Computational Comparison of Hydrostatic Guide Way Regulators

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Abstract
In this contribution operation principle of machine tools hydrostatic regulators are described and based on mathematical models a case study comparing a performance of hydrostatic regulators connected to a pocket without preload is evaluated.

Key words
Hydrostatics, Constant flow rate regulation, Flow divider, Constant flow resistance regulation, Capillary, Progressive flow rate regulation, PM regulator, Frequency driven pump control, Proportional pressure valve control.

1. Introduction

Hydrostatic (HS) guide ways are considered as one of the basic types of sliding guide ways, however it is ranked as a younger type of guide way. Hydrostatics has been with exceptions used in the mechanical engineering since seventies of twentieth century and frequently has been applied in the machine tool industry as a guide way of Z axis of middle size and large lathes, as a guide way of Y axis of horizontal milling machines and as a guide way of turntables of vertical turning lathes and milling machines.

Hydrostatic guide ways are nowadays used for precise machine tools, for machines with requirement of minimal positioning steps, for hard cutting machines (high dynamical loads from cutting forces require high damping capability of a MT structure) and for big machine tools, where it is difficult to solve movable connection with conventional guide ways with rolling elements.

Principle of the hydrostatic guide way is based on continuous supply of pressurized hydraulic oil between guiding faces. Supply of pressurized oil allows for fluid friction of guiding faces which is distinguished by a very small friction coefficient \( f = 5 \times 10^{-6} \) with a broad speed range. The low coefficient of friction is a significant advantage of this type of guide-way; during the movement the coefficient of friction reaches \( f = 10^{-4} \div 10^{-5} \) which is up to 100 times less than what is achievable with roller guides and up to 10000 times less than slide ways. Stability at high speeds and precision at slow movement is enabled by increasing a friction force with increasing speed of movement. The HS guide way excels also with high service life, damping capabilities and a high stiffness in the direction perpendicular to the motion direction.

The stiffness of the hydrostatic guide way depends on the height of a throttling gap and especially on the type of regulation of flow of pressurized oil in to the pocket hollow. It is the aim of this research paper to compare performance of different hydrostatic systems based on computational comparison of HS regulation systems.
2. Hydrostatic systems

The hydrostatic system generally consists of a hydraulic medium flow regulator, a hydrostatic cell and a medium supply system (Fig. 1). The hydrostatic system is usually formed from more than one HS pocket which need to be independently supplied by hydraulic medium in order to reach a steady-state of force equilibrium at differently and variably loaded pockets. From this reason a few, in this chapter outlined, methods of regulation are used.

![Fig. 1: Components of hydrostatic system](image)

2.1 Hydrostatic cell

The hydrostatic cell is formed by the hydrostatic pocket and a guiding prism. On the Fig. 2 displaying a bottom view of a simplified HS pocket can be recognized a cavity in the middle and a land lining the cavity. During the operation hydraulic medium (usually mineral hydraulic oil of known viscosity) is fed into the cavity creating a region of a constant pressure even at higher speeds [2]. Between the land and the guiding prism is by action of pressurized oil created a narrow gap throttling flow of pressurized oil outside of the hydrostatic cell. If we assume linear decline of the pressure thru a width $l$ of the land we can define an effective area $A_{eff}$ of the pocket which is critical for determining a reaction force of the hydrostatic cell [1].

The model of the throttling of oil in a real hydrostatic pocket with radii in the corners is based on Navier-Stokes equation under an assumption of laminar flow [3]:

![Fig. 2: Scheme of HS pocket](image)
where \( r_p, l, a, b \) are dimensions of the HS pocket, \( h \) is the height of the throttling gap and \( \mu \) denotes a dynamical viscosity of hydraulic oil used to feed the pocket. A simplified model neglecting an effect of radii in the corners is based on an exact description of a hydraulic conductivity of a rectangular tube [4]:

\[
C = \frac{ab^3}{12} f \left( \frac{a}{b} \right)
\]

Where \( a \) and \( b \) denotes the dimensions of cross section of the tube and:

\[
f \left( \frac{a}{b} \right) = 1 - \frac{192b}{\pi^5 a} \sum_{n=0}^{\infty} \frac{\tanh \left( (2n+1)\pi \frac{a}{2b} \right)}{(2n+1)^5}
\]

Function \( f = 0.422 \) for \( a/b = 1 \) and approaching \( 1 \) for \( a>>b \) [4]. In a case of the hydrostatic pocket where a circumference of the effective area of the pocket is much larger than the height of throttling gap we can assume that:

\[
C_t = \frac{ab^3}{12}
\]

The hydraulic resistance of the simplified hydrostatic pocket is than expressed as [5]:

\[
R_t = \frac{12\mu l}{bh^3}
\]

Based on the experimental results described in [6] is apparent that the simplified model gives good results when compared to experiment.

### 2.2 Constant flow rate regulation

Constant flow rate regulation is based on feeding of oil in each pocket at constant flow rate which is carried out by means of a single pump for every hydrostatic pocket or by a gear flow divider (Fig. 3). A main advantage of this regulation method is a high loading capacity given by no restriction of oil flow into the HS pocket. The oil pressure in the pocket is limited only by a pressure available on the pump – this method’s energy efficient.

![Fig. 3: Connection of the constant flow rate regulation with gear flow divider](image)

### 2.3 Constant flow resistance regulation
Constant flow resistance regulation exploits the constant value of the pump pressure and is carried out by connecting a geometrically stable hydraulic resistance in front of each pocket (Fig. 2). This method is very cost-effective since the hydraulic resistance usually constitutes of a capillary tube or other narrow groove.

A model of the hydraulic resistance of the capillary tube has been built up on the basis of Hagen-Poiseuille equation which exactly describes flow thru a circular tube with inner radius $a$ \[ 4 \]:

$$ q = - \frac{\pi a^4 dp}{8\mu dz} $$

The hydraulic resistance $R_k$ of the capillary tube is expressed as \[ 3 \]:

$$ R_k = \frac{P_p}{Q} = \frac{8l\mu}{\pi r_c^2} $$

Where $r_c$ denotes capillaries inner radius, $l$ denotes length of the capillary and $P_p$ denotes the pump pressure. An experimental verification of a model of the hydraulic resistance has been done in \[ 7 \].

### 2.4 Progressive flow rate regulation

Progressive flow rate regulators are among others mostly experimental devices represented by a PM flow regulator from Hyprostatik GmbH. The PM regulator aims at high stiffness operation of the hydrostatic guide way. The regulator is designed in such a way that at low pocket pressures the flow is restricted limiting the tendency to increase the throttling gap when the HS pocket is at light-duty. On the other hand when the pocket pressure is high the regulator is opened allowing the pocket pressure to raise up to 95% of the pump pressure.

The behavior of the PM regulator is assured by a pressure difference driven variable hydraulic resistance, which is formed by a thin diaphragm (a control element) comparing the pump pressure with the pocket pressure. A displacement of the control element is changing the hydraulic resistance in opposite manner to the pocket pressure which assures very high stiffness of the hydrostatic system with PM regulator. \[ 8 \]; \[ 9 \]; \[ 10 \]; \[ 11 \]

Since PM regulators are manufactured in a way that their pressure – flow rate characteristics is in a working range almost linear it is possible to characterize a regulator only with two constants given by manufacturer and the flow rate thru the regulator $Q_r$ is expressed as \[ 12 \]:

$$ Q_r(p_T) = Q_0 \left( 1 + (K_r - 1) \frac{p_T}{P_p} \right) $$

Where $P_T$ denotes the pocket pressure, $Q_0$ is the minimum flow thru the regulator and $K_r$ is a ratio between maximum and minimum flow rate thru the regulator.

### 3. Computational comparison of hydrostatic regulation systems

Computations have been carried out in order to compare a behavior of described methods of regulation on a specific example of a hydrostatic linear guide way which is not preloaded by opposite HS pocket. Computations are done in numerical manner. The guiding prism has been moved by small steps from a zero throttling gap up to twice the design height, computing oil flow, pocket pressure, reactions, stiffness and hydraulic power for each step. The results for each method of regulation are plotted into graphs illustrating load carrying, stiffness, oil flow,
pocket pressure and hydraulic power characteristics of the hydrostatic cell with respect to the height of the throttling gap. Plotted characteristics of hydraulic power represent the power consumed by throttling in hydrostatic system from the view of the oil supply system thus as would be measured on the hydraulic line from the oil supply system.

Parameters of HS pocket geometry and oil supply system have been restricted to values in Tab. 1. Dimensions of pocket geometry is denoted as illustrated in Fig. 2.

### 3.1 Constant flow rate regulation

For this regulation method, which was described in chapter 2.2, was assumed a condition of constant flow rate for all heights of the throttling gap until the nominal pump pressure was reached. Smaller throttling gaps keep the nominal pump pressure and the flow rate is decreasing with the throttling gap. Results are plotted in Fig. 4.

### 3.2 Constant flow resistance regulation with capillaries

The capillary for this regulation method, which is described in chapter 2.3, was designed according to an approach which is optimized for preloaded guide ways. The hydraulic resistance of the capillary is 1.5 times higher than the hydraulic resistance of the hydrostatic pocket. Results are plotted in Fig. 5. If the hydraulic resistances ration would be increased the load carrying capacity would decrease at specific height of throttling gap and the maximum stiffness would increase and shift to lower heights of the throttling gap, considering no change in the oil supply system. If the ration would decreased, the load carrying capacity would increase at specific height of the throttling gap and the stiffness would decrease.

### 3.3 Progressive flow rate regulation with PM flow regulators

The regulator for this method of regulation, is characterized by two constant $Q_0$ (the minimum flow) and $K_r$ (the ratio between maximum and minimum flow rate) as described in chapter 2.3. There is no general rule in designing those constants; however there are common values for the $K_r$ ratio, for most applications ranging from 1.6 to 3.5 (for steeper stiffness characteristics) [12]. The minimum flow $Q_0$ is usually designed according to the hydrostatic pocket size and desired height of the throttling gap. With the flow rate is increased also the height of the throttling gap and decreased the guide way stiffness in direction perpendicular to the movement. The maximum pocket pressure is limited to 95% of the pump pressure due to the residual resistance of the fully opened PM regulator. Results of a system with regulator with $K_r = 2.3$, which is the most often encountered value giving reasonable results and with $Q_0 = 31 \text{ cm}^3/\text{s}$ designed to utilize the maximum flow rate of the pump, are plotted in Fig. 6.

<table>
<thead>
<tr>
<th>Tab. 1: Fixed parameters of hydrostatic system</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>HS pocket geometry</strong></td>
</tr>
<tr>
<td><strong>Outside dimension</strong> $A = 60 \text{ mm}$</td>
</tr>
<tr>
<td><strong>Throttling gap length</strong> $l = 8 \text{ mm}$</td>
</tr>
<tr>
<td><strong>Oil supply system</strong></td>
</tr>
<tr>
<td><strong>Maximal flow rate</strong> $Q = 0.6 \text{ l/min}$</td>
</tr>
<tr>
<td><strong>Oil viscosity grade</strong> ISO VG 46</td>
</tr>
<tr>
<td><strong>Operating temperature</strong> $T = 30^\circ\text{C}$</td>
</tr>
</tbody>
</table>
Fig. 4: Characteristics of the hydrostatic system with constant flow rate regulation

Fig. 5: Characteristics of the hydrostatic system with constant resistance regulation
4. Results discussion

As is illustrated in previous chapter, a slope of load carrying and stiffness characteristics is distinctly steeper with the constant flow rate regulation and with the PM regulator than with the constant flow resistance regulation which results in significantly smaller magnitude of a maximum stiffness of the constant resistance regulation. The PM regulator has the highest magnitude of stiffness in the direction perpendicular to the direction of motion and also has the narrowest working range – a difference between maximum and minimum usable throttling gap.

Since the oil supply system and the hydrostatic pocket geometry are same for all three methods of regulation, the maximum load carrying capacity, which is only a function of the effective area of the pocket and the pocket pressure, is similar for every method. The only differences are the regulators residual resistances which were not evaluated.

All three regulation methods make use of the maximum flow rate and the maximum pump pressure. Constant flow rate regulation and PM regulator utilize the full hydraulic power (P_{max} and Q_{max}) when fully loaded and decrease the power demand with increasing throttling gap (unloading). Constant resistance regulation exhibits the highest power demand when unloaded due to constant supply of oil at the maximum pressure and the flow rate rising with the height of the throttling gap.
Values of the load carrying capacity and stiffness at a nominal height of the throttling gap \( h = 50 \ \mu m \) as well as their maximum values and working range of the throttling gap are summarized in Tab. 2.

Tab. 2: Summary of nominal and maximum values of load carrying capacities and stiffness of exemplary hydrostatic guide ways

<table>
<thead>
<tr>
<th>Regulation method</th>
<th>Load carrying capacity at ( h=50 \ \mu m ) [kN]</th>
<th>Maximum load carrying capacity [kN]</th>
<th>Stiffness at ( h=50 \ \mu m ) [kN/\mu m]</th>
<th>Peak stiffness [kN/\mu m]</th>
<th>Working range of the throttling gap [\mu m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant flow rate</td>
<td>4,5</td>
<td>8,3</td>
<td>0,28</td>
<td>0,58</td>
<td>49,0</td>
</tr>
<tr>
<td>Constant resistance</td>
<td>2,8</td>
<td>-</td>
<td>0,11</td>
<td>0,17</td>
<td>85</td>
</tr>
<tr>
<td>PM regulator</td>
<td>3,5</td>
<td>7,5</td>
<td>0,35</td>
<td>0,97</td>
<td>31</td>
</tr>
</tbody>
</table>

* Working range is the difference in the height of the throttling gap between 100% and 10% of the load carrying capacity

5. Conclusion

On the basis of computational comparison of regulation methods of exemplary hydrostatic guide way without preload was determined that for same geometry of the pocket and the same supply system each of the regulation systems has different behavior, which could be beneficial for different applications.

The guide way with constant flow regulation excels with highest load carrying capacity, reasonable stiffness and lowest overall energy consumptions. Those properties are given by mechanical splitting of the fluid flow by means of gear flow divider which function is not based on throttling the fluid. The regulation system is supplied with constant flow of oil with pressure corresponding to the load. This regulation method would be suitable for applications with high loads and on larger machines where larger throttling gaps are required due to lower machining and assembly tolerances of the guiding faces, consequently it is advantageous to pursue energy demands.

The guide way with PM regulator excels with the highest stiffness, very good load carrying capacity and energy demand, however with higher throttling gap the stiffness is decreasing. Therefore PM regulated guide way is suitable for application where superior stiffness is required and additional costs for precise manufacturing and assembly of guiding faces with small throttling gap are acceptable.

The guide way with constant resistance regulation by capillaries has very gradual load carrying capacity with no sharp transitions and low stiffness. Stiffness could be improved by capillary with higher resistance but load carrying capacity at nominal throttling gap would be lowered and vice versa. On the other hand capillaries are easy to manufacture and easy to modify in order to correct design errors. Therefore capillary regulated guide way is low cost and low tech solution which is perfect for less demanding applications.

\[ f \] coefficient of friction \((-\))
\[ F \] pockets reaction force \((N)\)
\[ h \] height of the throttling gap \((\mu m)\)
\[ P_t \] pocket pressure \((Pa)\)
\[ P_p \] pump pressure / supply pressure \((Pa)\)
\[ A \] first outer pocket dimension \((m)\)
\[\begin{array}{ll}
C & \text{second outer pocket dimension} \\
a & \text{first inner pocket dimension} \\
c & \text{second inner pocket dimension} \\
l & \text{length of the throttling gap} \\
b & \text{circumferences of the effective area} \\
A_{\text{eff}} & \text{effective area} \\
\mu & \text{dynamical viscosity} \\
r_p & \text{corner radius} \\
R_t & \text{pocket hydraulic resistance} \\
r_c & \text{capillary inner radius} \\
R_k & \text{capillary hydraulic resistance} \\
Q & \text{oil flow rate} \\
Q_0 & \text{minimum flow rate of PM regulator} \\
K_r & \text{PM regulator constant} \\
T & \text{oil temperature}
\end{array}\]
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