

# Simulation of the electromagnetic torque ripple of an asynchronous locomotive traction motor powered from an inverter

Vojtěch Dybala<sup>1,\*</sup>

<sup>1</sup> ČVUT v Praze, Fakulta strojní, Ústav automobilů, spalovacích motorů a kolejových vozidel, Technická 4, 166 07 Praha 6, Česká republika

## Abstract

The development of techniques and technologies in the branch of electrical drives for modern railway vehicles has enabled us to increase the installed power and subsequently improve the performance of these vehicles. As a consequence of the development there are newly arising problems. One of these areas is torsion oscillations and electromechanical phenomenon related to torsion dynamics of traction drives. And just in this area the research within PhD program and SGS grants at CTU in Prague has been focused. This contribution presents the improvement of the previous version of the asynchronous traction motor model powered from a sinus-power supply, which has been utilized within simulations and work of colleague of mine Ing. Tomáš Fridrichovský. The main goal of the model improvement is the ability to simulate the ripple of the electromagnetic torque. Within the article the focus will be put on two main features in the new model (the pulse-width modulator and the inverter), their realization and simulation results.

*Keywords:* asynchronous motor; inverter; PWM; electromagnetic torque; ripple

## 1. Introduction

The previously developed model of the asynchronous traction motor has been based on the general description of the machine by equations in  $\alpha$ - $\beta$  and d-q coordinate systems. As a first version the model of the motor was supplied from a sinus-power source and the control structure was simplified appropriately, which was relevant to at the time intended simulations.

The general goal for application of the model is to study electromechanical phenomenon in traction drives. Therefore, the next step has been done – modification of the model with respective improvement of the control structure. The idea of these improvements has been to allow simulation of a ripple of motor currents and electromagnetic torque respectively. Harmonic components of the ripple could be a kind of an excitation source of torsion oscillations. Fig. 1 presents parts of the model which had to be updated or newly built and added to the control structure of the complete model.

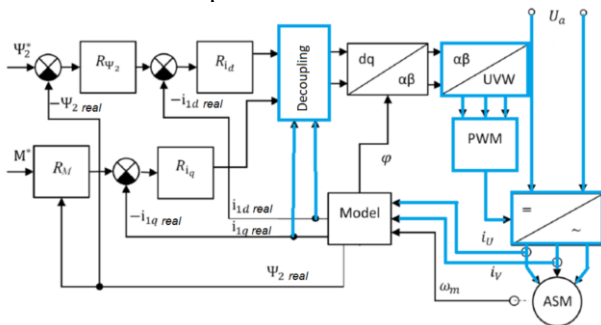


Fig. 1 - Fundamental control scheme

Next paragraphs deal with main features – the pulse-width modulator (PWM) and the 3-phase inverter models, built in MATLAB Simulink. Results of the simulation of the 3-phase asynchronous motor powered by

the inverter are presented as well to show the complete model functionality.

## 2. Theoretical background for implementation of improvements

The principle why currents and subsequently electromagnetic torque are rippled, when the traction motor is supplied from an inverter, is based on a well-known behaviour of a current in a R-L or R-L- $U_i$  type of load. The inverter generates voltage pulses which power the motor. In the principle description below is the motor represented by R-L- $U_i$  load. The very basic knowledge is that the inductance behaves as a resistor for any current change or could be called a current inertia, see Fig. 2.

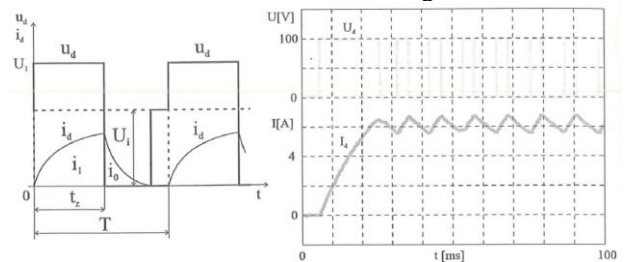


Fig. 2 - Phase current in R-L- $U_i$  load type [1]

- Switching component (e.g. IGBT) is switched-on, pulse of voltage is put on the load and following formula is valid:

$$R \cdot i_d + L \cdot \frac{di_d}{dt} + U_i = U_1 \quad (1)$$

Solution below describes current behaviour in the load:

$$i_d = I_{01} \cdot e^{-\frac{t}{\tau_d}} + \frac{U_1 - U_i}{R} \cdot \left(1 - e^{-\frac{t}{\tau_d}}\right) \quad (2)$$

\* Kontakt na autora: vojtech.dybala@fs.cvut.cz

- Switching component is switched-off, there is no voltage pulse on the load and following formula is valid:

$$R \cdot i_d + L \cdot \frac{di_d}{dt} + U_i = 0 \quad (3)$$

Solution below describes current behaviour in the load:

$$i_d = I_{02} \cdot e^{-\frac{t}{\tau_d}} - \frac{U_i}{R} \cdot \left(1 - e^{-\frac{t}{\tau_d}}\right) \quad (4)$$

- Current ripple is then defined as:

$$\Delta i_d = I_{02} - I_{01} \quad (5)$$

## 2.1. Three-phase inverter

The real inverter as well as the simulation model consists of more switching elements and diodes. These switching elements are GTO or IGBT type semiconductors in modern inverters. The inverter in question is a two-level three-phase inverter with the structure as presented in the Fig. 3 below.

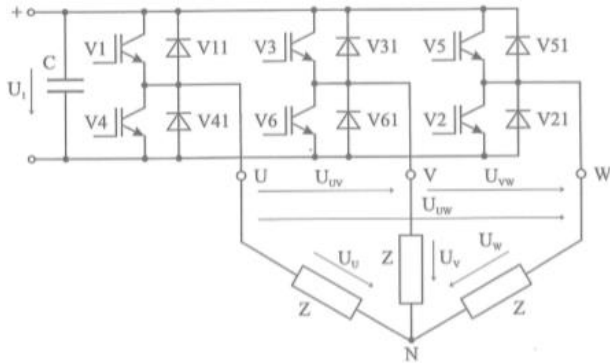


Fig. 3 - Scheme of a two-level three-phase inverter configuration [1]

The principle of current behaviour in a load supplied from an inverter is equivalent to the basic theory described in the chapter 2. The practical difference is in the way of switching elements control and current conduction through these elements and diodes, e.g. Fig. 4

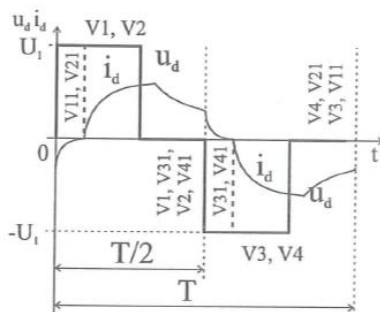


Fig. 4 - Output quantities of one phase of an inverter [1]

For purposes of supposed simulations, the three-phase inverter was built from IGBT elements, which are included in MATLAB Simulink library, see Fig. 5. This figure also shows the connected model of the asynchronous motor, PWM block which controls IGBT switching and source of DC voltage. The DC voltage source is a very

simplified replacement of an intermediate circuit of a frequency inverter.

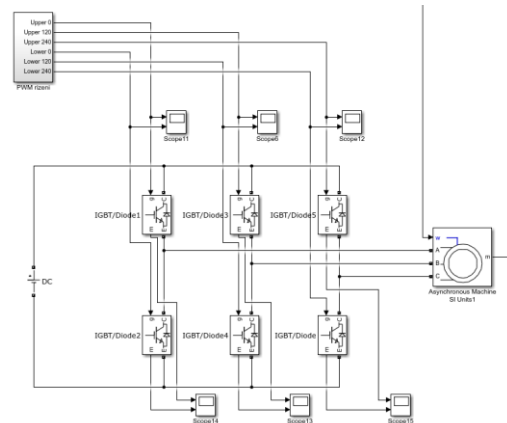


Fig. 5 - The three-phase inverter model

The function of the inverter requires a controlled switching of the semiconductor elements. The used switching algorithm is the Pulse-Width Modulation (PWM). The PWM is described more in detail in the next paragraph.

## 2.2. Pulse-width modulation

This algorithm belongs to the group of asynchronous modulations. PWM is called asynchronous modulation because of the fact, that switching frequency of the carrying signal is constant, while the frequency of the modulation signal changes according to the signal of the required 1st harmonic of the output signal. Just comparing of the carrying signal e.g. saw shape signal with the modulation signal is the principle how to generate the switching pulses for an IGBT. Specifically, it means when the modulation signal reaches higher value than the carrying signal, then the IGBT is switched on. In the inverse situation the signal is to switch the semiconductor off. Examples of pulse generating and PWM block built in MATLAB Simulink are presented in following pictures, Fig. 6 and Fig. 7.

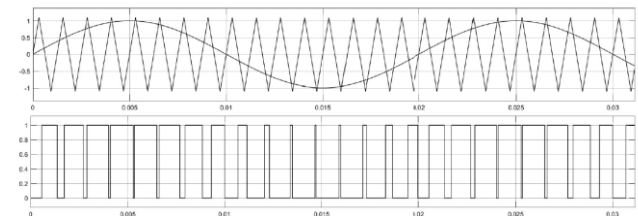


Fig. 6 - PWM principle and pulse generating

Fig. 7 presents how the algorithm of pulse generation is realised via comparison of a carrying signal (saw shape signal) and a modulation signal (sinus shape signal).

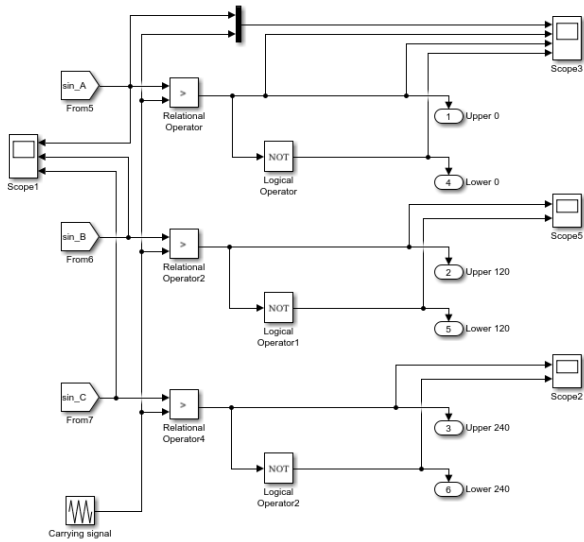


Fig. 7 - PWM block

### 3. Complete model assembly

After independent testing of sub-models of the inverter and PWM block these can be added to the complete model structure as presented in Fig. 8. The input of this model is required torque  $M_R^*$  and as the output is the electromagnetic torque, it is necessary to connect the output to a mass of inertia, which represents the rotor and enables to simulate dynamic behaviour of motor under external load.

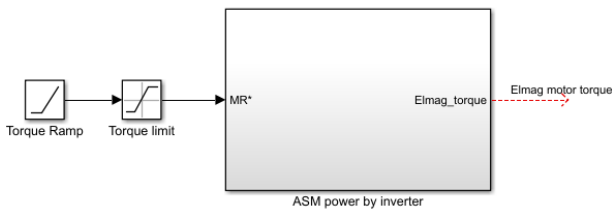


Fig. 8 – Complete model with input and output representation

The rotor was modelled according to formulas below, which describes its dynamic behaviour.

$$M_D = M_{elmag} - M_{load} \quad (6)$$

$$M_D = J \cdot \varepsilon \rightarrow \varepsilon = \frac{M_D}{J} \quad (7)$$

$$\omega = \int \varepsilon dt \quad (8)$$

The load torque  $M_{load}$  was modelled also in a simplified manner as a quadratic function of mechanical speed of the rotor. For purposes of model testing is this simplification reasonable. When the complete model was put together necessary adjustment of controller's set-up, currents sampling etc. was done. Because of the complete model extensiveness there will not be put any attention on more details in this contribution.

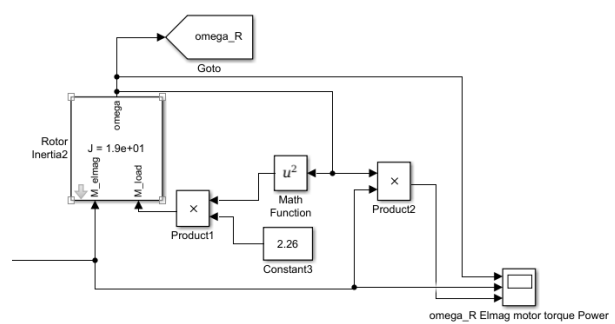


Fig. 9 - Rotor and its load model

### 4. Simulations and results

After a series of simulations and necessary adjustment the final set-up of the model was found. The result of the effort is the ability to simulate rippled motor currents and rippled electromagnetic torque respectively, as presented in following figures.

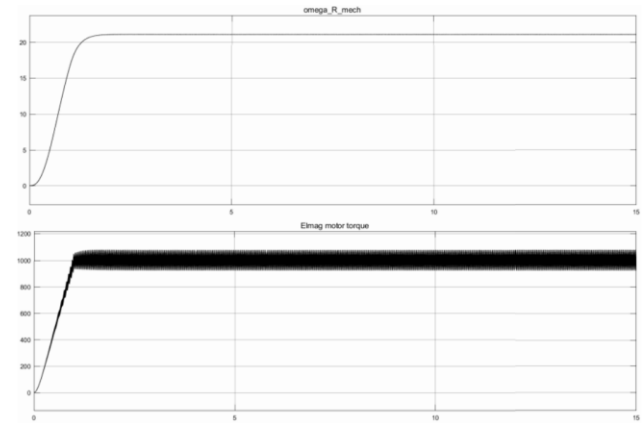


Fig. 10 – Motor mechanical speed and electromagnetic torque

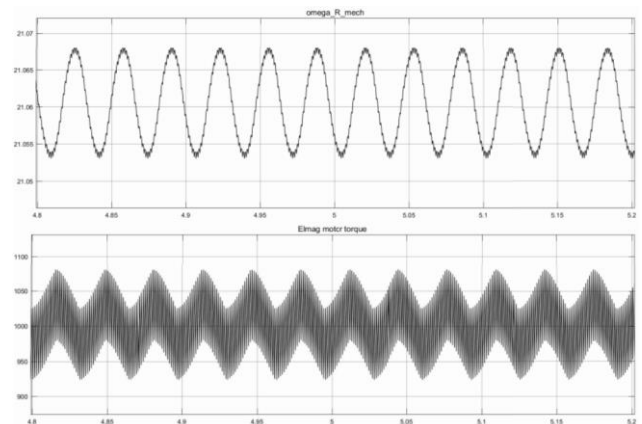


Fig. 11 - Motor mech. speed and elmag. torque – detail

Because the complete calculation model as well as its sub-models are simplified, it was also important to proceed kind of a verification. And as the main goal of the effort is to simulate torque and current ripple, which means to take the presence of harmonic components of current into account, the verification was conducted via a frequency analysis of stator phase current, see Fig. 12, Fig. 13, Fig. 14 and Fig. 15.

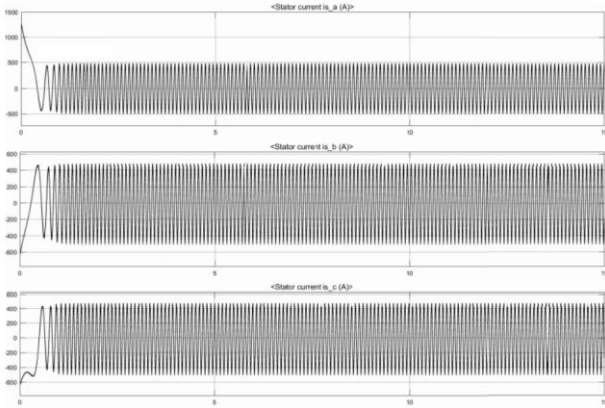


Fig. 12 - Stator phase currents

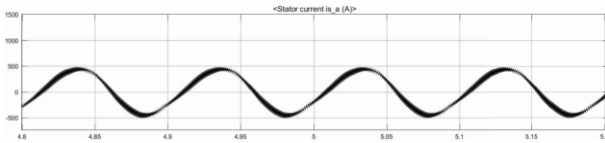


Fig. 13 - Stator phase current – detail

The basic idea of the verification when comparing harmonic components of the current is based on information published in [3]. Authors in this article presents results of measurements of coil current. That coil is powered by an inverter which works with pulse-width modulation. The important information from this article is, that generally the frequency spectrum of such a current consists of harmonic components which are combinations of multiples of PWM frequency and multiples of first harmonic. Mathematical formulation is below:

- For odd multiples of PWM frequency

$$f_i = k \cdot f_{PWM} \pm 2 \cdot l \cdot f_1 \quad [3] \quad (9)$$

$$(k = 1, 3, 5, \dots); (l=1, 2, 3, 4, \dots)$$

- For even multiples of PWM frequency

$$f_i = m \cdot f_{PWM} \pm (2 \cdot n + 1) \cdot f_1 \quad [3] \quad (10)$$

$$(m = 2, 4, 6, \dots); (l=0, 1, 2, 3, \dots)$$

These frequencies can be observed within the frequency analysis as a side-bands around multiples of PWM frequency.

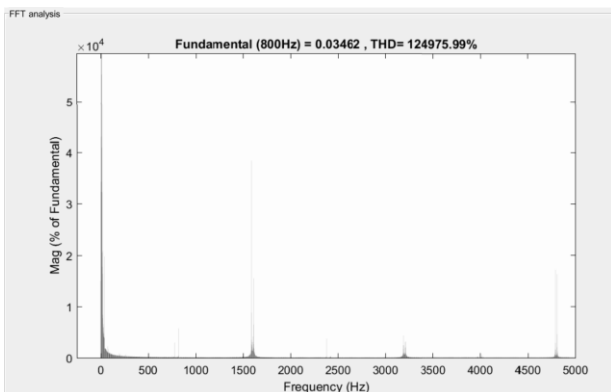


Fig. 14 - Phase current FFT analysis

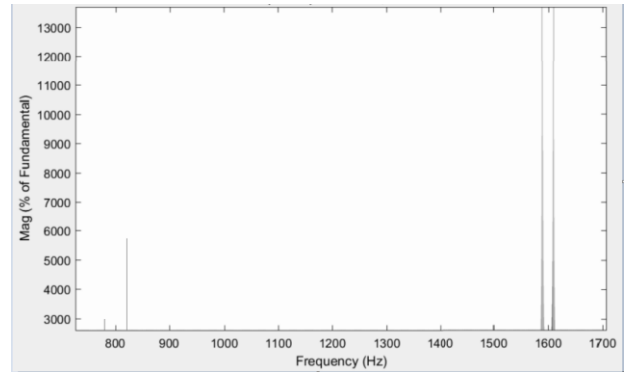


Fig. 15 - Phase current FFT analysis – detail

Fig. 14 and Fig. 15 presents frequency analysis of the simulated stator phase current. These pictures show presence of expected side-bands. As the PWM frequency is 800 Hz, in the detailed picture it is presented for the first and the second multiple of this frequency. The frequency analysis also gives the information that the first harmonic of the current is 10,25 Hz.

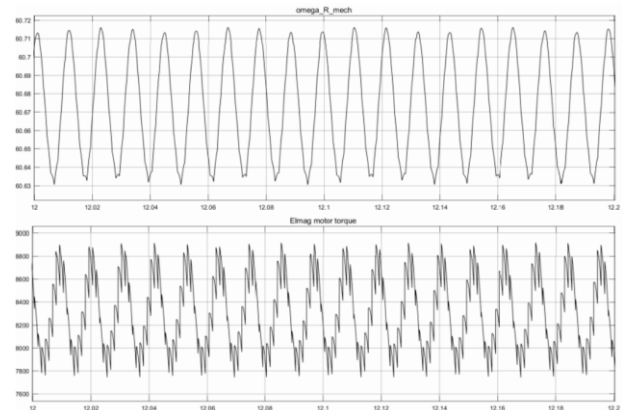


Fig. 16 - Motor speed and elmag. torque;  $\omega_{R,mech} \approx 1/3\omega_N$ ;  $M_{el-mag} \approx M_N$

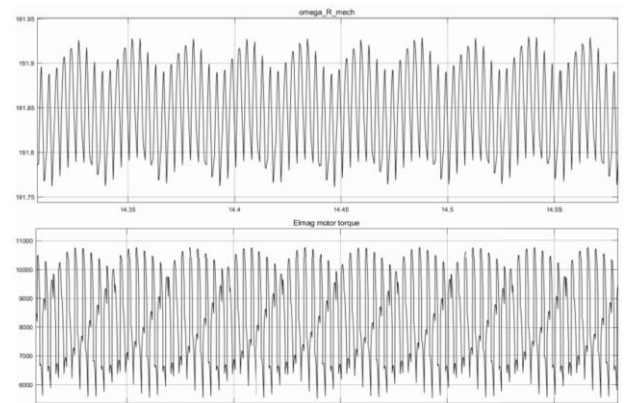


Fig. 17 - Motor speed and elmag. torque;  $\omega_{R,mech} \approx \omega_N$ ;  $M_{el-mag} \approx M_N$

Another characteristic within the model behaviour, which point to in principal correct results of simulation is a growing amplitude of current and as well as torque ripple. It has been known, that with increasing speed and thus increasing frequency of the first harmonic of power voltage the ratio of PWM frequency to the first harmonic frequency  $f_{PWM}/f_1$  decreases [4]. That is why the signal modulation gets worst and it cause bigger ripple.

Because this behaviour was expected two simulations were carried out. Fig. 16 shows a situation when the required electromagnetic torque of the motor is 8300 Nm, which is a nominal value for this motor. The motor speed is approximately 60 rad.s<sup>-1</sup>, which is app. 1/3 of nominal speed. In this case the amplitude of torque ripple is app. 7% of the nominal value. The second case see Fig. 17, was to simulate nominal torque 8300 Nm at approximately nominal speed of 192 rad.s<sup>-1</sup>. The second case simulation shows the amplitude of torque ripple app. 30% of the nominal value.

## 5. Conclusion

The idea to improve the previous model of the asynchronous motor with its complete control structure should lead to approximate its simulation results more to the real drive. For this purpose, the model was equipped with the inverter with PWM modulator. After implementation of these components and necessary set-up of model parameters the model as whole provides simulation results, which are in principal correct. Therefore, another step will be to connect it to the model of mechanical part of a traction drive system.

## Acknowledgement

The creation of this paper was supported by The Czech Technical University and the related grants – No. SGS18/130/OHK2/2T/12

## List of symbols

$f_i$	frequency (Hz)
$f_{\text{PWM}}$	PWM frequency (Hz)
$f_1$	1 <sup>st</sup> harmonic frequency (Hz)
$i_d$	load current (A)
$I_{01}$	minimum current (A)
$I_{02}$	maximum current (A)
$\Delta i_d$	current ripple (A)
$J$	rotor mass of inertia (kgm <sup>-2</sup> )
$L$	inductance (H)
$M_D$	dynamic torque (Nm)
$M_{\text{elmag}}$	electromagnetic torque (Nm)
$M_{\text{load}}$	load torque (Nm)
$M_N$	nominal torque (Nm)
$R$	ohmic resistance ( $\Omega$ )
$U_i$	inductive voltage (V)
$U_1$	input direct voltage (V)
$\varepsilon$	rotor angular acceleration (rad.s <sup>-2</sup> )
$\tau_D$	load time constant (H. $\Omega$ <sup>-1</sup> )
$\omega$	rotor angular speed (rad.s <sup>-1</sup> )
$\omega_N$	nominal rotor angular speed (rad.s <sup>-1</sup> )

## Disclosure statement

The content of this paper does not infringe any copyrights, patents, know-how of anybody else or rules of the grant provider.

## Reference

- [1] J. Javůrek, Regulace mdoerních elektrických pohonů, Praha: Grada Publishing, a.s., 2003.
- [2] J. Novák, Elektromechanické systémy v dopravě a ve strojírenství, Praha: Vydavatelství ČVUT, 2004.
- [3] J. Novák a J. Chyský, „Vyšetřování ztrát sinusových filtrů pro frekvenčně řízené pohony,“ v XXXIII. konference o elektrických pohonech, Plzeň, 2013.
- [4] P. Stejskal, Modulační techniky pro víceúrovňové střídače, Praha, ČVUT v Praze, 2015, Dostupné z: <https://dspace.cvut.cz/handle/10467/61749>