The issue of saturation in control systems using a model function with delay

Ing. Jaroslav Bušek

Supervisor: Prof. Ing. Pavel Zítek, DrSc.

Abstract

This paper deals with the issue of input saturation of controller with a more complicated dynamics using complex structure with both integration and time-delays included. The effect of control input saturation on process control is shown using simulation model in Simulink. Methods used to deal with this issue for conventional controllers are mentioned and tested suggestions for modification of control algorithms for meromorphic functions are introduced.

Keywords

Saturation, linear time-delay systems, control design, anti-windup, internal model control, meromorphic function.

1. Introduction

Most control systems are based on linear models because of easy control design using frequently used methods. These linear models are represented by a set of ordinary linear differential equations although the real plant dynamics usually contain time-delays or nonlinear parts – given by own attributes (i.e. friction) or added during the control system design (i.e. saturation). Then the linear model fits only in a vicinity of operational point at which the linear system has been identified. Therefore it is suitable to reflect these properties also in the model. Linear time-delay systems are widely used because of the presence of time-delays in the control systems is common feature of many physical systems, i.e. network communication, electrical circuits, chemical processes, heat transfer. Involving the delays in the model structure provides accurate results in control design. However time-delays are a source of performance degradation or instability in general. Therefore the control design has to be more precise for a performance improvement which mostly leads to more complicated structure of the controller.

Common phenomena of real systems are also actuator saturation caused by physical or other actuator limitations. Both properties of real plant and actuator properties should be taken in mind when the control system is designed. The existence of input saturation might wind up the pre-designed linear controller and thus deteriorates the performance of the system, and even leads to the loss of stability. Actuator saturation and time-delays may be observed together in control systems; even so it appears that the most of anti-windup results are mainly focused on undelayed systems.

This paper briefly reviews the windup/saturation phenomenon and different ways to compensate it for conventional controllers and especially shows a currently tested modification for controller given by a meromorphic function. This controller is a simplified version of the controller which is designed for areal laboratory time-delayed heat transfer system control using the finite spectrum assignment method (FSA) intended for time-delay systems control design.

2. Control input saturation phenomenon and integral anti-windup

The control input saturation is probably the most usual nonlinearity encountered in control system design because of the physical impossibility of applying unlimited control signals and/or of safety constraints. The effect of saturation is noticeable when the controller is an unstable system. This is always the case for a controller with integral action. Since the feedback loop is broken when the actuator saturates the integrating modes of the regulator may be then drift to undesirable values. The consequences are that it may take a long time for the system to reach equilibrium after an upset (see fig. 1). This phenomenon first noticed in conventional PID control is therefore called integrator windup. It was well known to practitioners of automatic control but has not received much attention from scholars. A basic summary of anti-windup schemes was introduced in [1].

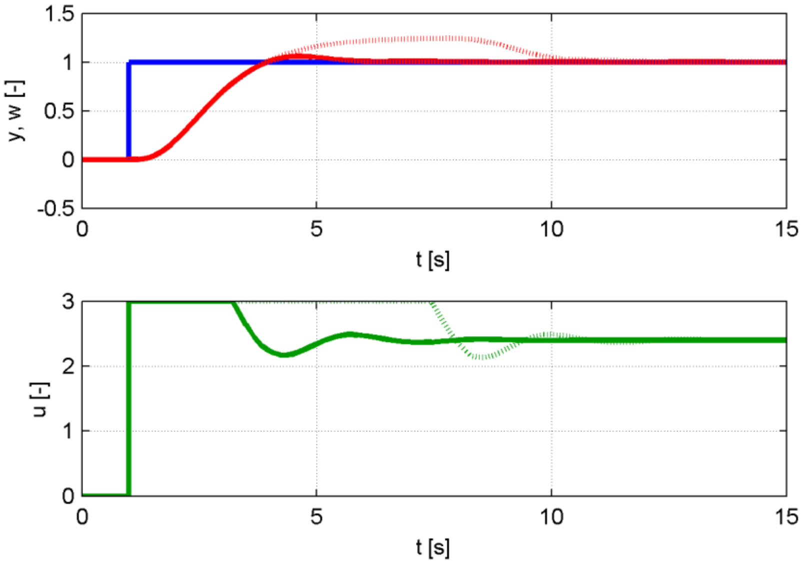

Fig. 1: Illustration of integrator windup (dashed line) and control with anti-windup (solid lane).

Three ad hoc methods how to compensate the integrator windup are listed in the following paragraphs. These methods were used in early years of anti-windup implementation nevertheless they have good properties and they provide the basis for further research.

Incremental algorithms were used in early phase. The control integral action was implemented with the actuator with motor driving the valve directly. Wind up is then overcome by stopping the integration whenever the actuating variable saturates – it means integration stops when the valve stops. When the controllers are implemented by computers a discrete translation of the mechanical design is used.

Back-calculation and tracking [2] generates controller which can be interpreted as having two modes namely, the common control mode when there is no saturated signal so it operates like ordinary controller, and a tracking mode when the integrator is tracked.

Implementation of back-calculation is shown in figure 2. When the output saturates the integral is recomputed so that its new value gives output at the saturation limit.

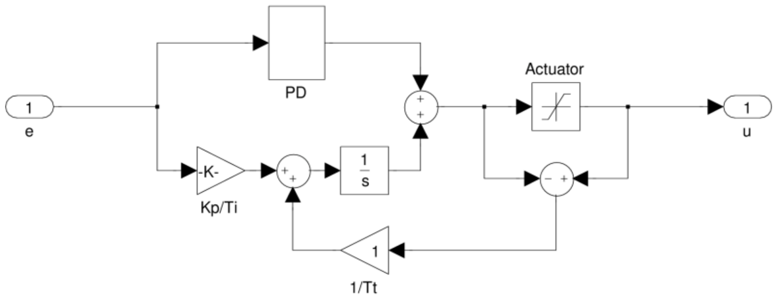

Fig. 2: PID controller with anti-windup based on back-calculation.

Conditional integration is based on applying the integration only when certain conditions are fulfilled – typically: control error is small and actuator does not saturate. The method is usually implemented with integration applied only when the predicted process output is in the proportional band.

3. Heat transfer system control motivation

The motivation for anti-windup research arose when the FSA control design was used on a laboratory plant. The plant is a laboratory heat transfer system which has only one closed heating circuit. It consists of electric flow heater, pump, water/air exchanger and 15 metres long thermally insulated pipeline (see fig. 3). The heat medium (distilled water) is pumped through the electric flow heater which power can be controlled by PWM signal. Water flows from the heater through the long pipeline providing delay from 50 to 300 seconds (depends on the actual pump speed) to the exchanger with two fans (one is controlled by digital signal and the other by analog signal). The pump is located between the exchanger and the heater and has adjustable speed by analog signal.

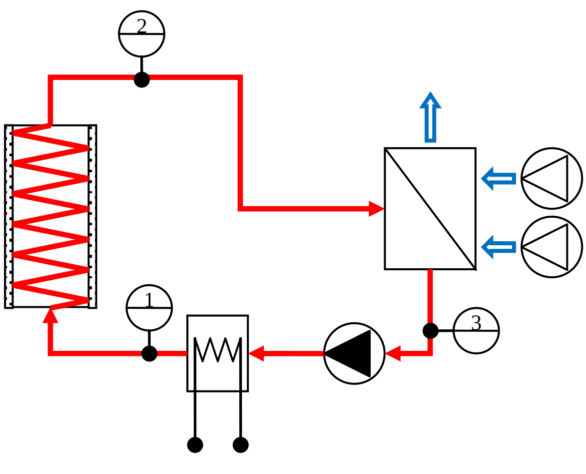

Fig. 3: Scheme of the laboratory heat transfer system (left) and its photo (right).

Three measured temperatures are located at the output of the heater (position 1, ϑ_1), at the end of the long thermally insulated pipeline (position 2, ϑ_2) and at the output of the exchanger (position 3, ϑ_3). The controlled variable $y(t)$ is temperature $\vartheta_3(t)$ at the output of the exchanger. The chosen control signal $u(t)$ is the power of the heater represented by the value ranging from 0 to 100 % of power supply. The values of two remaining control signals are fixed for simplicity.

The system model consists of dynamics of the three main system units – heater, exchanger, and pipeline. The units are described by first order anisochronic model in order to obtain as low order model as possible [3].

$$\begin{aligned} &\text{heater:} && T_1 \frac{d\vartheta_1(t)}{dt} + \vartheta_1(t) = \vartheta_3(t) + K_u\, u(t) \\ &\text{pipeline:} && \vartheta_2(t) = \vartheta_1(t-\tau_2) \\ &\text{exchanger:} && T_3 \frac{d\vartheta_3(t)}{dt} + \vartheta_3(t) = K_3\vartheta_2(t-\tau_3) \end{aligned} \tag{1}$$

Coefficients K_u, K_3 are obtained from the measured static characteristics of the system units. Time delay τ_2 is simply determined from the time shift between the step responses of the temperature $\vartheta_1(t)$ and $\vartheta_2(t)$ because the insulated pipeline has negligible heat loss and therefore the step responses have almost the same behaviour (see fig. 4).

$$T_1 = 36{,}5s \quad K_u = 20{,}3°C \quad \tau_2 = 119s \quad T_3 = 28{,}5s \quad K_3 = 0{,}4 \quad \tau_3 = 23{,}5s \tag{2}$$

The least-square fitting of the model is used to obtain the values of remaining three coefficients T_1, T_3, τ_3. The step response of temperature $\vartheta_3(t)$ is chosen as fitting curve (see fig. 4) [4].

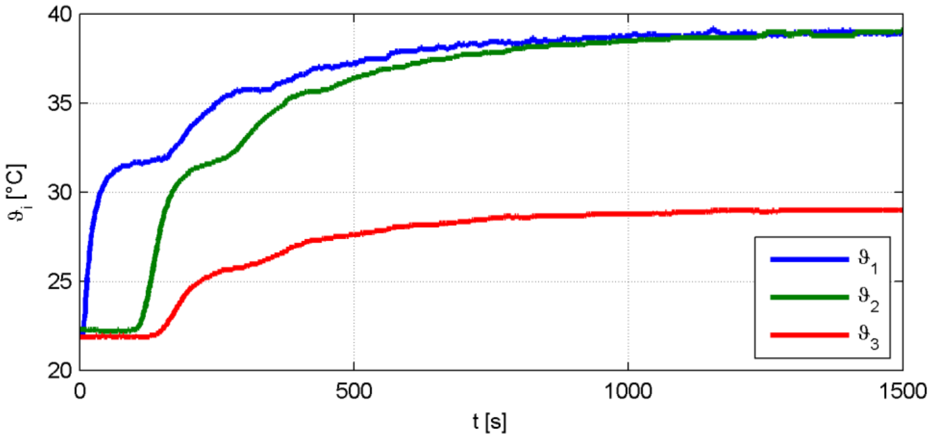

Fig. 4: Step responses of the measured temperatures for control input change.

Substituting the identified coefficients and joining the equations (1) together gives the transfer function of the system.

$$G_3(s) = \frac{8{,}1e^{-143s}}{1040s^2+65s+1-0{,}4e^{-143s}} \tag{3}$$

Application of the finite spectrum assignment (FSA) method [5] to controller design leads to a more complicated form of transfer function where it is hard to correctly implement an anti-windup compensation.

$$U(s) = G_U(s)U(s)e^{-Ls} + G_Y(s)Y(s) + G_W(s)W(s) \tag{4}$$

Without considering an anti-windup compensation the control process with the controller (4) becomes even unstable due to control input saturation (see fig. 5). To obtain a more simple anti-windup compensation another control strategy using the internal model in controller has been used.

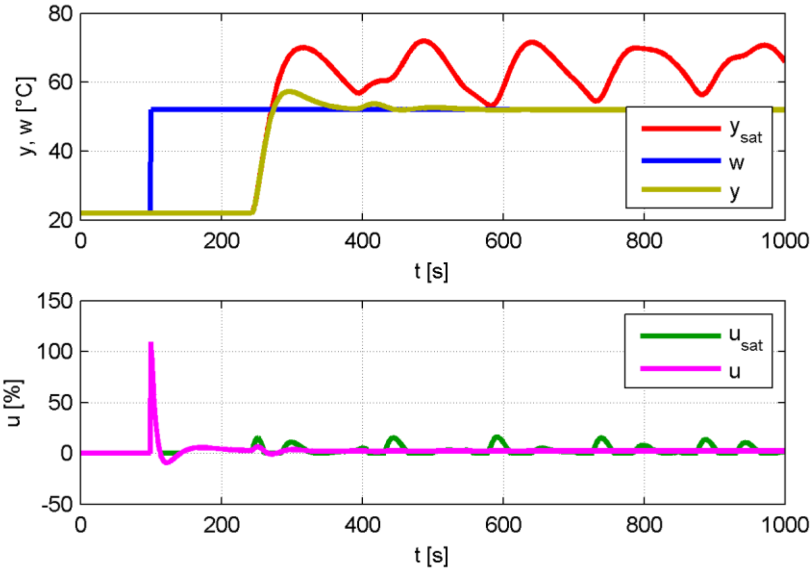

Fig. 5: Example of unstable behaviour with control input saturation u_{sat} (from 0 to 100 %) in comparison with behaviour without saturated u.

4. Internal model control (IMC) with anti-windup compensation

IMC is a popular design procedure in the process industries, particularly as a means for tuning single loop, PID-type controllers. Extensive use of IMC in system control has generated great interest among researches in various aspects [6, 7]. Unless the IMC controller is designed to optimize the nonlinear performance, it will not give satisfactory performance for the saturating system. IMC is a model based method as FSA but it gives a simpler structure of the controller and so it is more transparent to test this method with an anti-windup compensation.

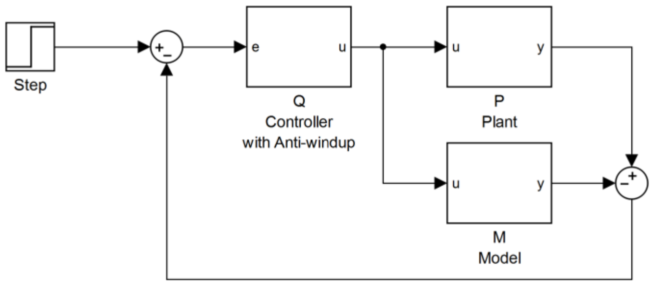

Fig. 6: Block scheme of IMC method.

The system model given by following transfer function is used for IMC design and simulation [8]. The model has five parameters by that it is possible to obtain system with required properties even so it is simple enough to implement the anti-windup compensation clearly.

$$P(s) = \frac{Ke^{-s\tau}}{(T_1 s+1)(Ts+e^{-s\vartheta})} \tag{5}$$

If IMC design is applied to system (5) the controller $Q(s)$ is given by following form. The second order filter $F(s)$ is chosen with the time constant equal to time constant T_1 of the system in order to reduce fraction.

$$Q(s) = \frac{(Ts+e^{-s\vartheta})}{K(T_1 s+1)} \tag{6}$$

If it is used the system model in form (5) IMC gives similar controller structure as FSA method for single input variable. Assume for simplicity an exact match of controlled system (5) and system model with given parameters that are designed only for demonstration and therefore they don't correspond with parameters of the heat transfer system.

$$T = 8{,}8s \quad T_1 = 1{,}22s \quad \tau = 1{,}73s \quad \vartheta = 3{,}6s \quad K = 1 \tag{7}$$

Integral square error of closed-loop controlled output is used as criterion comparing the performance of the controller without ($y(t)$) and with the anti-windup compensation ($y_{sat}(t)$) [6]. It reflects an effort to achieve the closest match between the anti-windup modification and designed behaviour of the closed-loop control.

$$J_{ISE} = \int_0^t (y(\tau) - y_{sat}(\tau))^2 d\tau \tag{8}$$

Back-calculation and tracking method is chosen for the first introduction to the saturation problem in IMC but the result is not satisfying as shown in figure 7. The criterion has minimum for $k_Q = 0$ and it means that the back-calculation and tracking structure is unnecessary and it is sufficient to let the actuator saturates.

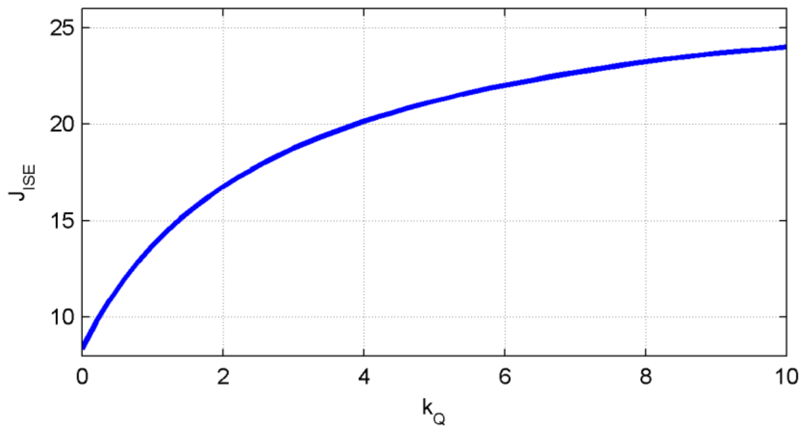

Fig. 7: Integral square error of back-calculation and tracking modification of IMC for various values of coefficient k_Q.

Better result is achieved when the back-calculation and tracking method is modified. Sum at the input of the integrator is changed in order to get positive feedback (with gain k_Q) from the saturation block (i.e. actuator) as shown in figure 8.

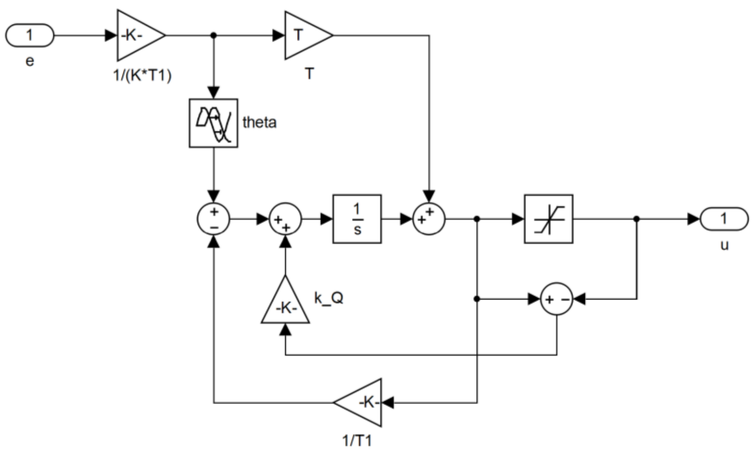

Fig. 8: Simulink model of the controller $Q(s)$ with anti-windup.

Based on the simulation the modification causes changing of course of the integral square error so that the minimum is not located at $k_Q = 0$ but at $k_Q = 0{,}74$. The value of the integral square error grows rapidly for higher values of k_Q which means poorer behaviour than for original back-calculation method. But value of coefficient k_Q around 0,74 brings improvement for the closed-loop control with the anti-windup compensation.

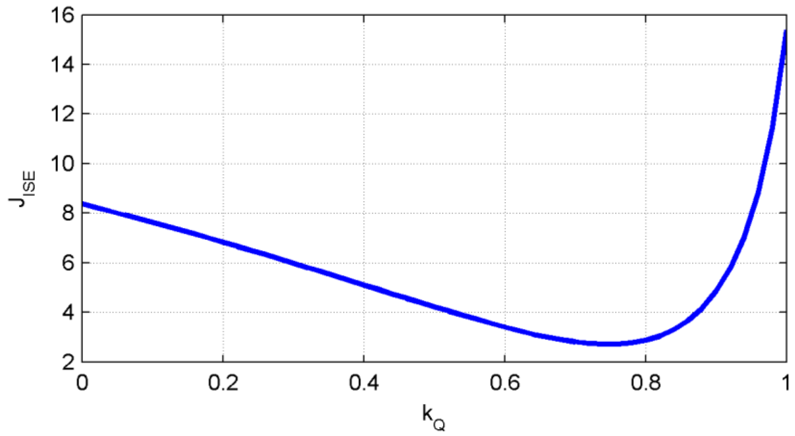

Fig. 9: Integral square error of modification of back-calculation method for various values of coefficient k_Q.

The step response and appropriate control input for $k_Q = 0{,}74$ are shown in figure 10. Behaviour of the controlled output $y_{aw}(t)$ satisfies the pre-designed control output $y(t)$ with noticeable delay caused by control input inactivity during the saturation. It happens probably due to catch up the missing control action. Nevertheless the modification gives better result than for the structure without the anti-windup with slow growth of $y_{sat}(t)$.

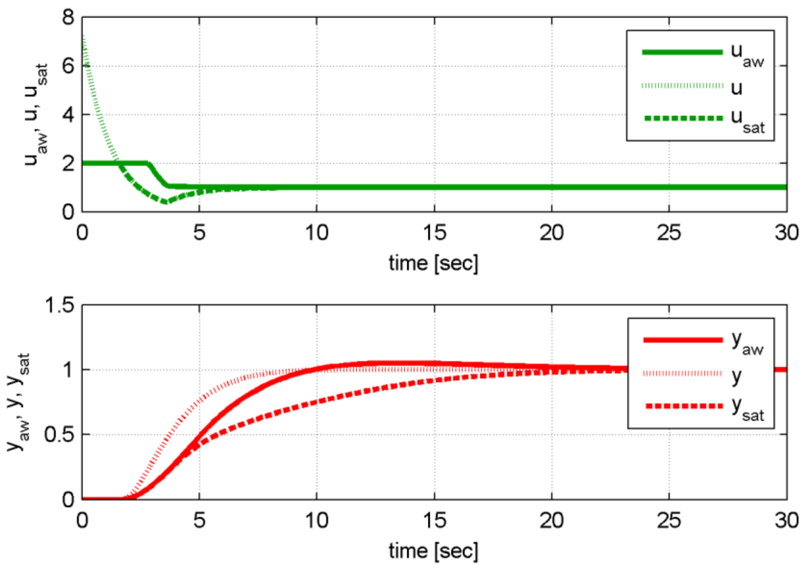

Fig. 10: Control input behaviour (top) and step response (bottom) of the IMC system without (dashed, y_{sat}) and with anti-windup (solid, y_{aw}) for $k_Q = 0,74$.

5. Conclusion

Basic ad hoc methods for compensating the integrator windup were presented in this paper. IMC design with the anti-windup compensation was tested based on the motivation to design appropriate controller (with anti-windup compensation) for the real plant using FSA method. The simulation of the tested anti-windup modification shows improvement for the chosen model.

Further work will focus on generalization of the anti-windup modification for general systems and also it is necessary to prove observed improvement analytically. The next step will continue with application of the anti-windup compensation to the FSA controller. Finally the designed controller will be used for the laboratory plant control.

List of symbols

$\vartheta_{1,2,3}$	temperatures	[°C]
T_1	time constant of the heater model	[s]
K_u	gain of the heater model	[°C]
τ_2	time delay of the pipeline model	[s]
T_3	time constant of the exchanger model	[s]
K_3	gain of the exchanger model	[]
τ_3	time delay of the exchanger model	[s]
y	controlled output	[]
u	actuating variable	[]
T	time constant of the model	[s]
τ	time delay of the model	[s]
ϑ	time delay of the model	[s]
K	gain of the model	[]
J_{ISE}	integral square error	[]
k_Q	anti-windup feedback gain	[]

References

[1] ÅSTRÖM, K.; RUNDQWIST, Lars. Integrator Windup and How to Avoid It. *American Control Conference, 1989*, pp. 1693 - 1698, 21-23 June 1989

[2] FERTIK, H. A, C. W. ROSS, Direct digital control algorithms with anti-windup feature. *ISA Trans*. 6, No. 4, pp. 317 - 328.

[3] VYHLÍDAL, Tomáš. *Analysis and Synthesis of Time Delay System Spectrum*. Prague, 2003. Ph.D. Thesis. Czech Technical University in Prague. Supervisor Prof. Ing. Pavel Zítek, DrSc.

[4] HOFREITER, Milan. *Identifikace systémů I*. Vyd. 1. Praha: Česká technika - nakladatelství ČVUT, 2009. 202 s. ISBN 978-80-01-04228-1.

[5] WANG, Qing-Guo; LEE, Tong H.; TAN, Kok K. *Finite-spectrum assignment for time-delay systems*. Springer, 1998

[6] CHUNG SENG Ling; BROWN, M.D.; WESTON, P.F.; Roberts, Clive, Gain tuned internal model control for handling saturation in actuators, *American Control Conference, 2004. Proceedings of the 2004* , vol. 5, pp. 4692,4697 vol. 5, June 30 2004-July 2 2004

[7] ZHENG, Alex; KOTHARE, Mayuresh V.; MORARI, Manfred. Anti-windup design for internal model control. *International Journal of Control*, 1994, 60.5: 1015-1024.

[8] ZÍTEK, Pavel a Antonín VÍTEČEK. *Návrh řízení podsystémů se zpožděními a nelinearitami*. 1. vyd. Praha: ČVUT, 1999. 165 s. ISBN 80-01-01939-X.