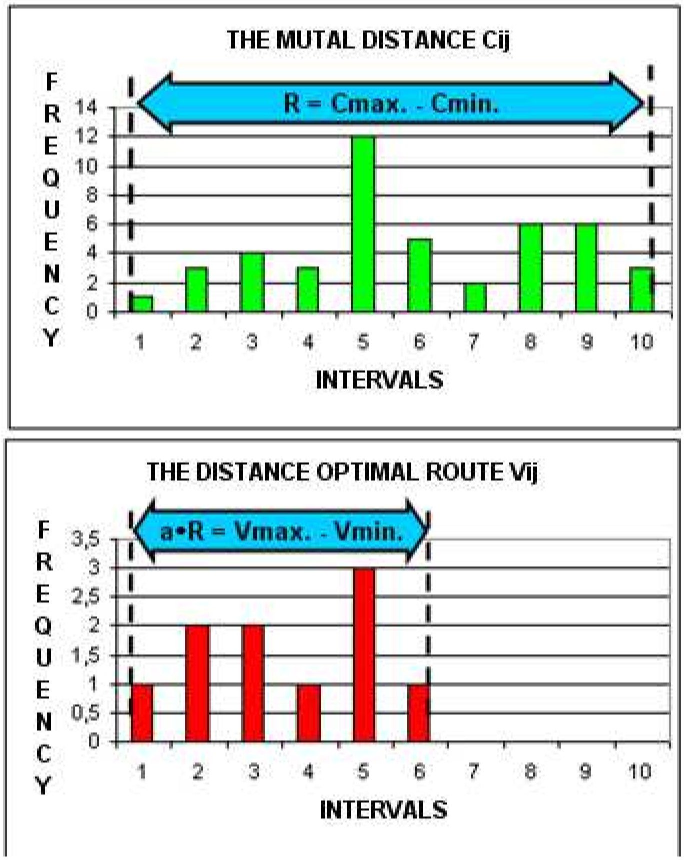
The main idea of this paper also described model is empirically discovered regarding the legality of the traveling salesman problem. I wondered distance matrix C between ten real points (tab. 1). Because the traveling salesman problem has various modifications, it is noted that it is a symmetric matrix, and apply here triangle inequality (e.g. to find the optimal order of processing orders these conditions apply). The distance matrix was determined by the maximum and minimum value, the difference represents the variation range R. For these ten points were determined by solving a mathematical calculation and distance respectively. paths forming the optimal route eg. from point 1 to point 10 etc. are shown in red, as further illustrated in table 1.

Tab. 1: *Distance matrix showing the distances forming the optimal route*

	1	2	3	4	5	6	7	8	9	10
1		**9,43**	15,30	10,44	14,76	9,22	4,12	5,00	13,00	8,06
2	9,43		20,61	19,70	13,00	18,60	11,40	14,21	**7,07**	17,09
3	15,3	20,61		17,69	12,80	**13,45**	11,18	12,53	18,03	17,80
4	10,44	19,70	17,69		23,35	4,24	11,05	7,07	23,20	**2,83**
5	14,76	13,00	**12,81**	23,35		20,03	12,37	16,28	7,00	21,84
6	9,22	18,60	13,45	**4,24**	20,03		8,25	4,47	20,88	5,10
7	**4,12**	11,40	11,18	11,05	12,37	8,25		4,00	12,65	9,49
8	5,00	14,21	12,53	7,07	16,28	4,47	**4,00**		16,49	5,83
9	13,00	7,07	18,03	23,20	**7,00**	20,88	12,65	16,49		21,02
10	8,06	17,09	17,80	2,83	21,84	5,10	9,49	**5,83**	21,02	

Lenght of the route: 70,79 m

The variance of R was divided into 10 intervals, which is the number, which is easy to work and in terms of use for other purposes has proved (some references e.g., that the appropriate number of intervals is the square root of the number of elements, as shown in ([7], p. 10), which corresponds to approximately 10 intervals).



Graph 1: *Comparison of the frequency distribution, Source: [6]*

In the thus defined intervals were determined frequency and the first elements of the distance matrix c_{ij} and second paths ten v_{ij} forming an optimum route as illustrated in Figure 1. From Figure 1 it is evident a very important finding: all elements v_{ij} frequency, i.e. the resulting paths is concentrated in the first five alternatively six intervals. This phenomenon was confirmed by a number of experiments (random points) and can state that has the force of roughly ninety percent of cases. And it is a success because it can be hypothesized that there is a high probability decisive about which path belongs and who does not belong to the optimal route. The criterion for this division is the upper limit of the sixth interval, which can be expressed as one which can be generalized to any upper limit of the interval see formula 2.

$$C_{\min} + 0{,}6 \bullet R = \text{Critical limit} \qquad (1)$$

$$C_{\min} + a \bullet R = \text{Critical limit} \qquad (2)$$

The existence of such a criterion proved to be a potential tool to guide the search of feasible solutions random blocking less likely route, which favors promising path. The result is multiple acceleration searching optimum solution.

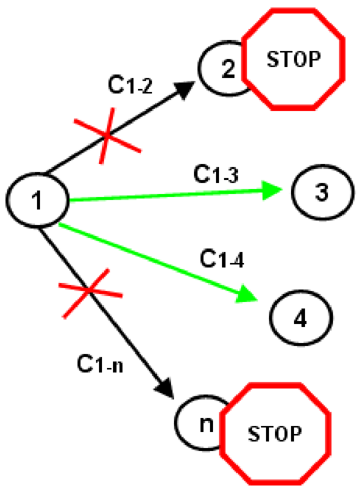


Fig. 2: *Sample rejection of some routes declining levied range of options*

At this point it is necessary to make a few comments. The determination of the correct interval, ie. the value of operator selection and will even mention later. The frequency was determined as absolute, empty values on the diagonal into it and were not counted because it is a symmetric matrix C, the frequency divided by the value of 2, which does not affect the proportionality graph 1.

2.2 BENEFITS OF SELECTION IMPROBABLE SOLUTIONS

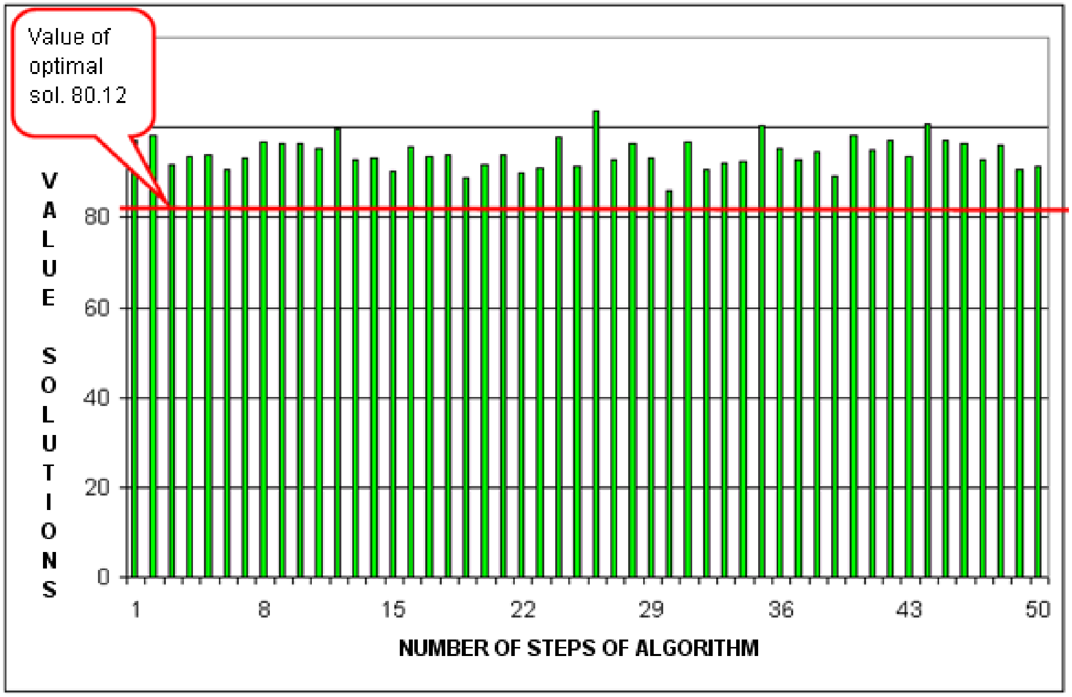
If they are randomly selected feasible solution of the traffic problem of finding the shortest route will succeed only after many attempts. Let us now quantify. The total number of possibilities is given by 3 and is based on the number of combinations at each step, which is halved, as has been said distance matrix is symmetric, which implies that each task has two solutions differ in the direction of passing along the same route (two solutions affects one combination).

$$k = \frac{1}{2}(n-1)! \qquad (3)$$

Then, the number of combinations amounts to 10 181 seats 440. Thus, the probability of finding an optimal solution in the first step is 1: 181,440, and in general it can be expressed depending on the number of steps (formula 4).

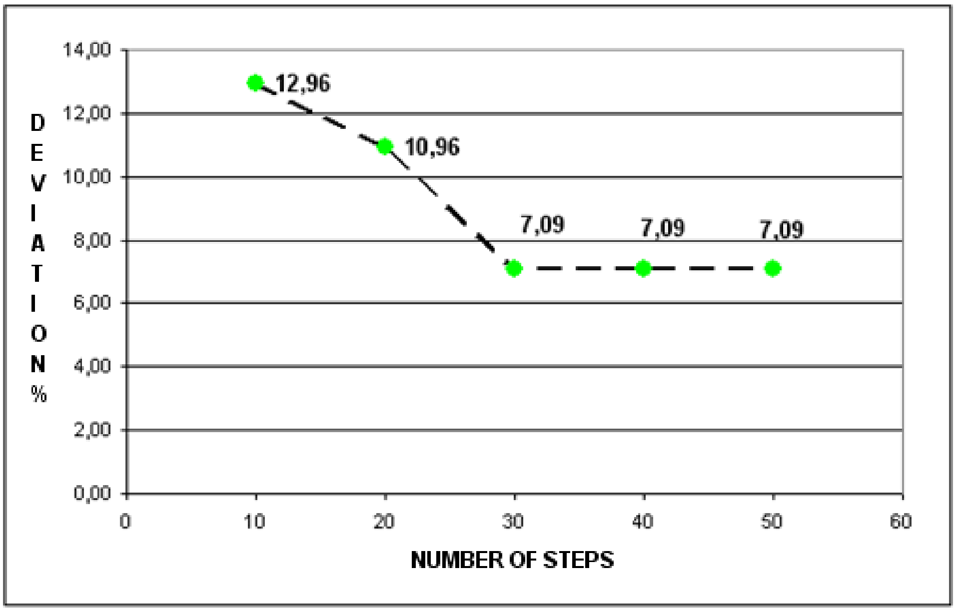
$$P(k) = \frac{k}{181\,440} \qquad (4)$$

This probability can be increased, parallel search. In my model in each step 500 finds feasible solutions from them to choose the best ie. With the shortest route (1 step represents 500 solution) will be considered throughout the rest of this article. This will likely started up the optimal solution in the first step increases to P (500) = 0.00276, which is still weak instrument for practical deployment.



Graph 2: *Values in the search for optima solutions*

After fifty steps (where each step represents 500 solution), from which it selects the best is the best solution reached deviation α = 7.09% of the optimal solution, which was the setting for the points calculated using the exact model published in [4]. Another way to interpret the values from this experiment provides in Figure 3. It is important to note that on the basis of a single test can not be concluded, therefore, the test was repeated for the other points having different mutual distances and the results were always similar. Thus assembled engine found not entirely bad solution, but not optimal solution.



Graph 3: *Deviation from finding the optimal route (%)*

Offering the course, the question of increasing the numbers of parallel solutions, however, this quantitative approach will increase the likelihood hence the number of steps linearly.
More effective is a qualitative change in the search engine that brings sudden reduction in the number of steps and the implementation of knowledge represented by a higher degree of probability of those roads that form the optimal route. The principle is then a simple matrix of the values of C are calculated variation range R, which can be reduced by the aforementioned selection, and operator. If this is set to a value of 0.5 is the upper limit of the fifth interval between critical and completely random ceases to be entirely random, because ways higher than the critical threshold have a lower probability of being selected, all within acceptable solution - the result must form a route, which is ensured by a series of conditional functions. These principles can be used in any environment, for simplicity was elected as user-friendly and transparent MS Excel, which is not primarily intended to complex simulations and optimization propočtům, which has just developed to emphasize the principle of quality solutions.
From point 1 is the future direction of a randomly determined using a random number generating function. The following points 2-10 are listed in the column and a random number before each item makes a selection, chosen such a solution that fits the highest random number. This principle is supplemented by a system of double regulation: if the journey is inadmissible solutions random number is reduced by the value of 4, if the path length is a critical limit, the random number is reduced by the value of 2 if the path is a feasible solution, and its length is below the critical limits, then it is only a random number. It is obvious that the "battle highs" has the greatest chance of a random number, a random number value reduced by the two has a chance „only fight" with other random numbers decreased by the value of 2 (the situation when a selected combination of the first points „ exhausted" a path with less than the critical length), it is logical that the numbers humiliated by the value 4 in this „figur“ can not succeed and thus ensures that the result will always feasible solution. This principle is repeated in ten steps behind determining the route between the ten points where the resulting point is already in the following steps may occur, which is ensured by reducing the mentioned values of the random number by the value 4. Such a solution is underneath 500 and the result is the achievement of optimal solution have an average of over 4 steps and a maximum of seven steps. Correct identification improbable waveforms optimal solution

would reduce the number of options from which to choose factorial - see formula 3. The functioning principle of the key elements of the program illustrated in Fig. 3.

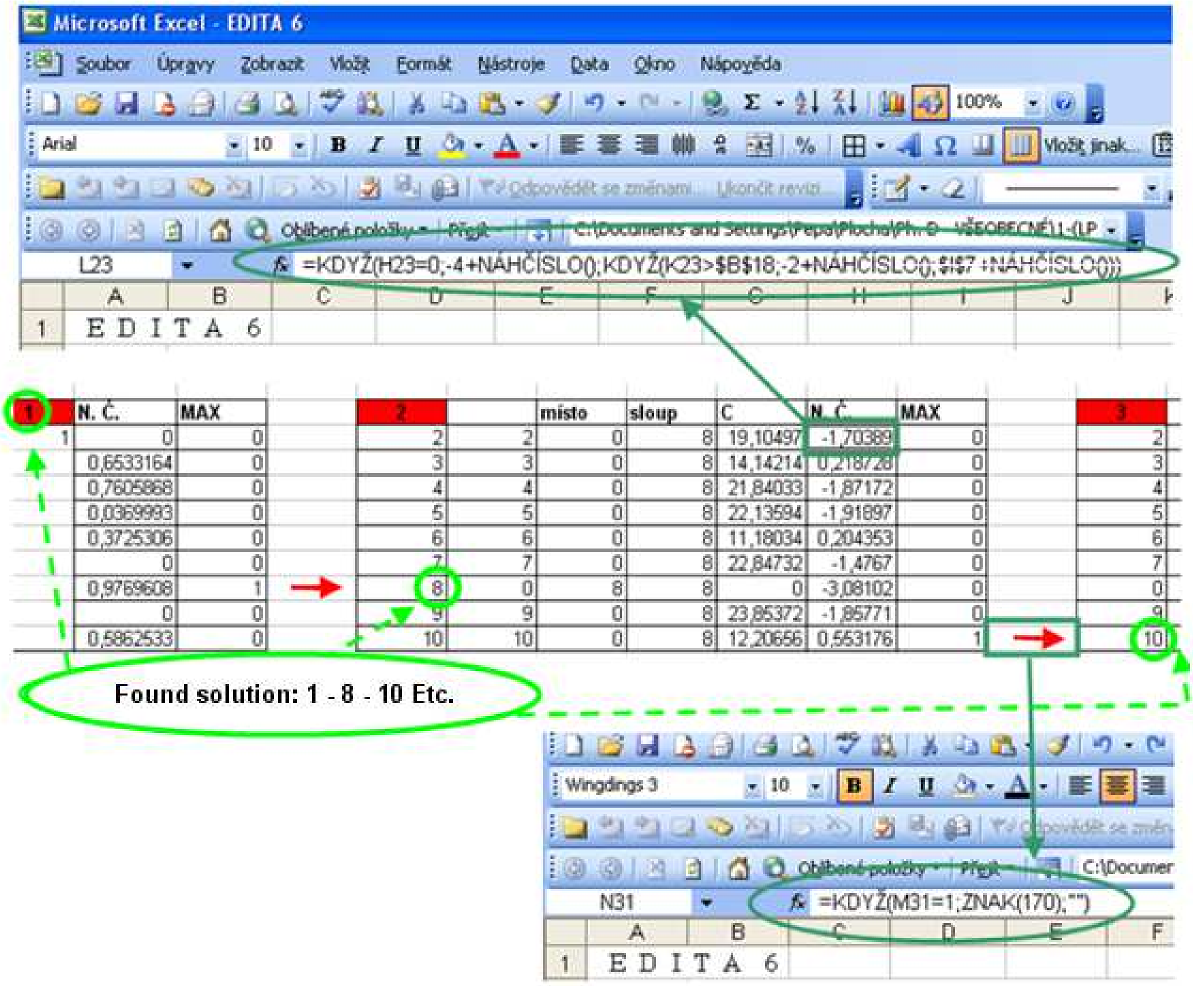


Fig. 3: Sample the main principles of the functioning of the search program

Source: own - model Edit 6.2

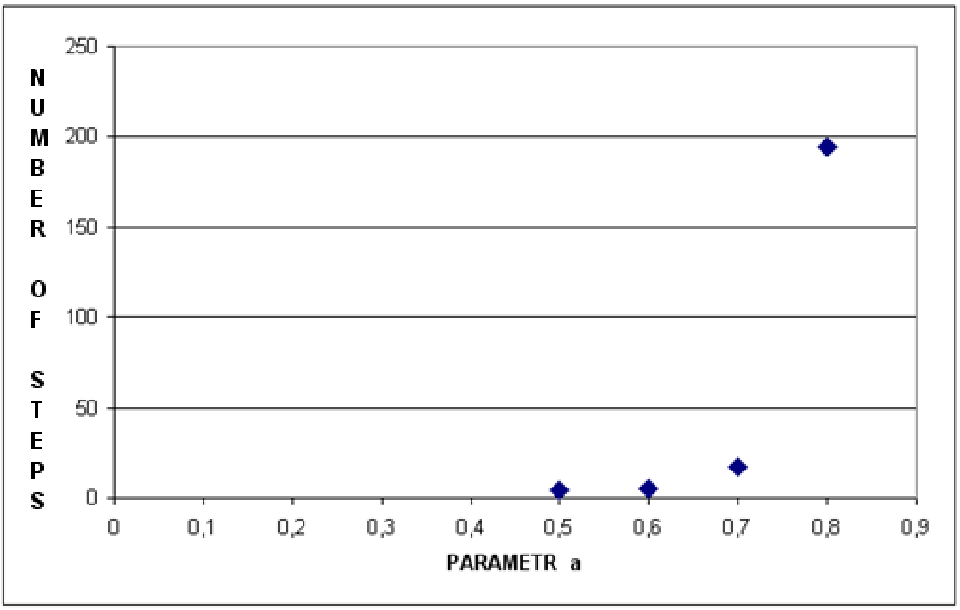
3. VALUE OPERATOR SELECTION

3.1 USE BASIC REGULARITIES

Determination of the critical limits which reduces the variation range of values of path lengths (the distances) depends on the value and operator selection. When a = 0.5, and this is a case where the integrity path lengths forming the resultant trace really only the first five intervals found solution very quickly. However if the frequency distribution of these trips into six intervals, it means slowing the speed of finding the optimal solution, because the path or paths that lie above the critical level, when choosing the optimal solution disadvantage. Therefore, it is safer to set this operator to a = 0.6 representing shot six intervals, which is approximately the same number of steps and increased certainty that the optimal solution not prepare. Of course, there are infrequent cases when the optimum route consists of a relatively long path, which lies in the interval from 7 to 8. The problem is that when the operator selection setting to a higher value begins to increase sharply the number of options through which the resulting route guide and therefore is to find the optimal solution requires more steps, as demonstrated by the test results summarized in table 2 and graph 4.

__Tab. 2:__ Influence value operator selection on the number of steps the optimum solution

Operator selection – a	0,5	0,6	0,7	0,8
Numer of steps - k	4	5	17	194



***Graph 4:**Effect of operator selection of the number of steps to the optimum solution*

If the operator selection directly equals one, a situation where there is no regulation as is evident from Figure 4 the values a = 0.8 and 0.9 already very close to this condition.

3.2 TO USE SPECIÁL STATISTICAL CHARACTERISTICS

The value of operator selection General can not be determined as a = 0.6 for all cases. The arrangement of points in a plane can be of various types and it is necessary to classify the different types and each type assign the appropriate value to operator selection. As a very beneficial showed asymmetry coefficient α_3. The coefficient of asymmetry is given by formula 5 and tells us how much the frequency distribution deviates from the normal distribution ([7], p. 29).

$$\alpha_3 = \frac{m_{3,x}}{s_x^3} = \frac{\dfrac{\sum_{i=1}^{k}\left(x_i - \overline{x}\right)^3 \bullet n_i}{\sum_{i=1}^{k} n_i}}{\left(\sqrt{\dfrac{\sum_{i=1}^{k}\left(x_i - \overline{x}\right)^2 \bullet n_i}{\sum_{i=1}^{k} n_i}}\right)^3} \qquad (5.)$$

Many attempts I was able to confirm the hypothesis that the operator selection should be set depending on the value of the coefficient of asymmetry. Dependence on other indicators such as the coefficient of variation or the concentration factor I could not confirm. The coefficient of variation Vx expresses the ratio between the standard deviation of lengths and their average value see formula 6, concentration ratio tells you whether the frequency distribution of lengths in matrix C sharper or flatter than the normal distribution described in formula 7.

$$V_x = \frac{s_x}{\bar{x}} = \frac{\sqrt{\dfrac{\sum_{i=1}^{k} \left(x_i - \bar{x}\right)^2 \bullet n_i}{\sum_{i=1}^{k} n_i}}}{\dfrac{\sum_{i=1}^{k} x_i \bullet n_i}{\sum_{i=1}^{k} n_i}} \qquad (6.)$$

$$\alpha_4 = \frac{m_{4,x}}{s_x^4} = \frac{\dfrac{\sum_{i=1}^{k} \left(x_i - \bar{x}\right)^4 \bullet n_i}{\sum_{i=1}^{k} n_i}}{\left(\sqrt{\dfrac{\sum_{i=1}^{k} \left(x_i - \bar{x}\right)^2 \bullet n_i}{\sum_{i=1}^{k} n_i}}\right)^4} \qquad (7.)$$

Demonstration experiment described in table 3 and 4.

Tab. 3: *The number of steps of the algorithm depending on the value and effect parameters α3, α4, Vx*

α_3	-0,3519				-0,3131				-0,0316			
α_4	1,7116				1,9107				1,9632			
V_x	0,423				0,4029				0,397			
a ➔	**0,3**	**0,4**	**0,5**	**0,6**	**0,3**	**0,4**	**0,5**	**0,6**	**0,3**	**0,4**	**0,5**	**0,6**
k	No solution	No solution	2	10	1	8	2	6	2	1	1	2
			2	10	1	7	1	4	9	1	1	10
			1	5	2	2	1	19	3	1	2	4
			1	8	2	4	1	12	10	2	1	3
			2	5	1	7	1	8	2	1	2	8
$\bar{k}$	-	-	1,6	7,6	1,4	5,6	1,2	9,8	5,2	1,2	1,4	5,4

Tab. 4: *The number of steps of the algorithm depending on the value and effect parameters α3, α4, Vx*

α_3	0,0395				0,2199				0,5825			
α_4	1,7815				1,9424				1,9795			
V_x	0,515				0,458				0,671			
a ➔	**0,3**	**0,4**	**0,5**	**0,6**	**0,3**	**0,4**	**0,5**	**0,6**	**0,3**	**0,4**	**0,5**	**0,6**
k	No solution	7	33	67	2	4	11	14	40	60	90	No solution
		1	39	51	2	12	13	40	36	52	111	
		7	29	74	2	8	4	27	31	78	84	
		2	30	82	2	8	9	43	43	64	102	
		7	35	59	1	6	11	19	33	59	98	
$\bar{k}$	-	4,8	33,2	66,6	1,8	7,6	9,6	28,6	36,6	62,6	97	-

Tables 3 and 4 are sample from a larger experiment on them and explains the principle of the experiment. There is also clearly visible outcome of the experiment. Each arrangement of a set of points gives different distances between points of which is determined by the coefficient of asymmetry and its amendments related to the change selection operator, so that the number of steps for finding the optimal solution as low as possible. This observation is summarized in table 5.

Tab. 5: *Empirical findings appropriate value operator a minimize the number of steps for different types of asymmetry α3*

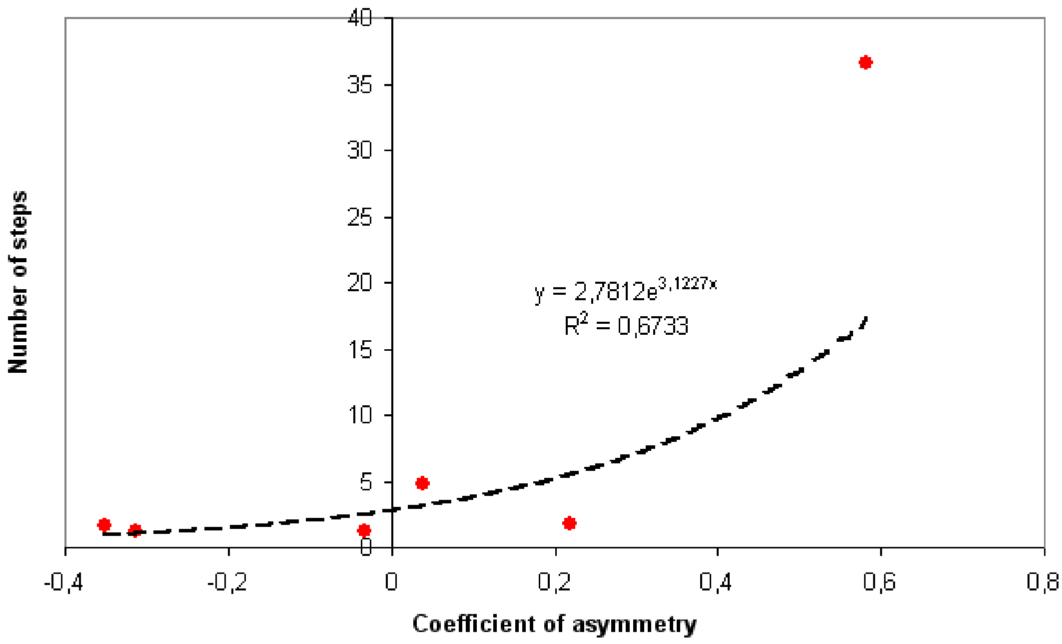
α_3	(- ∞; - 0,2)	‹-0,2; +0,2›	(+ 0,2; + ∞)
a	0,5	0,4	0,3

The incorporation of this knowledge in the described model will mean a lower number of steps in solving a algorithm. Model based on a calculation of the asymmetry coefficient α_3 automatically sets the appropriate selection operator a.

3.3 DETERMINATION EXPECTED NUMER OF STEPS OT THE ALGORITHM

For each heuristic method, the known solution, which is the lowest of all, we assume that it is optimal, but definite assurance that this really is not. Therefore crafted model I test so that its results are compared to results from designated exact calculation and determine what session model number of steps needed to reach the optimal solution value. If the optimal solution is found eg. On average in ten steps, then if the model in solving the tasks carried out 15 steps, there is a certain degree of probability, that the best solution found is the optimal solution.

From table 4 it is evident interesting fact, if the frequency distribution of the distances in matrix C which indicates rightward asymmetric negative coefficient α_3 then a low number of steps, with increasing α_3 number of steps increases. For α_3 positive representing the leftward asymmetric frequency distribution is therefore the number of steps higher. This phenomenon describes the graph 5.



Graph 5: *Number of steps of the algorithm (k) depends on* α_3

Values are interleaved exponential curve describing the trend where the accuracy of the correlation between the points and the curve is given by the value of R^2 = 0.6733, which corresponds to the value for extracting correlation index I = 0.821 (the value closer to the value I = 1, the curve of the correlation dependence tighter). I value = 0.821 can be considered sufficient when estimating the number of required steps of the algorithm detected using an exponential curve (function) the formula 8.

$$k = 2{,}7812 \bullet e^{3{,}1227 \bullet \alpha_3} \qquad (8.)$$

This knowledge will help the user to identify whether the best solution reached already represents optimum or whether it is necessary to look.

3.4 TESTING FUNCTIONALITY OF THE MODEL

Principles have been described as model works and what uses algorithms. Now make a test model, which will be tested whether it is able to find in any situation optimal solution with an average of ten steps with a probability of 95 %. The test was modeled 36 random positions of points for which the optimal solution and the number of steps were statistically processed in order to carry statistical testing see tab. 6.

Tab. 6: *Test model - the number of steps in the search for solutions*

Number	n	36
Average	$\bar{k}$	7,87
Standard deviation	s	12,56

I will test mean values using the formula 9.

$$U = \frac{\overline{(k} - \mu o)}{s} \bullet \sqrt{n} = \frac{7{,}87 - 10}{12{,}56} \bullet \sqrt{36} = -1{,}018 \qquad (9.)$$

Hypothesis is true, if the inequality see formula 10.

$U \geq u_{(1-0,05)}$, where $u_{(1-0,05)} = -u_{(0,05)} = -1{,}65$; then $-1{,}018 \geq -1{,}65$ (10.)

Test this hypothesis, so a 95 % probability model can solve any problem with the number of steps lower than 10.

4. CONCLUSION

The paper describes a mathematical abstraction problem of finding a route between a set of points, where the points represent the mutual distances at which they are applied statistical regularities paying for sightseeing transportation problem. These patterns help guide the random choice of solutions, so the best possible or optimal solution is found quickly. This is supported by a classification problem again carried out using statistical values calculated from the matrix of mutual distances with such valuable information showed asymmetry coefficient frequency distribution of distances mentioned. Then the model can be programmed to automatically respond to the appropriate type of problem that can adjust itself operator selection, whose value affects the number of steps by extension, the speed of finding solutions.

In all tests, see the generated model most optimal solution with a very low number of steps, which represents a time less than a minute, which is faster than the time it needs a tool solver to find a solution using a binary linear programming.

All tests are performed on the points whose coordinates were determined as random numbers.

The result is a practical tool, able to solve this combinatorial problem that can be encountered in practice in many industries, mostly in the optimization of logistic routes. Human imagination and intuition can do with a certain imprecision bring a solution to this and similar problems, as stated in ([5]), but this problem with more places we would intuitively correct solution they could find.

Post considers other issues for potential research eg. How the program could work with the application of these principles to the higher number of points than 10. Alternatively, as it increased the number of steps a number of points. Very interesting issue is the application of statistical methods to the distance matrix and results and looking for connections that contribute to the effectiveness of heuristic methods for solving this problem. Another way is to confirm or refute the determination of new approaches in other modifications orbital traffic problem like. Asymmetrical problem, void the triangle inequality, dynamization and reoptimalizační approaches to this problem as stated in ([2], pp. 60 - 61) and the special area comprises more circuit variants of this problem and many other variations depending on the application this problem.

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Symbols:

a	- Operator selection	(1)
α	- Deviation from the optimal solution	(%)
α_3	- Coefficient of asymmetry	(1)
α_4	- Concentration factor	(1)
C	- Distance matrix	(m)
c_{ij}	- Element distance matrix	(m)
I	- Correlation index	(1)
k	- Number of steps of the algorithm	(numer of steps of algorithm)
$\bar{k}$	- Average number of steps of the algorithm	(numer of steps of algorithm)
P(k)	- Probability of finding the optimal solution in k steps	(%)
R	- Variation range	(m)
s	- Sample standard deviation	(numer of steps of algorithm)
V	- Matrix resulting distance	(m)
v_{ij}	- Element matrix resulting distance	(m)

References:

[1] COOK, W., J. *Po stopách obchodního cestujícího. Matematika na hranicích možností.* 1. vyd. Praha: Dokořán, 2012. 255 s. ISBN 978-80-7363-412-4.

[2] FIALA, P. a kol. *Operační výzkum nové trendy.* 1. vyd. Praha: Professional Publishing, 2010. 239 s. ISBN 978-80-7431-036-2.

[3] JABLONSKÝ, J. *Programy pro matematické modelování.* 1. vyd. Praha: VŠE, 2007. 260 s. ISBN 978-80-245-1178-8.

[4] KOŠŤÁLEK, J. *Matematické algoritmy jako nástroje efektivního řízení procesů v podnicích*. In sborník: Mezinárodní vědecké konference, Karviná: SU, 2013. ISBN 978-80-7248-901-5.

[5] KOŠŤÁLEK, J., REJF, L., VANIŠ, L. *Limity lidského myšlení v rozhodovacích procesech spojených s podnikovým řízením.* In sborník: 15. konference, Integrované inženýrství v řízení průmyslových podniků, Praha: ČVUT v Praze, Ústav řízení a ekonomiky podniku, Brno, 2014. ISBN 978-80–01–05537-3.

[6] KOŠŤÁLEK, J., STIEBEROVÁ, B., KOŽÍŠEK, J. *Aplikace speciálních metaheuristických metod na řešení okružního dopravního problému*. In sborník: Mezinárodní Masarykova konference pro doktorandy a mladé vědecké pracovníky, Hradec Králové: Magnanimitas, 2014. ISBN 978-80-87952-07-8.

[7] KOŽÍŠEK, J., STIEBEROVÁ, B. *Statistická a rozhodovací analýza*. 1. vyd. Praha: ČVUT, 2008. 252 s. ISBN 978-80-01-04209-0.

[8] KOŽÍŠEK, J. *Statistická analýza*. 4. vyd. Praha: ČVUT, 2002. 249 s. ISBN 80-01-02496-2.

[9] TÖPFER, P. *Algoritmy a programovací techniky.* 2. vyd. Praha: Prométheus, 2010. 300 s. ISBN 978-80-7196-350-9.

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