Identification of relaxation parametr composite tube from fluid transient experiment

Ing. David Hromádka

Prof. Ing. Rudolf Žitný, CSc.

Abstract

This paper presents a fluid transient inflation experiments with a viscoelastic composite tube and its numerical simulation. Mathematical description of experimental setup (inflated tube, piping and pressurized vessel) is based on the windkessel model and a nonlinear viscoelastic constitutive model of the inflated tube, which is derived from the principle of maximised dissipated energy. The governing system of equations is solved by means of the implicit Euler method. Fluid transient simulation was used for relaxation parameter identification of the constitutive viscoelastic model of the specimen.

The viscoelastic tube was tested by inflation and a fluid transient test. The tested tube had a composite structure with layer reinforced by fibre oriented in helical structure with limited extensibility. The purely elastic part of constitutive model was obtained from the inflation test. Results obtained from simulations were compared with experimental measurements carried out on a viscoelastic tube and relaxation parameter was obtained.

Simulations with increased viscosity and local losses and the purely elastic response of the tube wall were carried out (no wall damping function). These results were compared with experimental data to confirm the hypothesis that wall viscoelasticity plays an important role in damping pressure pulsations within the tested specimen.

Keywords

Viscoelasticity, fluid transient, blood vessel, pressure pulsation, evolution equation, maximum dissipation

1. Introduction

Viscoelastic processes are non-equilibrium time dependent processes. Some energy is reversibly stored during loading and some is dissipated to heat. Several approaches exist for viscoelastic behaviour description of solids.

One approach is utilization of the hereditary integral formulation based on Boltzmann superposition principle for modelling nonlinear viscoelastic behaviour developed by Coleman and Noll [3] and used for soft tissue by Fung [4] who named this approach as Quasi-linear viscoelasticity (QLV). Many researchers adopted and adapted QLV theory to fit the responses of soft tissues Abramowitch and Woo [1], Funk et. al. [5], Lynch et. al. [13], Sarver et. al. [14], Toms et. al. [15], Valdez-Jasso [17]. There are also phenomenological models that are derived from parallel or serial connection of elastic springs and viscous dampers Valdez-Jasso [16], Bessems [2]. The transversely isotropic viscohyperelastic material was introduced in Limbert and Middleton work [12]. The mechanical formulation is based on a definition of a general Helmholtz free energy function which is a sum of hyperelastic and viscous potential. Their approach is capable to describe anisotropic viscous behaviour also. Other approach is formulation viscoelastic processes in the state variables which lead to evolution equations and it is presented in Holzapfel work [10, 11] where the state variables are presented as inelastic strains or stresses. The foundations of this approach are still derived from phenomenological models of connected springs and dashpots. Haslach [6, 7, 8, 9] introduced a new class non-equilibrium thermoviscoelastic evolution equation based on long-term behaviour and a maximum dissipation principle for polymers, rubbers and soft tissues. The non-linear evolution equations (1) for thermoviscoelastic behaviour in terms of state variables, x_i, and control variables, y_i, are generated from long-term constitutive models represented by an energy function Ψ used for elasticity, see below.

$$\begin{bmatrix} \dfrac{dx_1}{dt} \\ \dfrac{dx_2}{dt} \\ \cdots \\ \dfrac{dx_n}{dt} \end{bmatrix} = -k \begin{bmatrix} \dfrac{\partial^2\psi}{\partial x_1^2} & \dfrac{\partial^2\psi}{\partial x_1 \partial x_2} & \cdots & \dfrac{\partial^2\psi}{\partial x_1 \partial x_n} \\ \dfrac{\partial^2\psi}{\partial x_2 \partial x_1} & \dfrac{\partial^2\psi}{\partial x_2^2} & \cdots & \dfrac{\partial^2\psi}{\partial x_2 \partial x_n} \\ \cdots & \cdots & \cdots & \cdots \\ \dfrac{\partial^2\psi}{\partial x_n \partial x_1} & \dfrac{\partial^2\psi}{\partial x_n \partial x_2} & \cdots & \dfrac{\partial^2\psi}{\partial x_n^2} \end{bmatrix}^{-2} \begin{bmatrix} -y_1(t) + \dfrac{\partial\psi}{\partial x_1} \\ -y_1(t) + \dfrac{\partial\psi}{\partial x_1} \\ \cdots \\ -y_n(t) + \dfrac{\partial\psi}{\partial x_n} \end{bmatrix} \tag{1}$$

This approach reduces the number of experiments and there is no need to obtain creep or relaxation function for description of these phenomena. Moreover, the classical spring and dashpot linear models were recovered from this hypothesis (Kelvin-Voight model, Standard Linear Solid model).

This paper deals with a fluid transient experiment on a composite tube. Mathematical model formulation is simplified by the fact that only a small part of system boundary is flexible (tested section of elastic tube is very short comparing with a long rigid piping in experimental setup). Thus the effect of a moving pulse wave can be neglected and the whole system can be approximated by the 'windkessel' (0-D) model that is by a system of ordinary differential equations. Only one specimen of elastic tubes was tested: a simple tube having a composite three layers structure. The primary aim is to obtain a relaxation parameter of the constitutive model for description of the tested specimen during transient loading. Haslach's construction of thermostatic nonlinear evolution equation was utilized for this purpose. The investigated parameter is relaxation parameter.

2. Methods

2.1 Manufacture of physical model blood vessel

Physical model was developed as a composite tube with three layers. The first (intima) layer was formed from a thin wall latex tube. The second layer (media) was formed from rubber band helically wounded on the outer surface of the first layer. The rubber band increases its stiffness significantly when a large deformation is achieved (tested bands have the limiting stretch ratio 2). The connection between the first and the second layer was realized by a silicone matrix. Silicone matrix formed also the third layer.

2.2 Experiment - Inflation test

Inflation test was carried out to obtain the dependence between pressure and volume, i.e. to provide information on elastic behaviour of the tested specimen at equilibrium state. The physical model was inflated by a small predefined increment of volume and the corresponding pressure was recorded by a pressure transducer.

2.3 Experiment – Fluid transient test

Experiment was carried out using the experimental setup shown schematically in Fig.1. Experiment proceeded by this way: at the beginning the pressure vessel and the tested specimen are pressurized by the compressor (C). When the demanded pressure level is reached inside the specimen, p, and in the pressure vessel, p_a, the compressor is turned off. After the water level and pressures were stabilized the electromagnetic valve (stop-cock) is opened and almost instantaneously closed so that the air pressure p_a drops to a prescribed value (still above the atmospheric pressure, this value is specified so that to avoid a possible collapse of the elastic tube during oscillations). The sudden drop of pressure initiates oscillation of the water column, accompanied by pressure pulses, $p(t)$, recorded by a pressure transducer. The water column motion is driven by gravity forces (height h_N+h_P) and by pressure forces caused by compression and expansion of air volume, (V_a). The tested elastic pipe has approximately the same initial inner radius r_0 as the connected piping and the valve, the length of tested elastic section is about 5% of the connected rigid pipes. Parameters of water hammer experiment are presented in Tab. 1.

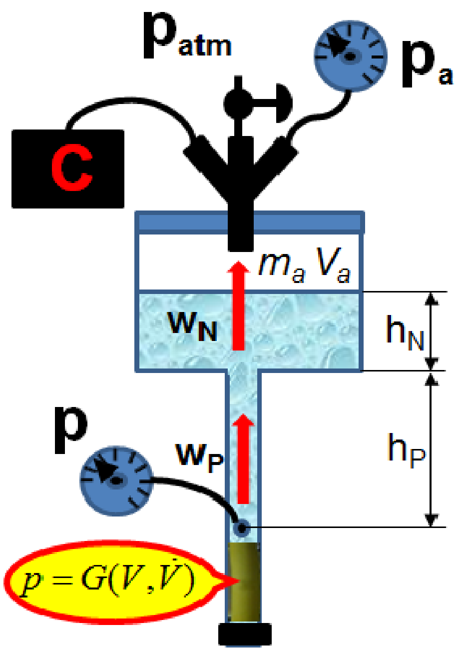

Fig. 1. *Scheme of experimental setup with reservoir*

Table 1. *- Water hammer experiment parameters*

Specimen	**Value**
Inner diameter r_0	0.0174 m
Wall thickness h_0	0.0023 m
Length	0.05 m
Piping	**Value**
Inner pipe diameter	0.023 m
Wall thickness of pipe	0.0028 m
Height h_N	0.17 m
Height h_P	1.08 m
Air volume V_{a0}	$2.96\text{x}10^{-4}$ m^3
Volume V_0	$3.41\text{x}10^{-5}$ m^3
Minimum valve diametr	0.00132 m

The volume, V_{a0}, V_0, corresponds to the state with atmospheric pressure within the pressure vessel and the tested composite tube.

2.4 Mathematical model

Mathematical model is reduced to 5 ordinary differential equations and continuity equation for 6 unknowns: mass of air, $m_a(t)$, pressure, $p_a(t)$, of air in the pressure vessel, pressure, $p(t)$, in the viscoelastic pipe, flow velocities, $w_P(t)$, $w_N(t)$, in the rigid piping and the pressure vessel and the flowrate, $\dot{V}$, through the rigid piping.
The mass flowrate of air through the stop-cock (2, 3) follows from the St.Venant-Wanzel equation for subsonic flow,

$$\dot{m}_a = -S(t)\sqrt{\frac{2\kappa}{\kappa-1}p_a\rho_a\left[\left(\frac{p_{atm}}{p_a}\right)^{\frac{2}{\kappa}}-\left(\frac{p_{atm}}{p_a}\right)^{\frac{\kappa+1}{\kappa}}\right]} \tag{2}$$

or

$$\dot{m}_a = -S(t)\sqrt{\kappa p_a\rho_a\left(\frac{2}{\kappa+1}\right)^{\frac{\kappa+1}{\kappa-1}}} \tag{3}$$

for the supersonic flow as soon as the ratio of outlet/inlet pressures drops below the threshold

$$\frac{p_{atm}}{p_a} < \left(\frac{2}{\kappa+1}\right)^{\frac{\kappa}{\kappa-1}}, \tag{4}$$

where κ is the specific heat ratio, p_{atm}, is the atmospheric pressure and ρ_a is the actual air density. $S(t)$, is the flow-through area of the electromagnetic valve and its time course, controlled by a computer, is shown in Fig.2.

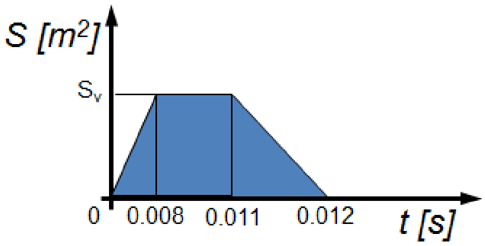

Fig. 2. *Valve actuating function S(t)*

Pressure changes inside the decompressed vessel depend upon the mass flowrate of air and upon the change of the internal volume, V_a. Assuming adiabatic expansion of air in the pressure vessel the pressure changes can be calculated as

$$\frac{dp_a}{p_a} = \left(\frac{1+\frac{dm_a}{m_a}}{1+\frac{dV_a}{V_a}}\right)^{\kappa} \cong \kappa\left(\frac{dm_a}{m_a}-\frac{dV_a}{V_a}\right) \tag{5}$$

Continuity equations for incompressible liquid

$$w_N = w_P\frac{A_P}{A_N} \tag{6}$$

$$\dot{V} = -w_P A_P \tag{7}$$

where, w_N, w_P, are velocities in the pressure vessel and in the rigid pipe and A_N, A_P, are corresponding cross-sectional areas.
The Bernoulli's equation (8) takes into account inertia of liquid, change of cross-sections, friction, a pressure and gravity forces

$$
\begin{aligned}
& h_N \frac{dw_N}{dt} + h_P \frac{dw_P}{dt} + \frac{w_N^2 - w_P^2}{2} = -\frac{1}{\rho}\left(p_a - p\right) - g\left(h_N + h_P\right) + \cdots \\
& \cdots + \left(\frac{\lambda_N h_N}{4R_N}\right) w_N \left|w_N\right| + \left(\frac{\lambda_P h_P}{4R_P} + \frac{\zeta_{VzP}}{2} + \frac{\zeta_{PN}}{2}\right) w_P \left|w_P\right|
\end{aligned} \tag{8}
$$

λ_P, λ_N, are the dArcy's friction factors calculated as $64/Re$ in laminar and as $0.316/Re^{0.25}$ in the turbulent flow regime. Coefficients, ζ_{VzP}, ζ_{PN}, determine local pressure losses with sudden change of velocity and transition section.
The Eqs.(9,10) describe relationship between pressure, volume and rate of the volume change of the viscoelastic section (therefore viscoelastic behaviour of wall) assuming a parallel arrangement of a dashpot and an elastic unit

$$
p\left(V,\dot{V}\right) = G_e\left(V\right) + \frac{1}{k} G_v\left(V\right)\dot{V} = G_e\left(V\right) + \frac{1}{k}\frac{1}{2}\frac{r_0}{h_0}\frac{1}{V}\left[\left(2G_e^{'}V + G_e\right)\right]^2 \dot{V} \tag{9}
$$

$$
\begin{aligned}
& \frac{dp\left(V,\dot{V}\right)}{dt} = \frac{\partial p\left(V\right)}{\partial V}\dot{V} + \frac{\partial p\left(V\right)}{\partial \dot{V}}\frac{\partial \dot{V}}{\partial t} = \cdots \\
& \cdots = G_e^{'}\dot{V} + \frac{1}{k}\frac{1}{2}\frac{r_0}{h_0}\left(\frac{\left(2G_e^{'}V + G_e\right)}{V}\left(4G_e^{''}V + 4G_e^{'} - \frac{G_e}{V}\right)\right)\dot{V}^2 + \frac{1}{k}\frac{1}{2}\frac{r_0}{h_0}\frac{1}{V}\left[\left(2G_e^{'}V + G_e\right)\right]^2 \frac{\partial \dot{V}}{\partial t}
\end{aligned} \tag{10}
$$

The elastic part G_e can be described by a hyper-elastic model (assuming symmetrically deformed circular pipe) or by using an experimentally determined function from the quasi-static inflation test. The relaxation coefficient k has in this case the unit [Pa/s] and must be evaluated from the observed attenuation of oscillations. The form of $G_v(V)$ was derived according to the idea of Haslach [13] (the idea assumes that the viscous characteristic can be expressed in terms the elastic behaviour using only one relaxation parameter) in the form

$$
G_v\left(V\right) = \frac{1}{2}\frac{r_0}{h_0}\frac{1}{V}\left[\left(2G_e^{'}V + G_e\right)\right]^2 \tag{11}
$$

$$
G_v^{'}(V) = \frac{1}{2}\frac{r_0}{h_0}\left(\frac{\left(2G_e^{'}V + G_e\right)}{V}\left(4G_e^{''}V + 4G_e^{'} - \frac{G_e}{V}\right)\right) \tag{12}
$$

where, r_0, is the reference inner radius, h_0, correspond to reference thickness of tested specimen.
To make the model description more readable it is possible to rewrite the Bernoulli's equation (8) into the form

$$
\frac{d\dot{V}}{dt} = f(p, p_a, \dot{V}, V) \tag{13}
$$

using Eqs.(6,7) for elimination of velocities, and the volume V replaces the water level h_N.
Evolution of pressure inside the elastic tube follows from the Eq.(10) combined with the Eq.(13)

$$
\frac{dp}{dt} = G_e^{'}\dot{V} + \frac{1}{k}G_v^{'}\dot{V}^2 + \frac{1}{k}G_v f(p, p_a, \dot{V}, V) \tag{14}
$$

while the evolution of pressure inside the pressure vessel p_a is described by Eq.(5). The last differential equation

$$\frac{dV}{dt} = \dot{V} \tag{15}$$

completes the system of four differential equation for unknown variables $p, p_a, \dot{V}, V$.
Initial conditions for the system (2-10) are pressures in the pressure vessel and zero velocities (flowrate). The initial air density ρ_{a0} is given by equation of state for ideal gas. The volume of air in the pressure vessel and water volume in tested specimen are given by equations below,

$$V_a = V_{a0} + V(p) \tag{16}$$

$$V = V_0 + V(p) \tag{17}$$

The volumes V_{a0}, V_0 correspond to state with atmospheric pressure within the pressure vessel and the composite tube. When the pressure vessel and specimen are pressurized the volumes V_{a0}, V_0 increase by the same value $V(p)$ (volume of displaced water).
Fully implicit Euler method was used for the numerical integration of differential equations (12),(13),(14),(4) and implemented as a simple Fortran program.

2.5 Frequency analysis

Frequency analysis was carried out on time interval 0-1 second. The main frequency was determined as the number of peaks per time interval.

3. Results

3.1 Elastic response

The inflation test of the blood vessel physical model revealed nonlinear pressure-volume relationship, see Fig. 2.

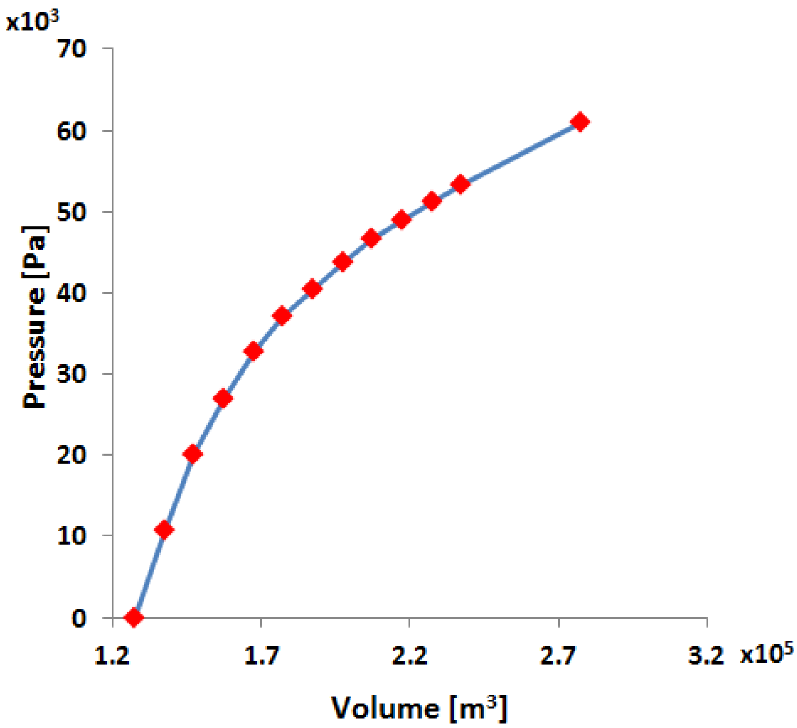

Fig. 3. *Nonlinear pressure-volume relationship fitted by linear spline model.*

The model (linear spline model) was adopted for description of the pure elastic behaviour. The model fit experimental data successfully.

3.2 Dynamic response

Results from experiment and simulation are shown on Fig. 4. , Fig. 5. Points represent recorded pressure during the fluid transient experiment and the blue line is numerical prediction. The coefficient *k* of the attenuating function G_v which was used in simulations was estimated manually, see the Fig. 4.

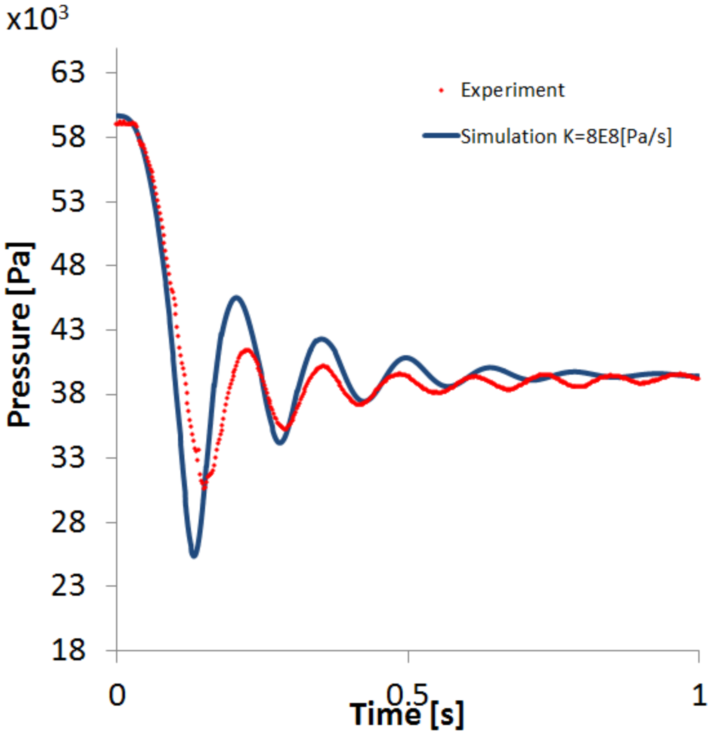

Fig. 4. *Pressure responses after almost instantaneously closed valve and simulation with three different constants k.*

The attenuation of pressure on Fig.5. is caused only by enlargement of local pressure losses by thousand fold (purple line) and enlargement of dynamic viscosity by hundred fold (blue line), in this case is function G_v=0.

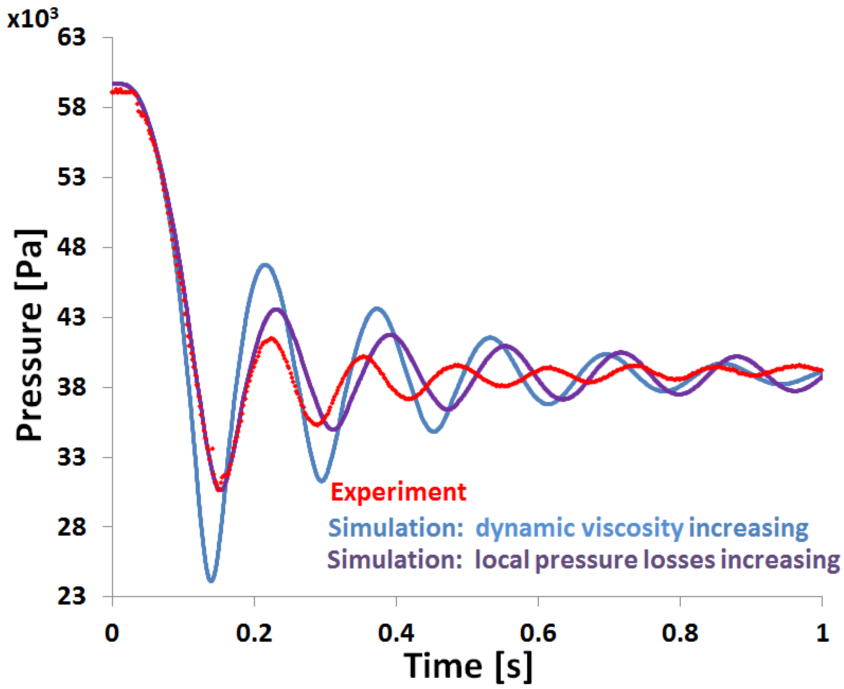

Fig. 5. *Pressure responses with zero G2*

Measured natural frequency was approximately 7 Hz. The simulation based on the linear spline model gave the frequency approximately 6.5 Hz.

4. Conclusion

The approximation of static inflation experiment *p(V)* by linear splines was used for modelling a transient response of the system consisting in viscoelastic pipe+rigid pipe+pressure accumulator, filled by water. Comparison of recorded pressure pulsations with simulations enables us to identify the value of relaxation parameter of the attenuation function. The best fit corresponds to simulation with relaxation coefficient k=8E8*Pa/s*. Simulations with increased viscosity and local losses and the purely elastic response of the tube wall (no wall damping function) revealed that the viscoelasticity of the tube wall is important for pressure pulsation attenuation. In order to achieve of similar attenuation at purely elastic response of the wall, the dynamic viscosity have to be increased approximately hundred times, or local losses have to be increased thousand fold.

It was demonstrated that the viscoelastic response of the wall is essential for attenuation of pressure pulsations and that the experimental setup design is suitable for measurement of viscoelastic behaviour of the straight tubes.

Symbols

ζ_{PN}	local pressure losses with sudden change of velocity	[-]
ζ_{VzP}	local pressure losses with transition section	[-]
κ	specific heat ratio	[-]
λ_N	pressure-vessel dArcy's friction factor	[-]
λ_P	piping dArcy's friction factor	[-]
ρ	water density	[kg/m^3]
ρ_a	air density	[kg/m^3]
ρ_{a0}	initial air density	[kg/m^3]

Ψ	strain energy density function	[J/m^3]
A_N	pressure-vessel cross-sectional area	[m^2]
A_P	pipe cross-sectional area	[m^2]
g	gravitational acceleration	[m/s^2]
G_e	stiffness characteristic of tested specimen	[Pa]
G_v	attenuation function	[Pa^2/m^3]
h_0	initial wall thickness	[m]
h_N	water level height within the pressure level	[m]
h_P	pipe height	[m]
k	relaxation parameter	[Pa/s]
m_a	air mass	[kg]
p	pressure within tested specimen	[Pa]
p_a	air pressure in pressure vessel	[Pa]
p_{atm}	atmospheric pressure	[Pa]
r_0	initial radius of tested specimen	[m]
Re	Reynolds number	[-]
R_N	pressure-vessel radius	[m]
R_P	pipe radius	[m]
S	cross-sectional area of outflow valve	[m^2]
V	volume within tested specimen	[m^3]
V_0	initial volume within tested specimen	[m^3]
V_a	air volume within pressure vessel	[m^3]
V_{a0}	initial air volume within pressure vessel	[m^3]
w_N	velocity within pressure vessel	[m/s]
w_P	velocity within pipe	[m/s]
x_i	state variable	
y_i	control variable	
$\dot{m}_a$	mass flow rate	[kg/s]
$\dot{V}$	flow rate	[m^3/s]

REFERENCES

[1] ABRAMOWITCH, S. D. and S. L. WOO. An improved method to analyze the stress relaxation of ligaments following a finite ramp time based on the quasi-linear viscoelastic theory. *J. biomech. Eng.* 2004, 126, p. 92–97.

[2] BESSEMS, D. Experimental validation of a time-domain-based wave propagation model of blood flow in viscoelastic vessels. *J. of Biomech.* 2008, č. 41, p. 284–291.

[3] COLEMAN, B.D. and W. NOLL. Foundations of linear viscoelasticity. *Rev. Mod. Phys.* 1961, 33(2), p. 239–249.

[4] FUNG, Y. C. *Biomechanics: Mechanical Properties of Living Tissues*. Berlin: Springer, 1981.

[5] FUNK, J. R. and G.W. HALL. Linear and quasi-linear viscoelastic characterization of ankle ligaments. *J. Biomech. Eng.* 2000, 122, p. 15–22.

[6] HASLACH, H.W. Jr. Geometric Structure Of The Non-Equilibrium Thermodynamics Of Homogenous System.*Reports Of Mathemathical Physics*. 1997, Vol.2, p. 147-162.

[7] HASLACH, H.W. Jr. Nonlinear viscoelastic, thermodynamically consistent models for biological soft tissue.*Biomechan Model Mechanobiol.* 2005, 3, p. 172–189.

[8] HASLACH, H.W. Jr. A maximum dissipation thermodynamic multi-scale model for the dynamic response of the arterial elastin–water system. *Acta Mechanica*. 2010, 213, p. 169-188.
[9] HASLACH, H.W.Jr. a N. ZENG. Maximum dissipation evolution equations for non-linear thermoviscoelasticity.*International Journal of Non-Linear Mechanics*. 1999, 34, p. 361-385.
[10] HOLZAPFEL, G.A. A new viscoelastic constitutive model for continuous media at finite thermomechanical changes. *I. Solids Structures*. 1996, No. 20-22, p. 3019-3034.
[11] HOLZAPFEL, H:W. and T.C. GASSER. A viscoelastic model for fiber-reinforced composites and finite strains: Continuum basis, computational aspects and applications. *Comput. Methods Appl. Mech. Engrg.* 2001, 190, p. 4379-4403.
[12] LIMBERT, G. A transversely isotropic viscohyperelastic material Application to the modeling of biological soft connective tissues. *International Journal of Solids and Structures*. 2004, 41, p. 4237–4260.
[13] LYNCH, H.A. a W. JOHANNESSEN. Effect of fiber orientation and strain rate on the nonlinear uniaxial tensile material properties of tendon. *J. Biomech. Eng.* 2003, 125, 726–31.
[14] SARVER, J. J. and P. S ROBINSON. Methods for quasi-linear viscoelastic modeling of soft tissue: application to incremental stress-relaxation experiments. *J. Biomech. Eng.* 2003, 125, 754–8.
[15] TOMS, S. R. and G. J. DAKIN. Quasi-linear viscoelastic behavior of the human periodontal ligament. *J. Biomech.*2002, 35, p. 1411–5.
[16] VALDEZ-JASSO, D. Analysis of Viscoelastic Wall Properties in Ovine. *IEEE TRANSACTIONS ON BIOMEDICAL ENGINEERING*. 2009, NO. 2, p. 210-219.
[17] VALDEZ-JASSO, D. Linear and Nonlinear Viscoelastic Modeling of Aorta and Carotid. *Annals of Biomedical Engineering*. 2011, No. 5., p. 1438-1456.