Air source heat pump model with defrosting

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Summary
The paper presents air source heat pump model. The heat pump works in varying conditions on the air source side. The general aim of the paper is to describe the model of finned tube evaporator working in frosting conditions and impact of growing frost layer on working efficiency of heat pump. The model of whole system consists sub models of compressor, condenser, refrigerant, expansion valve, evaporator, ventilator and the others. Each of the sub models is described separately. The results of the model can be used for heat exchanger optimization in order to achieve high seasonal working efficiency of heat pump.

Key words: heat pump; evaporator; refrigeration system; modeling

1. Introduction
Heat pumps are nowadays broadly used as space and water heaters in households. They are considered as one of the renewable energy sources and therefore they are subject of government support programs. The heart of refrigerant system in heat pump is compressor, in most cases propelled by electromotor. Since the electricity in the grid is produced by nonrenewable sources, there is important demand on energy efficiency of systems with heat pumps. Accordingly to EU legislative, the heat pump is renewable energy source if its SPF is higher than SPF_MIN from equation (1).

\[
SPF_{MIN} = 1,15 \cdot \frac{1}{\eta_{el}}
\]

The value \(\eta_{el}\) is an average efficiency of electricity production in EU and can be found in data from EUROSTAT. SPF is an efficiency of whole system, which is different in every application. The heat pumps are nowadays assessed by SCOP value. It is defined in European Directive 811/2013 and standard EN 14825. The standard EN 14825 defines the condition for heat pumps measurement and the procedure of SCOP calculation. From the SCOP value the seasonal energy efficiency \(\eta_s\) can be estimated and the heat pump can have its energy label.

The air source heat pumps obtains usually lower \(\eta_s\) than water or brine source heat pumps. The major problem which air source heat pumps have to deal with is the frost formation on surface of evaporator. The growth of frost layer leads to lower \(U_{ev}\), higher \(\Delta t_{ev}\). The process is in time nonlinear since the frost layer insulate the surface against the heat transfer, but also leads to lower airflow through heat exchanger.

The majority of air source heat pumps uses the reversed refrigerant cycle of heat pump for defrosting. During defrosting heat pump doesn’t produce any heating capacity for the heating system, but compressor is still consuming electric power therefore air source heat pumps have to optimize time between defrosting. It is usually done by measurement of temperature difference between ambient temperature \(t_a\) and evaporating temperature of refrigerant. Its value of is set by manufacturer to regulator and it is based on empirical knowledge or some measurement. The bigger difference means lower \(t_e\), in the same ambient temperature, which leads to approximately 2 % lower COP [1]. The problem of setting correct difference is much bigger when heat pump has a variable speed compressor.

The model presented in this paper can be used for defrosting optimization since it can predict the speed of frost formation and its impact on \(U_{ev}\) value.

The expansion valve sets the superheat temperature difference \(\Delta t_{sup}\). It controls the refrigerant flow rate from condenser to evaporator in order to achieve designed temperature difference between \(t_{ev}\) and \(t_{ev, out}\). If the \(\Delta t_{sup}\) is too low the expansion valve will not set stable refrigerant mass flow and \(\Delta t_{sup}\) would vary. On the other hand if \(\Delta t_{sup}\) is too high \(t_{ev}\) will be lower than optimal and COP will get lower. The model of evaporator and refrigerant circle can be used to find optimal \(\Delta t_{sup}\).

2. Models of heat pump
Mathematical model of heat pump was developed. It consist sub model of every major component – condenser, evaporator, compressor and expansion valve. The components are connected though refrigerant cycle as shown in Fig. 1.

Since the model is intended to describe universal heat pump, its main inputs can be derived from each components manufacturer data. In best scenario the air source heat pump would not have to be measured at all, or in just in couple working point, just to verify the model.

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2.1. The model of refrigerant cycle

The described model of heat pump works with refrigerant as its working fluid. The refrigerant connects all major parts of heat pump in thermodynamic cycle shown in Fig. 2.

The refrigerant flows from the outlet of evaporator 1 to the suction of compressor. The heat pumps suction pipe is usually short enough to neglect additional heat gain and pressure losses. The pressure losses are neglected in whole model since they are usually not significant comparing to pressure drop in expansion valve. The refrigerant is compressed in compressor and discharged as superheated steam to condenser. In condenser refrigerant transfer its heat energy to water and comes from condenser in liquid phase as slightly subcooled liquid.

From the condensed the refrigerant flows to expansion valve. The expansion is considered as adiabatic since the expansion valve is usually at the entrance to the evaporator. The refrigerant is evaporated and superheated in evaporator.

The refrigerant thermodynamic properties were modeled by Refprop library. The use functions were implemented to heat pump thermodynamic cycle model.

2.2. The model of compressor

Compressor has the similar function as human hearth. It forces the refrigerant movement by increasing its pressure and temperature and transferring it from the evaporating to condensing side of the refrigerant cycle.

Since the majority of heat pumps, which were newly introduced to the market, have inverter driven compressors, the variable speed model of compressor was developed. Simplified model of real compressor is derived from the theory of piston compressors. It describes the mass flow of refrigerant and power input of compressor in every working conditions and can be derived from the manufacturer’s compressor data. The refrigerant mass flow is described as follows:

$$m_{ref} = V_m \cdot \lambda_s \cdot \rho_{ref} \cdot s \cdot n$$

(2)

The swept volume in the case of compressors with complicated geometry can be defined as volume of sucked gas by the compressor with pressure ratio 1. In the case of compressors with variable speed it is not just a constant. It depends also on n and can be described as follows:

$$V_m = C_1 \cdot n^3 + C_2 \cdot n^2 + C_3 \cdot n + C_4$$

(3)

The volumetric efficiency $\lambda_s$ is changing with different pressure and temperature state of refrigerant in suction and discharge of compressor. It depends on pressure ratio. In the foregone applications of vastly used piston compressors was the volumetric efficiency function of clearance volume when the piston is at the top dead center. Today’s mostly used compressors in heat pumps applications are without mentioned issue and volumetric efficiency is close to 1 and can be described as follows:

$$\lambda_s = 1 - C_s \cdot (\sigma - 1)$$

(5)

The isentropic efficiency describes the compressor’s energy efficiency. The real compression process can be defined as polytrophic with varying exponent of compression. The simplified description of real compression is by the difference between real and isentropic compression. The isentropic efficiency in model is described as follows [4]:

$$\eta_s = D_1 + D_2 \cdot n + D_3 \cdot n^2 + D_4 \cdot \phi + D_5 \cdot \phi^2 + D_6 \cdot n \cdot \phi + D_7 \cdot p_s$$

(6)

The coefficient $\phi$ is defined as follows:

$$\phi = \sigma^{\frac{\gamma}{\gamma - 1}}$$

(7)

Polytrophic index of compression can be derived from thermodynamics properties of refrigerant and is defined as follows:

$$n_{poly} = \frac{\ln p_s}{\ln \frac{\rho_i}{\rho_s}}$$

(8)
The model of compressor accordingly to pressure conditions at the suction and discharge counts the pressure ratio, than isentropic and volumetric efficiency. After that the enthalpy of refrigerant in discharge is computed as follows:

\[ h_2 = h_1 + \frac{h_{s,w} - h_i}{\eta_v} \]  

(9)

The compressor power input is defined by:

\[ P_{com} = \dot{m}_{ref} \cdot (h_2 - h_1) + \dot{Q}_{loss} \]  

(10)

The heat loss cannot be counted exactly from compressor data sheet, it is estimated by manufacturers to be 5 % of compressor power input. For insulated compressors used in heat pump applications can be neglected (it is no one of the inputs of model).

The model of compressor has been validated on variable speed compressor MITSUBISHI ANB33FBDMT. The model described compressor with sufficient precision. The result of test has been published in [5].

2.2.1. The compressor working envelope

Every refrigerant compressor has work inside its working envelope given by the operational limits. The typical working envelope is shown on Fig. 1. The working envelope is mainly limited by maximal and minimal evaporating and condensing pressure and pressure ratio. The compressor can’t work outside this envelope.

![Fig. 3. The working envelope of compressor.](image)

2.3. The model of condenser

For the steady state conditions the heat flows of liquid on both sides equals to heat flow transferred by exchanger working surface. It can be described as follows:

\[ \dot{Q}_{con} = \dot{m}_{ref} \cdot abs(h_{ref,in} - h_{ref,out}) \]  

(11)

\[ \dot{Q}_{con} = \dot{m}_{liq} \cdot c_{p,liq} \cdot abs(t_{liq,out} - t_{liq,in}) \]  

(12)

The eq. (11) and (12) describe heat transferred on the refrigerant side and secondary side. The eq. (13) describes heat transfer rate through heat exchanger surface. The heat transfer rate has to be the same.

The condenser is modeled as 3 successive heat exchangers as it is shown on Fig. 3. In the first heat exchanger the incoming hot refrigerant gas is cooled to condensing temperature. In the second section is refrigerant condensed and in the last part is subcooled. The numerical solver implemented to model changes the condensing temperature until the sufficient accuracy between eq. (11), (12) and (13) is reached.

![Fig. 4. The temperature profile of water and refrigerant in the model of condenser.](image)

The condenser in heat pump systems is usually brazed plate heat exchanger. The inside geometry in not known and the lack of information cause to consider the UA value of condenser to be approximately constant in all working conditions.

2.4. The model of evaporator

The model of evaporator is very complex. In involves processes like heat and mass transfer or pressure drop. The heat and transfer on the surface area is not unified. It varies with the temperature difference, frost layer thickness, the air velocity and the others. The mentioned problems led to modeling the evaporator in sections.

The refrigerant flows through distributor from expansion valve. The distributor divide the stream of refrigerant to different Loops of evaporator. The distribution of refrigerant is considered to be equal in every Loop of evaporator. The scheme of different Loops is in Fig. 5.
The inputs to the model of one section are:
- the surface area of section;
- the thickness and material of tube and fins;
- geometry – fin pitch, horizontal and vertical distance between tubes;
- frost layer properties – thickness, thermal conductivity;
- inlet temperature, mass flow rate and relative humidity and enthalpy of refrigerant and air.

The model firstly calculate the heat transfer coefficient and pressure drop of the section. The governing equation can be found in [6]. With the estimation of previously mentioned values the heat transfer rate is calculated both from calorimetric equation and heat transfer equation:

\[ \dot{Q}_{S,i} = \frac{A_S \cdot \delta_{in}}{\mu_{air} + \rho_{ice}} \cdot \frac{c_{\alpha}}{\lambda_{air}} \cdot \frac{m_{air,S} \cdot c_{p,air} \cdot (t_{air,in} - t_{air,out})}{S_{S}} \tag{14} \]

\[ \dot{Q}_{c} = m_{ref,in} \cdot c_{p,ref} \cdot (t_{ref,in} - t_{ref,out}) \tag{15} \]

The heat transfer rate from eq. (14) and (15) must be equal. To achieve conformity between eq. (14) and (15) the \( t_{air,out} \) is being changed by implemented solver. With the knowledge of heat transfer rate and properties of materials and frost layer the surface temperature is calculated. If the surface temperature is lower than dew point temperature of air, the condensation of air moisture has to be calculated. To work with moist air properties the Psych software [7] was implemented to model. With the estimation of mean surface temperature the heat capacity transferred by condensing moisture \( \dot{Q}_{c,S,j} \) is calculated. The total heat transfer in section is calculated by (16). The enthalpy of refrigerant at the outlet of section is given by (17).

\[ \dot{Q}_{S} = \dot{Q}_{S,i} + \dot{Q}_{c} \tag{16} \]

\[ h_{ref,out} = h_{ref,in} + \dot{Q}_{S} \cdot m_{ref} \tag{17} \]

The main outputs of the section model are the outlet enthalpies and temperatures of air and refrigerant, the heat transfer coefficient, pressure drop, the rate of frost formation, the rate of water condensation.

### 2.4.2. The model of one Loop

The models of each section are connected by model of Loop. The configuration of Loop is shown on Fig. 6. The Loop model starts the calculations of section models from the Section 1 to Section X and calculates the overall heat transfer coefficient of Loop. With the knowledge of number of Loops the overall heat transfer coefficient of whole evaporator \( U_{ev} \) and pressure drop \( \Delta p_{air,ev} \) is calculated. The model of Loop has implemented model of ventilator and numerical solver for volume flow of air estimation. The simplified model of ventilator calculates the air volume flow \( V_{air} \) and power input of ventilator \( P_{vent} \) by eq. (18) and (19):
\[ V_{air} = f(\Delta p_{air, ev}) \]  
\[ P = f(\Delta p_{air, ev}) \]  

The numerical solver firstly estimates \( V_{air} \), then calculates the pressure drop and consequently tries to find more precise \( V_{air} \) estimation, until the demanded accuracy is reached. Lastly the total cooling capacity of evaporator is calculated by (20)

\[ \dot{Q}_{ev} = n \sum_{i=1}^{X} \dot{Q}_{S,i} \]  

### 2.4.1. The model of whole evaporator

The last subsection of evaporator model connects it with model of refrigerant cycle. The evaporator is divided into two successive heat exchangers as is shown on Fig. 7. In the first one the refrigerant is evaporated. In the second one the refrigerant is superheated.

![Fig. 7. The temperature profile of air and refrigerant in the model of evaporator.](image)

With the estimation of \( U_{ev} \) and heat transfer area \( A_{ev} \), the eq. (20), (21) and (22) can be solved for both heat exchangers in the evaporator:

\[ \dot{Q}_{HX, ev} = m_{ref, abs}(h_{ref, in} - h_{ref, out}) \]  
\[ \dot{Q}_{HX, ev} = m_{air, com} \cdot c_{p, liq} \cdot abs(t_{liq, out} - t_{liq, in}) \]  
\[ \dot{Q}_{HX, ev} = U_{ev}A_{ev}\frac{\delta u - \delta l}{ln\frac{\delta u}{\delta l}} \]

The \( m_{air, com} \) is corrected mass flow of air. Since the part of heat capacity can be transferred by air moisture condensation, the air mass flow value in the equation (21) has to be changed to respect this effect. The eq. (23) describes mentioned correction.

\[ m_{air, com} = \frac{\dot{Q}_{ev}}{c_{p, air} \cdot (t_e - t_{X, air, out})} \]

The model of whole evaporator changes the evaporating temperature of refrigerant to reach balance between eq. (20), (21) and (22). Since with changing evaporation temperature the model of Loop and all sections is changed, the numerical solver was implemented to find correct evaporating temperature.

### 3. The model validation

The previously described model have not been validated completely yet. The precision of model can be (in the time when this paper was written) estimated only from compressor and evaporator validation.

#### 2.5. The model of compressor

The model of compressor from chapter 2.2 was validated on compressor MITSUBISHI ANB33FBDMT. The accuracy of model is shown in Fig. 8 and 9.

![Fig. 8. The accuracy of refrigerant mass flow prediction.](image)

![Fig. 9. The accuracy of compressor power input.](image)

The compressor was measured in the Honeywell laboratory in Brno [8]. Model described the compressor with sufficient accuracy.

#### 2.6. The model of condenser

To estimate the accuracy of evaporator model the special test-bench would be needed. Since it is not available the model of evaporator has to be compared with data from manufacturer. The manufacturer usually do not provide the data of evaporator in frosting conditions, therefore the model can be validated only with air moisture condensation.
The performance data from one nominal working point of evaporator Luvata 0722A3606090030EXX12 were available. The main geometry information is in Tab. 2. The comparison between model and datasheet data are in Tab. 3.

Table 2. Material and construction of evaporator.

<table>
<thead>
<tr>
<th>Description</th>
<th>Unit</th>
<th>Manufacturer</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fin type</td>
<td>-</td>
<td>25x21.65 staggered</td>
<td></td>
</tr>
<tr>
<td>Fin material</td>
<td>-</td>
<td>hydrophilic aluminum</td>
<td></td>
</tr>
<tr>
<td>Type of tube</td>
<td>-</td>
<td>CrossGrooved C</td>
<td></td>
</tr>
<tr>
<td>Fin spacing</td>
<td>n</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Number of rows</td>
<td>n</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>Number of circuits</td>
<td>n</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>Height of finned pack</td>
<td>mm</td>
<td>900</td>
<td></td>
</tr>
<tr>
<td>Length of finned pack</td>
<td>mm</td>
<td>900</td>
<td></td>
</tr>
</tbody>
</table>

Table 3. Evaporator model validation results.

<table>
<thead>
<tr>
<th>Description</th>
<th>Unit</th>
<th>Manufacturer</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air Entering temperature</td>
<td>°C</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>Air Relative humidity</td>
<td>%</td>
<td>87</td>
<td></td>
</tr>
<tr>
<td>Volumetric flow</td>
<td>m³/h</td>
<td>4400</td>
<td></td>
</tr>
<tr>
<td>Refrigerant Condensing temp.</td>
<td>°C</td>
<td>36</td>
<td></td>
</tr>
<tr>
<td>Refrigerant Liquid subcooling</td>
<td>K</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Refrigerant Total flow</td>
<td>kg/h</td>
<td>209</td>
<td>205</td>
</tr>
<tr>
<td>Refrigerant Outer area</td>
<td>m²</td>
<td>68.6</td>
<td>67.6</td>
</tr>
<tr>
<td>Evaporator temperature</td>
<td>°C</td>
<td>1.54</td>
<td>1.47</td>
</tr>
<tr>
<td>Total capacity</td>
<td>kW</td>
<td>10.4</td>
<td>10.1</td>
</tr>
<tr>
<td>Sensible capacity</td>
<td>kW</td>
<td>6.95</td>
<td>6.50</td>
</tr>
<tr>
<td>Latent capacity</td>
<td>kW</td>
<td>3.45</td>
<td>3.60</td>
</tr>
<tr>
<td>Condensed water</td>
<td>l/h</td>
<td>5.08</td>
<td>5.07</td>
</tr>
</tbody>
</table>

The major function of expansion valve is to prevent liquid refrigerant to enter the suction of compressor. The refrigerant entering the evaporator is the mixture of saturated gas and boiling liquid refrigerant. As the stream flows through evaporator tubes the mixture has more and more gas. In some point the share of gas in mixture is so dominant that boiling refrigerant is abducted by it. To prevent boiling refrigerant to enter compressor the expansion valve sets the refrigerant superheat. The expansion valve controls the temperature difference between evaporating temperature \( t_{ev} \) and refrigerant outlet temperature \( t_{ref,out} \). If the temperature difference is lower than demanded, the expansion valve throttles the refrigerant flow. The problem can appear when the set superheat temperature difference is low and the transferring heat capacity high. The drops of boiling refrigerant can land on the surface where the bulb controlling expansion valve is placed. The boiling refrigerant has the evaporating temperature and although the gas refrigerant stream is superheated the bulb would measure \( t_{ev} \). The expansion valve would throttle the refrigerant stream and when the boiling refrigerant close to bulb is evaporated, the bulb would measure bigger than set superheat and the expansion valve would open more. The whole process then repeats.

To prevent mentioned effect the sufficient superheat has to be set. Minimal stable superheat of refrigerant corresponds to heat capacity transferred by evaporator as shown on Fig. 11.

The model can predict the superheating temperature difference by estimating of place (tube) where the refrigerant should be theoretically evaporated. The example of such prediction is in Fig. 12, 13 and 14. On Fig. 12 is the temperature profile or air and refrigerant in one Loop of evaporator. The superheat is set 6 K and compressor rotational speed is set to 50 Hz. The evaporator is over dimensioned / the end of evaporation process is in Section 5. The sections 1 to 4 are just for refrigerant superheating – almost more than ¼ of surface area.
Fig. 12. Temperature profile of Loop – 50 Hz, 6 K superheat.

On Fig. 13 the compressor rotational speed was set to 120 Hz. The refrigerant was evaporated in Section 2. In that case the unstable behavior of expansion valve can occur.

Fig. 13. Temperature profile of Loop – 100 Hz, 6 K superheat.

The last Fig. 14 represents the rotational speed 120 Hz and superheat 8 K. The refrigerant was evaporated in Section 3. Although the superheat was 2 K higher the evaporating temperature stayed almost constant.

Fig. 14. Temperature profile of Loop – 100 Hz, 8 K superheat.

2.4.1. Compressor rotational speed setting

The model can be modified to set the rotational speed of compressor to get demanded heat capacity when needed or to consume demanded electricity input. The first case scenario can happened when in households when there is big hot water demand. The second scenario can happened in some buildings with photovoltaic systems when there is need to consume the electricity production instead of sending it to public electricity network. The model can predict the power input of compressor and ventilator and the information can be used in regulator of compressor. The power input of compressor as function of rotational speed for the same conditions on evaporator and condenser is in Fig. 15.

Fig. 15. Compressor power input as function of compressor speed.

2.4.1. Defrosting optimization

The air source heat pump needs to defrost in ambient temperatures close to 0 °C. The heat pump controller usually measures difference between evaporating temperature and ambient temperature and when it is bigger than set value the defrosting process start. During defrosting the whole evaporator surface has to be heated to 0 °C and overheated to set temperature in order to ensure that all frost was melted down.

The problem with such logic faces is that with modulated compressor the cooling capacity varies and the evaporating temperature is changing in accordance with it (see Fig. 12 and 13). The presented model can calculate the value of evaporating temperature with no frost and compare it with measured value. When the difference is bigger than set. The defrosting process can start.

The other option which can be made is to predict future evaporating temperature in time for given conditions. The heat transfer properties of evaporator don’t change in time linearly so the model can predict ideal time of defrosting with simple economic function. It is caused by insulation of heat transfer area by frost layer and growing pressure drop on air side leading to lower volumetric air flow. The change in evaporating temperature with grooving frost layer is in Fig. 16.
3. Final remarks and future work

The presented model was not validated yet. The validation will be possible in specialized laboratory. The mathematical model can be good for heat exchangers optimization, heating and cooling capacities prediction and prediction of frost formation.

The internal structure of model is easy for changing parameter. There are still some parts which need some improvement:

- thermal properties of frost layer;
- the overall heat transfer coefficient of condenser as function of refrigerant and water properties;
- the more geometry of evaporator – the internal loops of real evaporator don’t have the same number of tube;
- the heat transfer on refrigerant side of evaporator calculation.

The mentioned drawbacks do not make the model less unreal. The prediction of behavior is logical and model sufficiently precise.

Acknowledgement

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Nomenclature

- $A$ heat exchanger active area (m$^2$)
- $C$ constant (-)
- $COP$ coefficient of performance (-)
- $c_p$ specific heat at constant pressure (J·kg$^{-1}$·K$^{-1}$)
- $D$ constant (-)
- $h$ specific enthalpy (kJ·kg$^{-1}$); $m$ mass flow (kg·s$^{-1}$)
- $n$ rotational speed (Hz);
- $p$ pressure (Pa)
- $P$ power input (W)
- $\dot{Q}$ heat capacity (W).
- $SCOP$ seasonal performance factor of heat pump (-)
- $SPF$ seasonal performance factor of system with heat pump (-)
- $t$ temperature (°C)
- $U$ overall heat transfer coefficient (W·m$^{-2}$·K$^{-1}$)
- $V$ volume (m$^3$);
- $\eta$ efficiency (-)
- $\Delta$ deference (-)
- $\phi$ auxiliary coefficient (-)
- $\delta$ temperature difference (K)
- $\lambda$ volumetric efficiency, heat conductivity (-)
- $\sigma$ pressure ratio (-)
- $\rho$ density (kg·m$^{-3}$)
- $\kappa$ polytrophic index of compression (-).

Fig. 16. The time change of evaporating temperature.

References