Analysis of uncertainty in object positions measurement in digital image

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Abstract

Presented here is a method for uncertainty estimation in object positions measurement using digital image analysis. Objects (spots on a surface) are imaged using a scanner into a digital image. Image analysis provides object positions. The process of object imaging and image analysis is repeated and information about object positions is gathered. The statistical analysis if the information gives an estimate of the uncertainty in object positions measurement. The presented method can be considered as an instance of a measurement system analysis.

Key words

Uncertainty estimation, measurement system analysis, statistical analysis.

1. Introduction

Information about uncertainty in a measurement is essential for any reasonable interpretation of an experiment including such a measurement. It is not rare that as results of experiments are provided just numbers without any information about uncertainty. Such situations are common in student's works but even results of election preferences estimations in TV broadcasting are often displayed without necessary indicators of uncertainty.

There is no worldwide standard for evaluating uncertainty in a measurement. In this work we will refer to three guides where approaches to uncertainty can be found. Bell's Guide to Uncertainty of Measurement [1] contains very comprehensive formulations of important terms and clear step-by-step guide to assess uncertainty. Bernard's approach [2] is focused mainly to engineering audience and might be not as clear as Bell's but together with Zvárová's book [3] emphasizes one important aspect of data analysis. This aspect is simply the step where the experimenter *visualizes* the data, *looks* at that data visualization and creates initial conjectures about the data. It will be shown that this initial human assessment is very valuable and provides more „white-box" view on the data than the more „black-box" or rigor step-by-step approach.

Our measurement is motivated by the requirements of a future simulation project. In such a simulation we want to generate synthetic sets of points with similar properties as the properties of sets of points representing the real objects. So we will measure the displacement of real objects, estimate its size and uncertainty parameters and use them in the simulation.

2. Measuring the displacement

We want to measure the *displacement* in object positions. According to [1] it means that we want a result consisting of *three* numbers – the *size* of the displacement, the *magnitude* of its uncertainty and the *level of confidence* of the uncertainty. For example, if we measured the length of a rod, we would might have the result in the form: „The length of the rod is *23 cm*

+/- 1 cm and there is probability less or equal than *5 %* that the true length of the rod falls outside the interval (22; 24) cm.“ Here *23 cm* is the size, *+/-1 cm* is the uncertainty and *5 %* is the level of confidence.

The task formulation is as follows. Estimate the size and uncertainty of the displacement in object positions measurement. The object is a spot on a paper. Spots are formed in groups each containing N spots (Figure 1). A spot is represented as a point in 2D space and its position is expressed as a pair of coordinates (x, y). A group of spots is represented as a set of points $\{(x_1, y_1), (x_2, y_2), (x_N, y_N)\}$ where N is the number of spots in the group (Figure 2). Displacement is the distance between two corresponding points from two corresponding point sets (Figure 3, Figure 4). Corresponding point sets are two (generally different) representations of a single group of spots.

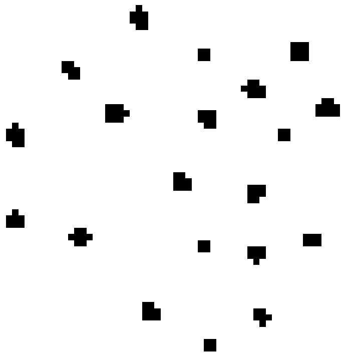

Figure 1. A group of 20 spots.

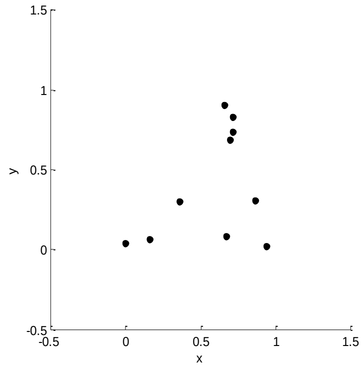

Figure 2. Set of 10 points representing a group of 10 spots.

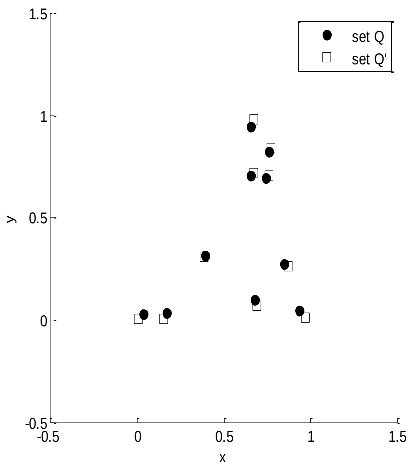

Figure 3. Two corresponding sets of points. Each set has 10 points. There is a displacement in each corresponding pair of points.

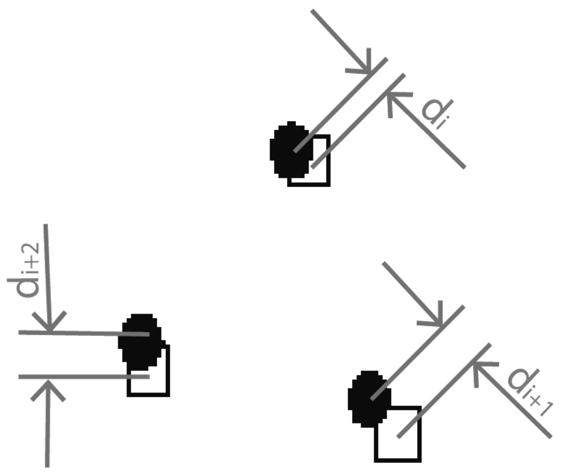

Figure 4. Displacement in corresponding pairs of points (enlarged lower right area in Figure 3).

3. State-of-the-art

As mentioned in the Introduction there is no single worldwide accepted standard for uncertainty evaluation. The approach in [1] somehow hides the statistical background of the matter. The classification of uncertainty evaluation into two types Type A and Type B where Type A „uses statistics“ and Type B „uses any other information“ might be somehow confusing. According to our point of view there *is* statistics behind both of the two types. The statistics in Type B uncertainty evaluation is just hidden behind magical numbers like $\frac{1}{\sqrt{3}}$.

Such numbers originate from distributions connected with measurement errors (i.e. $\frac{1}{\sqrt{3}}$ comes from the uniform distribution and expresses about 58 % of the width of the error interval). Strategies [2] and [3] point out the importance of visualizing the data and provide deeper insight into the statistical base of the uncertainty.

4. The method and results

To estimate the size of the displacement and its uncertainty we designed the following experiment. An object consisting of 40 spots was created (Figure 5). The object was imaged three times as three digital images (Figure 6). The digital images were processed so that positions of individual spots were recorded resulting in three sets with 40 point pairs each. Correspondences between points from the first and the second and from the first and the third set were found. Then the second and the third sets were aligned with the first set using the `procrustes` algorithm [4]. The displacements between corresponding point pairs were calculated. After this operation 80 values of displacement were obtained.
The relative location of the two corresponding point sets is determined by translating and rotating the two sets until the sum of the squares of the displacements is minimal. This was done using the `procrustes` algorithm [4].

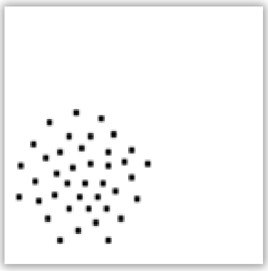

Figure 5. Object consisting of 40 dots.

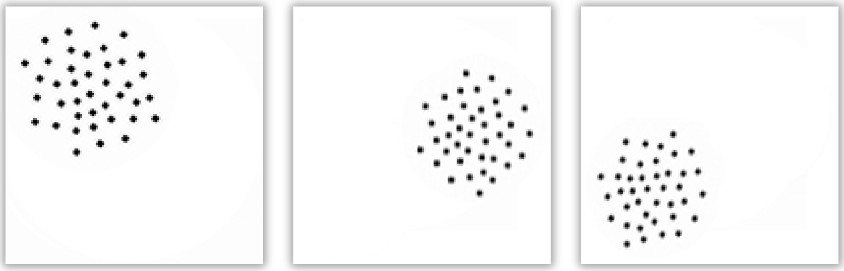

Figure 6. The object from Figure 5 imaged three times as three digital images. The images are translated and rotated.

The analysis started with looking at the data in the form of the histogram (Figure 7). The histogram indicates non-symmetrical distribution so we are in doubt that it is possible to treat the displacement as normally distributed. Next step was the box-and-whisker plot (Figure 8). Again the plot suggests non-symmetry in the data.

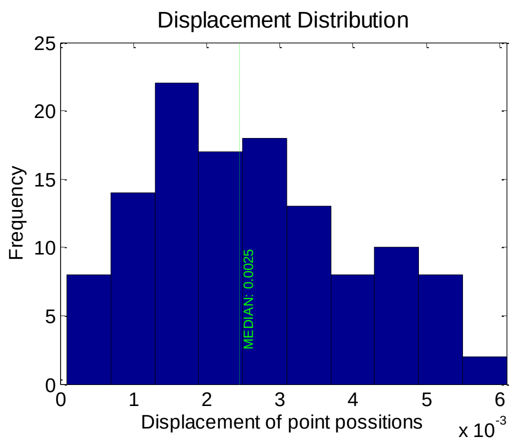

Figure 7. Displacement distribution.

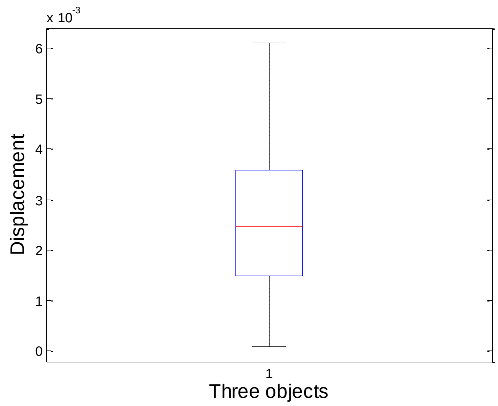

Figure 8. Boxplot of the displacement.

Further step was displaying the normal probability plot (Figure 9). Again the not-closeness of the blue points (the measured displacement) to the red dashed line indicates not-normal distribution. Finally the quantitative test for normality was performed. We used Jarque-Bera test [5] and it rejected the null hypothesis that the data (the displacement) comes from the population with normal distribution (p-value was 0.04).

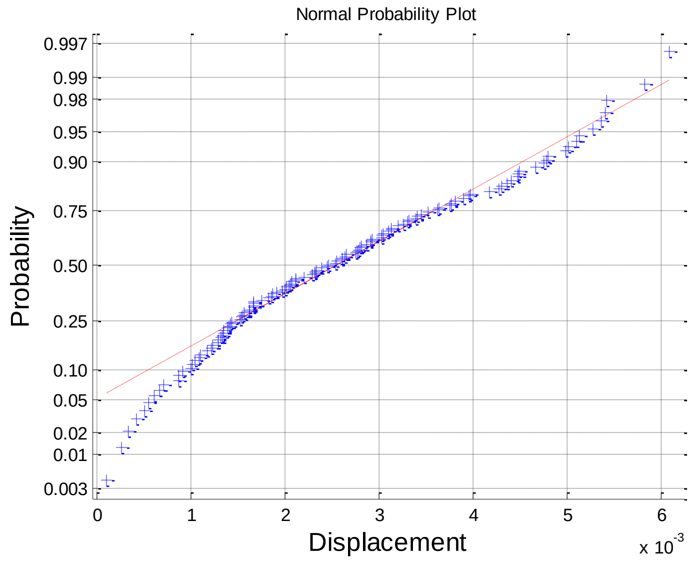

Figure 9. Normal plot of the displacement

We concluded that we could not treat the measured data as normally distributed. Because of the non-symmetry we picked the median (0.0025) as the measure of the *size* of the displacement and the interquartile range (0.0021) as the measure of the magnitude of the uncertainty. The related *confidence level* was 0.5.

5. Conclusion

Results of the analysis are depicted in Table 1. We provided a method for uncertainty evaluation for a case when more specific than standard treatment of the data is needed. The assessment of visualized data was shown.

Table 1. Results of the displacement measurement.

measured variable	size	uncertainty	confidence level
displacement	0.0025	0.0021	50%

The future work will be focused on finding suitable model for the displacement distribution and fitting the model to measured data. The fitted model will provide more convenient means for further usage in the simulation project.

Reference

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