The effect of axles´ flexibility to simulation´s results of the rolling stock ride

Vliv poddajnosti nápravy na výsledky simulace jízdy kolejového vozidla

Ing. Karel Zinke

Vedoucí práce: doc. Ing. Josef Kolář CSc.

Annotation

Mathematical model, describing the movement of the rail vehicle, could be very complex system of differential equation. One of the most important part of this system is mathematical description of the contact wheel and rail. Usually this problem is mathematically interpreted without any consideration to beam flexibility of the wheel sets´ axels. This publication is going to describe, how the flexibility of the axle is changing the shape of the wheel profile and how this shape´s change affects the mathematical simulation´s results of the riding vehicle.

Keywords

Wheel set, axle flexibility, wheel profile, railway vehicle axle bending

Abstrakt

Matematický model, popisující pohyb kolejového vozidla, může být velice komplexní systém diferenciálních rovnic. Jedna z nejvíce důležitých částí tohoto systému je matematický popis kontaktu kolo-kolejnice. Většinou je tento problém matematicky popsán bez ohledu na ohybovou poddajnost nápravy. Tato publikace popisuje, jak ohybová poddajnost nápravy ovlivní tvar profilu kola a jak se změna profilu promítne do výsledků matematické simulace jízdy vozidla.

Klíčová slova

Dvojkolí, poddajnost nápravy, profil dvojkolí, ohyb nápravy kolejového vozidla

1. Introduction - Problem formulation

Mathematical formulation of rail vehicle motions is very complex. Specific problem for railways vehicle is contact between wheel and rail, and its formulation. Mostly the wheel profile and rail are described by templates or measured profile on real vehicle, with no attention to axle flexibility. So the axle is expected to be rigid. Let´s have a look, what could happened, when we expect axle to be flexible.

Let´s apply static loading caused by the mass of the vehicle to a flexible axle and calculate how are the wheel sets dealing with static load. We expect the bogie is equipped with outer frame. The static load of the vehicle acting on both ends of wheel set (contact wheel set and axle box.

Let´s mark the forces F_a and F_b. For vehicle equipped with two bogies (each bogie has two wheel set) we can simply assume the value of the forces F_a and F_b are equal to mass of the vehicle reduced by a mass of the four wheel sets and divided by 8 (amount of static forces). This forces cause reaction forces (marked as R_a and R_b), which are acting between wheel and rail, and thanks to symmetry, they are equal. This interpretation is formulated as static, so no dynamic forces, etc. are taking into consideration. Scheme of this formulation can be seen in figure 1.

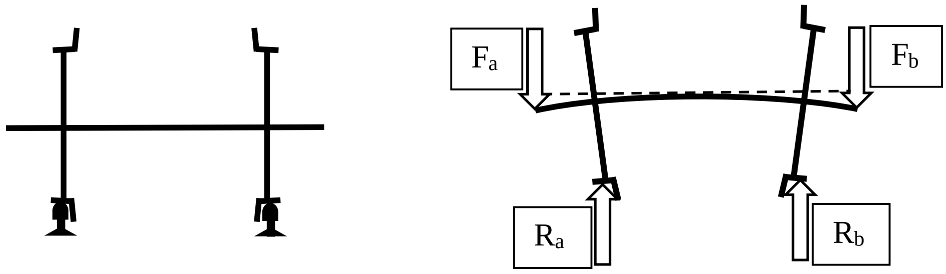

Figure 1. Scheme of wheelset and its expected bending deflection caused by forces F_a and F_b – these represent static load

Now we reformulate previous definition as a static load on the beam. The beam is overhanging beam and is loaded by two symmetric forces F_a and F_b on both ends and is resting on a pin support and a roller support, which are compensating the contact between wheel and rail. In these support the reacting forces are caused, we mark them R_a and R_b. Scheme for this formulation represents Figure 2.

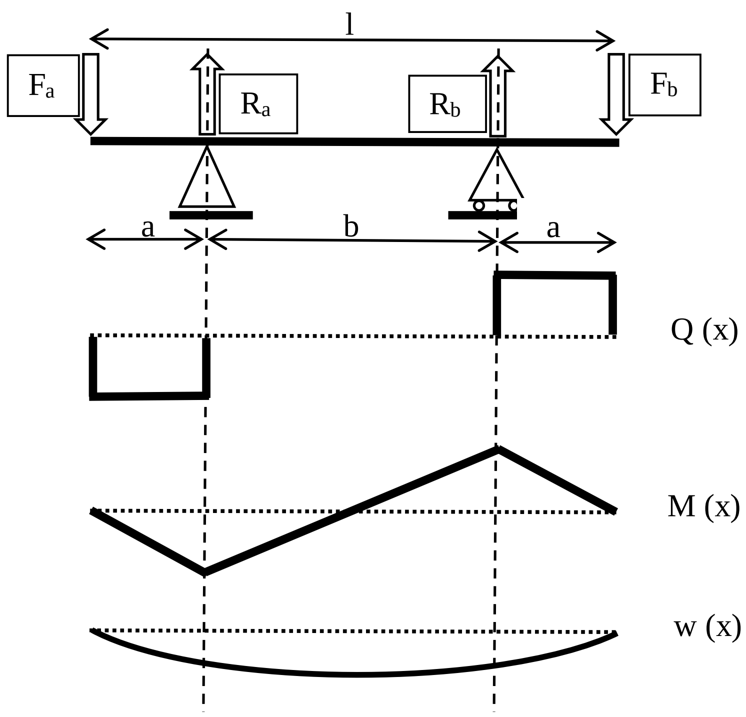

Figure 2. Overhanging beam representing static load acting on the axle with subjected shear force, bending moment and deflection.

1.1. Analytical solving

We apply the equation of equilibrium – the sum of forces should equal zero as in equation 1.

$$\sum Fi = 0 \tag{1}$$

By substitution the forces to the equation we obtain formula:

$$F_a - R_a - R_b - F_b = 0 \tag{2}$$

As we are expecting the full symmetry of the wheel set, the acting force will be equally high. To reaction forces this is applied also, the magnitude of the forces are the same, but they will have opposite direction. We formulate magnitude of the forces by equation 3.

$$F_a = -R_a = -R_b = F_b \tag{3}$$

Then we determine the course of the share forces *Q*, which is described by equation 4, and from thus we derive the formula for bending moment *M*. Bending moment is interpret by equation 5, where *l* represents distance between forces F_a and F_b (middle distance between axle boxes) and *a* represents distance between F_a and R_a or F_b and R_b (middle distance between axle box and wheel).

$$Q(x) = \begin{cases} Fa & 0 \leq x \leq a \\ 0 & a < x \leq (l-a) \\ Fb & (l-a) < x \leq l \end{cases} \tag{4}$$

$$M(x) = \begin{cases} Fa\,x & 0 \leq x \leq a \\ Fa\,x - Ra\,(x-a) & a < x \leq (l-a) \\ Fb\,(l-x) & (l-a) < x \leq l \end{cases} \tag{5}$$

The beam´s deflection *w* could be described by The Euler-Bernoulli theorem, which is shown in equation 6:

$$\frac{d^2}{dx^2}\left(EI\frac{d^2w(x)}{dx^2}\right) = q \tag{6}$$

$$I = \frac{\pi d^4}{64} \tag{7}$$

We expect the moment of inertia *I* is constant all over the length of the axle and is given by the equation 7. Also elastic modulus *E* is expected to be constant and for steel it equals: $E_{(steel)} = 2{,}1\,.\,10^{11}Pa$. Then we can determine the elastic deflection function for the beam as is described by equation 8 and angle deflection by equation 9.

$$w(x) = \begin{cases} -\dfrac{\frac{1}{6}Fa\,x^3 + C_1x + C_2}{EI} & 0 \leq x \leq a \\ -\dfrac{\frac{1}{6}Fa\,x^3 - Ra\,x^2(\frac{1}{6}x - \frac{1}{2}a) + C_1x + C_2}{EI} & a < x \leq (l-a) \\ -\dfrac{Fb\,x^2(\frac{1}{2}l - \frac{1}{6}x) + C_1x + C_2}{EI} & (l-a) < x \leq l \end{cases} \tag{8}$$

$$\varphi(x) = \begin{cases} -\dfrac{\frac{1}{2}Fa\,x^2 + C_1}{EI} & 0 \le x \le a \\ -\dfrac{\frac{1}{2}Fa\,x^3 - Ra\,x(\frac{1}{2}x - a) + C_1}{EI} & a < x \le (l-a) \\ -\dfrac{Fb\,x(l - \frac{1}{2}x) + C_1}{EI} & (l-a) < x \le l \end{cases} \tag{9}$$

To equation 9 we substitute initial condition φ (x=l/2 or x=a+b/2)=0, so we obtain equation 10, from which we can calculate the angle deflection of the axle.

$$\varphi(\mathrm{x}) = \frac{Fa}{2E\frac{\pi\, d_a^4}{64}}a\,b \tag{10}$$

1.2. FEM calculation

To verify previous calculation and making it more accurately (in reality the moment of inertia is not constant) another calculation by finite element method was used. The flexibility of the axle were simulated in ANSYS Workbench 13. CAD 3D model of the axle with wheels on both sides were created in Pro/Engineer 5.0 (from PTC). The model was imported to ANSYS environment and was mashed (solid tetrahedrons elements were used). On figure 3 we can see the mesh of the wheel set´s model that was created in ANSYS. The static forces acting to axle and supports of the wheel set were input. The initial condition for simulation correspond to previous analytical calculation.

The simulation was done for two kind of axles – first one was axle used on narrow gauge operating bogie and second one axle used on standard gauge operating bogie. First one represents cape gauge (1067mm) and simple conical wheel profile, second one standard gauge (1435mm) and advanced profile shape UIC-S1002. For both axles the deformations caused by static load (mass of loaded vehicle) were calculated and virtually measured. Figure 4 is showing the deflection of the wheel set.

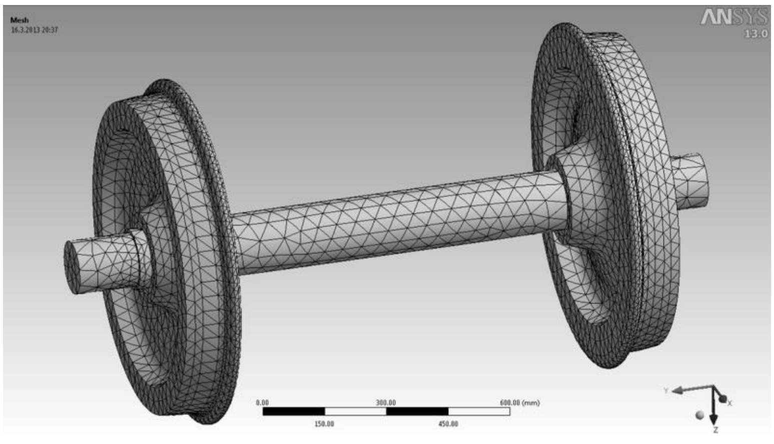

Figure 3 Model of wheel set in Ansys and its mesh.

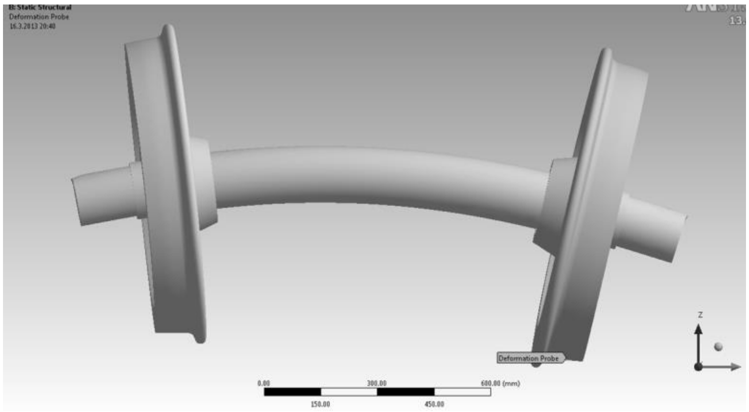

Figure 4 Deformed wheel set calculated by Ansys

2. Axle flexibility results

The results of both calculation (analytical and by finite element method) are summarized in table 1. The deflection calculated by each method are a little bit different. We expect these differences are caused by different moment of inertia (analytic method expected the moment of inertia to be constant). As the results difference are not so high, we assume the calculated deformation as verified. The deflection of the axle and its effect to wheel deformation can be seen in figure 5.

Table 1. – Summary of calculations - input values and the results of calculations

Method	Analytics		FEM	
Gauge [mm]	1067	1435	1067	1435
d_a - Axle diameter [mm]	182	140	182	140
a - distance [mm]	200	285	200	285
b - distance [mm]	1120	1500	1120	1500
I - moment of inertia [mm^4]	53858648	18857410	53858648	18857410
R_k - Wheel radius	400	400	400	400
Wheel direction deformation in y-axe [mm]	0.5151	0.6161	0.4132	0.5410
φ - Angle deflection [°]	0.0738	0.0882	0.0592	0.0775

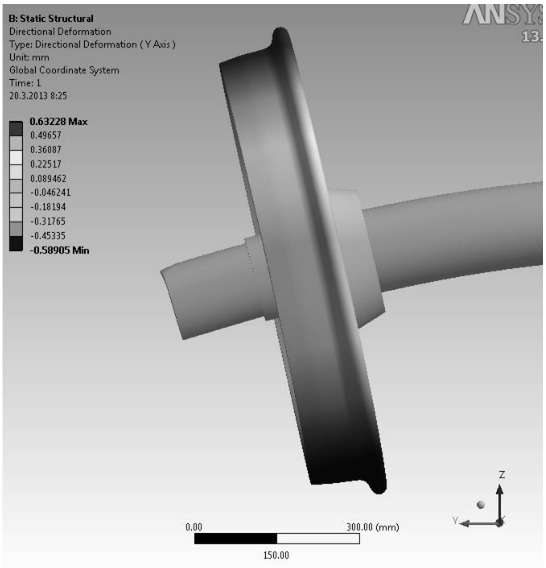

Figure 5 Example of Ansys graphic interpretation of deformation results on the wheel

3. Wheel profile transformation

Both results (by analytics method and finite element method) are adverting to axles´ deflection, which is canting the wheels. This cant can be calculated from lateral deformation of the wheels calculated previously. We estimate the angle of wheel deflection and by those we transform the wheel profile shape. The values of deflection calculated by finite element method have been taken. Two wheel profiles were chosen as origin – conical wheel profile used in Indonesia and mostly used profile on standard gauge in Europe UIC-S1002.

For estimation how the wheel profile looks after the axle is deformed the mathematical software Matlab 2010b (by MathWorks) was used. The profiles were input and rotated by the calculated angle of deflection. The center of rotation was estimated as a point of intersection axle lateral axe and its normal passing the contact wheel/rail point.

4. Simulation of the rolling stock ride

For comparison of effect wheel profile shape´s change by axle flexibility the simulation by Multi-body software was chosen. All the simulations were done in Simpack 8.95a, multi-body simulation software created by Simpack AG (previously known as INTEC GmbH). This software is specialized besides classic MBS problems for railways problems calculation such as contact wheel and rail.

The input model was created as two identical bogies connected to vehicle car body. Each of the bogies consists of two identical wheel sets, which are by the primary suspension (spring and damper) connected to frame and from this frame is joint to a bogie bolster by a secondary suspension.

As transformation for two different profiles were done, four different profiles (origin and transformed; conical profile and UIC-S1002) were input to Simpack. Four different tracks were created in order to cover more varied track shape possibility. Each of them begun with straight track and then is smoothly transitioning to the curve (each track has different curve radius – 300m, 750m, 1500m and 3000m). For each track combined with each profile the simulation has been done.

4.1. Simpack results

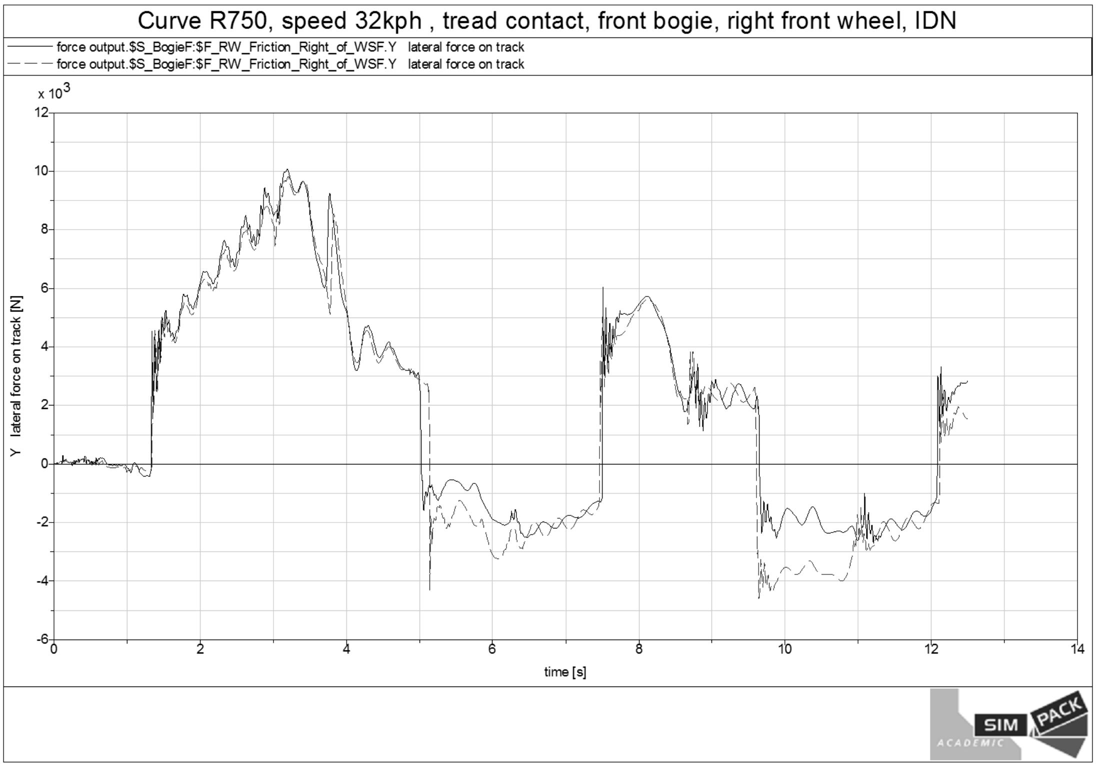

Figure 6. - Course of the lateral force acting in the tread contact between right front wheel of front bogie and rail (rail profile is UIC 54, its cant 1:40 and gauge 1067mm). Vehicle is passing the curve R750 at the speed of 32kph. The conical wheel profile was used.

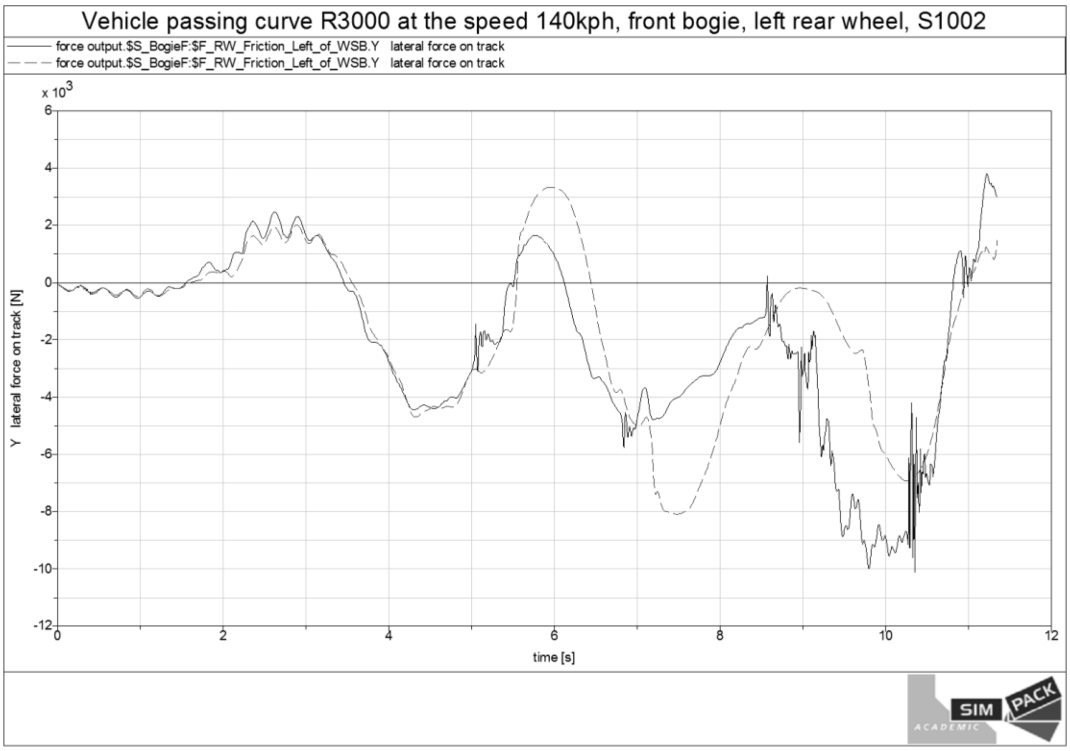

Figure 7. - Lateral force acting in the tread contact between right front wheel of front bogie and rail (rail profile is UIC 60, its cant 1:40 and gauge 1435mm). Vehicle is passing the curve R3000 at the speed of 140kph. The UIC-S1002 wheel profile was used.

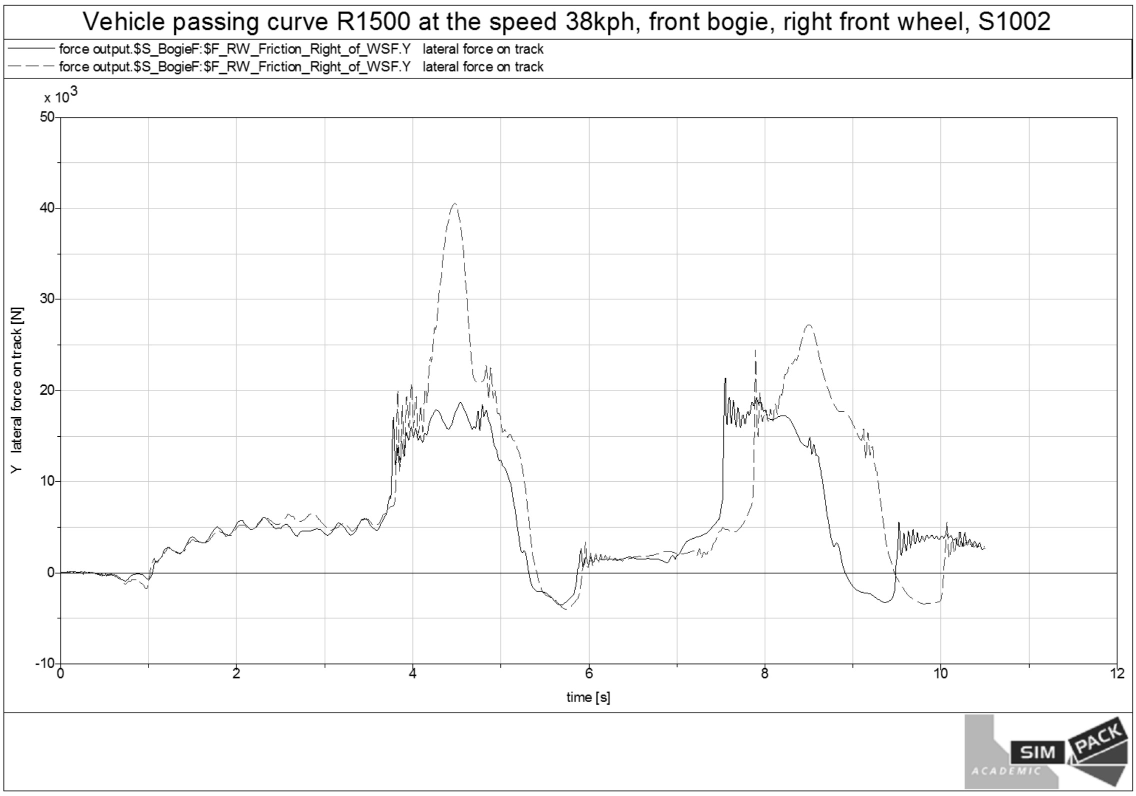

Figure 8. - Lateral force acting in the tread contact between right front wheel of front bogie and rail (rail profile is UIC 60, its cant 1:40 and gauge 1435mm). Vehicle is passing the curve R1500 at the speed of 38kph. The UIC-S1002 wheel profile was used.

5. Conclusion

On figures 6, 7 and 8 you can see three examples of calculated lateral force acting on a track. Mostly the output functions of origin profile and transformed profile are similar to each other, however in some parts they are different. This difference is not so much significant, but it is not negligible also.

We can claim (in reference to the results), that calculated axle flexibility has effect to vehicle´s rail behavior. It will be very interesting to compare the calculation to measured values and to assess, if adding wheel sets´ axle static bending flexibility to simulation has positive influence for correspondence of measured and simulated data. This is author´s future plan target.

List of symbols

I	moment of inertia	(mm^4)
E	elastic modulus	(Pa)
a	distance between Fa and Ra, corespond to distance between axle box and wheel	(mm)
b	distance between Ra and Rb, corespond to distance between tread datums	(mm)
Q	shear force	(N)
M	Bending moment	(N.m)
w	Elastic deflection	(m)

φ	Bending deflection	(°)
R_k	Wheel radius	(mm)
F_a	Static load force	(N)
F_b	Static load force	(N)
R_b	Reaction force	(N)
R_b	Reaction force	(N)
d_a	Diameter of the axle	(mm)
l	Distance between Fa and Fb, corespond to middle distance of axle boxes	(mm)

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