Abstract
This study disserts about the CFD optimization applied on a practical example, a curved diffuser. A numerical investigation has been made into the flow behaviour in a curved diffuser with the focus on local losses. Turbulence and other three-dimensional effects are considered. An investigation is made into the effects upon the flow of the curvature of the diffuser’s geometry. A global extreme (a minimum of local losses) is searched by an approximation of a target function.

Keywords
CFD, optimization, curved diffuser, MATLAB, ANSYS Fluent, global extreme, radial basis function, separation, vortex structure, secondary flow, losses.

1. Introduction
The optimization is the phenomenon of any engineering application in nowadays; it is a very effective procedure used to get the closest result of a searched optimum without trying a quantum of variations. On the other hand, it is a very difficult and sophisticated task. Therefore, the optimization process requires a leader or more often, a group of experienced people who are able to find an optimal compromise. However, software which perform an optimization of diverse tasks in many technical fields are developed and used presently. Hereupon companies which are facing optimization tasks have to make a classical economic decision of their capital’s investment in software or researchers.

A curved diffuser is the mechanical device designed to control a flow field; it slows down the fluid’s velocity at the entrance to an open thermodynamic system. Contrariwise, a mechanical device where the fluid’s velocity accelerates is called a nozzle.

The goal of this optimization task is to find the global extreme of a target function; the global minimum as the lowest local losses. To approach this point as close as possible, the genetic algorithms are often used or other methods to find local extremes possibly near to global extremes. Here, a local extreme location the closest to a global extreme is implemented by an approximation of a target function.

The finding of the optimal diffuser’s geometry is considered as a complicated and heavy problem since some unfavorable flow’s effects may occur in this task. A flow’s separation; even a double flow’s separation and vortex structures due to three-dimensionality could be expected.

2. Formulation
Since an optimal geometry with minimal losses at the output needs to be found, this task is specified as the Design Problem [1].

The geometry of a curved diffuser has two different cross sections connected by spline curves as it is shown in Fig. 1. These cross sections are connected tangentially for all of
searched optimal geometries of a curved diffuser in the selected interval $I_{X,Y} = <10;20> \text{ mm}$ (the interval of the horizontal length $X$ and the vertical height $Y$). The dimension of the input cross section is $5 \times 5 \text{ mm} (a \times d)$ and of the output cross section is $10 \times 5 \text{ mm} (b \times d)$. The curved diffuser’s height is $h = 50 \text{ mm}$ and its length is $l = 70 \text{ mm}$.

![Curved Diffuser's Geometry](image)

Fig. 1. Curved Diffuser’s Geometry.

Boundary and initial conditions for this curved diffuser are as follow. Flow is steady and three-dimensional. A viscous and incompressible liquid is expected. The flowing medium is water with the temperature of $T = 20 \, ^\circ \text{C}$ and with the inflow’s velocity of $v = 15 \text{ m/s}$. The standard atmospheric pressure $p_{\text{atm}} = 101,325.0 \text{ Pa}$ is set up.

3. Procedure

The optimization process shown in Fig. 2 is consisted of two main parts. The first part is a main program of mathematical functions implemented in the program MATLAB [2]. The second part is a geometry creation in the program Gambit followed by a CFD computation in the program ANSYS Fluent [3].

![Optimization Process – Flow Chart](image)

Fig. 2. Optimization Process – Flow Chart.

An appropriate generation of random numbers is provided by Quasi-Random Point Set [2]. In Fig. 3 are seen twenty generated points in the selected interval $I$ by Halton Sequences as
the chosen method of *Quasi-Random Sequences*. (Note that four points are added as the extreme values, in corners).

Variants of curved diffuser’s geometries are made from values of the created *Halton Point Set*. Afterwards, these variants are computed by the *CFD* method. Gained results of local losses are added to a target function as its substitution and subsequently solved in the matrix system (1.2). (Note that local losses are computed by a common way through the *Bernoulli’s equation* with a usage of the *Continuity equation* [4]).

The *Radial Basis Function (Linear)* [5] “the distance’s function of points” is chosen as a target function:

\[
\hat{f}(\vec{x}) = \sum_{i=1}^{N} a_i \cdot \varphi \left( \left\| \vec{x}_i - \vec{x} \right\| \right)
\]  

(1.1)

where

- \(\hat{f}(\vec{x})\) = approximant
- \(\left\| \vec{x}_i - \vec{x} \right\|\) = norm distance
- \(\vec{x}\) = independent point {point of a selected mesh}
- \(\vec{x}_i\) = centre (distance from some other point) {points of a *Halton Point Set*}
- \(a_i\) = weight coefficient
- \(\varphi\) = RBF type

The equation (1.1) can be solved as the matrix system:

\[
F = a \cdot D \quad \Rightarrow \quad a = F \setminus D
\]

(1.2)

where

- \(F\) = substitution of a target function
- \(a\) = weight vector
- \(D\) = distance matrix

(Note that for the computation of a weight vector \(a\) (1.2) are only added points of *Halton Point Set* to the distance matrix \(D\); and the substitution of a target function \(F\) is consisted of local losses as the gained results from a *CFD* computation).

This obtained weight vector \(a\) is used for an approximation of a target function (1.1) which contains points of *Halton Point Set* and also a selected mesh of an arbitrary amount of points.

![Fig. 3. Halton Point Set (20 points + 4 added points).](image-url)
The optimal curved diffuser’s geometry is found by the optimization procedure described above. The optimized geometry is situated in the location of \( X = 20 \text{ mm} \) and \( Y = 20 \text{ mm} \) (the extremes \( X_{\text{max}} \) and \( Y_{\text{max}} \) of the interval \( I \)), Fig. 4. This variation has local losses of \( \zeta = 4.97 \).

![Approximated RBF - Optimized Curved Diffuser’s Geometry (Mesh of 500x500 Points).](image)

\( \text{Fig. 4. Approximated RBF - Optimized Curved Diffuser’s Geometry (Mesh of 500x500 Points).} \)

\( \text{axis } x \text{ on the right bottom side, axis } y \text{ on the left bottom side} \)

4. Flow’s Effects

Flow’s effects have been reviewed in the optimized curved diffuser \( (X = 20 \text{ mm} \) and \( Y = 20 \text{ mm}) \). A cross section of the \( x-y \) plane has been made in the middle part of a curved diffuser \( d/2 \), Fig. 2, with a purpose of the analysis of general flow’s behaviours; next called the \( \text{CS-A} \). A cross section of the \( y-z \) plane has been made near to the outflow, for the reason of the analysis of three dimensional flow’s effects; next called the \( \text{CS-B} \).

![Velocity Vectors Colored By Velocity Magnitude (m/s).](image)

\( \text{Fig. 5. Optimized Curved Diffuser - Velocity Vectors (CS-B).} \)

Vortex structures have shown up in the optimized curved diffuser, presumably due to a boundary layer on the side walls; it is seen in the \( \text{CS-B} \) of the \( \text{Fig. 5} \). This effect is called the “Secondary Flow”.
The comparison of the computation in three dimensions against the computation in two dimensions has been accomplished in the optimized curved diffuser (the case of $X_{\text{max}}$ and $Y_{\text{max}}$); as it is shown in Figs. 6 to 9 of velocity behaviours. In Fig. 8 are seen vortexes and the flow separates on the lower wall of the curved diffuser in the 3D computation, whereas in the 2D computation the flow is continuous and without any separation, Fig. 9.

Fig. 6. Optimized Curved Diffuser – Velocity Magnitude by 3D Computation (CS-A); $\zeta = 1.63$.

Fig. 7. Optimized Curved Diffuser - Velocity Magnitude by 2D Computation; $\zeta = 0.57$.

Fig. 8. Optimized Curved Diffuser - Velocity Vectors by 3D Computation (CS-A).

Fig. 9. Optimized Curved Diffuser - Velocity Vectors by 2D Computation.

Therefore, the results gained by the computation in three dimensions are more valuable and flow’s effects are better described than it is possible to determine from the computation in two dimensions.

An evaluation of a possible flow’s separation is made in the cases of curved diffuser’s geometries – in the extremes of the interval $I$ (Chap. 2: Formulation), Figs. 6, 8 and Figs. 10 to 15.

Fig. 10. Velocity Magnitude of Curved Diffuser - Case of $X_{\text{max}}$ and $Y_{\text{min}}$ (CS-A); $\zeta = 1.97$.

Fig. 11. Velocity Vectors of Curved Diffuser – Case of $X_{\text{max}}$ and $Y_{\text{min}}$ (CS-A).
Fig. 12. Velocity Magnitude of Curved Diffuser – Case of $X_{\text{min}}$ and $Y_{\text{max}}$ (CS-A); $\zeta = 2.00$.

Fig. 13. Velocity Vectors of Curved Diffuser – Case of $X_{\text{min}}$ and $Y_{\text{max}}$ (CS-A).

Fig. 14. Velocity Magnitude of Curved Diffuser – Case of $X_{\text{min}}$ and $Y_{\text{min}}$ (CS-A); $\zeta = 2.03$.

Fig. 15. Velocity Vectors of Curved Diffuser – Case of $X_{\text{min}}$ and $Y_{\text{min}}$ (CS-A).

The flow separated on the lower wall in each of extreme’s variation in the interval $I$ as it is seen in Figs. 6, 8 and Figs. 10 to 15. Also in the cases of the interval’s extremes – the $X_{\text{min}}$ and $Y_{\text{min}}$, Fig. 15 and of the $X_{\text{max}}$ and $Y_{\text{min}}$, Fig. 11 – the flow separated on both sides of the curved diffuser’s wall: the so called “Double Flow’s Separation”.

5. Conclusion

The optimal curved diffuser’s geometry has been found by the approximation of a target function (RBF Linear) in the location of $X = 20 \text{ mm}$ and $Y = 20 \text{ mm}$. This variant has the lowest local losses of $\zeta = 1.63$.

The cause of local losses is mainly a dissipation of energy in vortex structures [4].

The vortex structures had occurred in the curved diffuser as it is seen in Fig. 5; this effect has been classified as the “Secondary Flow”. These vortex structures had occurred around the corners of the curved diffuser, presumably due to a boundary layer on the side walls.

The flow had been separated in the optimized curved diffuser on the lower wall, Fig. 8; vortexes had shown up. Also in the cases of interval’s extremes – the $X_{\text{min}}$ and $Y_{\text{min}}$, Fig. 15 and the $X_{\text{max}}$ and $Y_{\text{min}}$, Fig. 11 – the flow was separated in the both sides of the curved diffuser’s wall; the so called “Double Flow’s Separation”.

Generally for this curved diffuser, the lowest local losses could be found in the geometry of some large interval with the equal $X$ and $Y$.

The comparison of the computation in three dimensions against the computation in two dimensions was accomplished, Figs. 6 to 9. The flow was continuous and without any separation in the 2D computation, while the flow was separated on the curved diffuser’s lower
wall in the 3D computation. Therefore, it could be said in general, that results gained by a computation in three dimensions could be more useful for a flow’s description than results of a computation in two dimensions.

Attachment

**Testing Function**

A testing function has been created in a purpose to verify the suitability of the created optimization process.

In Fig. 16 is shown the chosen area where is created the *Rastrigin’s Function* [6] as the testing function:

$$Ras(x) = 20 + x_1^2 + x_2^2 - 10 \cdot (\cos 2 \cdot \pi \cdot x_1 + \cos 2 \cdot \pi \cdot x_2)$$

(2)

where \(x_1, x_2\) = independent variables

![Fig. 16. Rastrigin’s Function.](image)

The sample of one hundred points, generated by the *Halton Point Set* in the chosen interval \(I = \langle -5; 5 \rangle\), has been made. Figs. 17 and 18 show the similarity in general, with the known behaviour of the *Rastrigin’s Function*, Fig. 16.

![Fig. 17. Rastrigin’s Function - 100 points of Halton Point Set.](image)

![Fig. 18. Rastrigin’s Function - Approximation of RBF (100 points of Halton Point Set).](image)
In conclusion, the created optimization process is accurate and seems to be a right option. It has been proved by an easier location of extremes with a refinement (an approximation of the RBF) of the testing function, Fig. 18 against the function without any modification, Fig. 17.

**Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
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<tbody>
<tr>
<td>$p_{atm}$</td>
<td>Standard Atmospheric Pressure</td>
<td>[Pa]</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature</td>
<td>[K]</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Local Losses</td>
<td></td>
</tr>
<tr>
<td>$v$</td>
<td>Velocity</td>
<td>[m.s$^{-1}$]</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Mathematical Constant</td>
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**Shortcuts**

- **ANSYS Fluent**: Program for a general purpose of CFD code based on the finite volume method on collocated grid
- **AP**: Analysis / Direct Problem
- **CFD**: Computational Fluid Dynamics
- **CS-A**: Cross Section A of the x-y plane in the middle part of a curved diffuser
- **CS-B**: Cross Section B of the y-z plane near to the outflow
- **DP**: Design / Inverse Problem
- **Gambit**: Pre-processor of the program ANSYS Fluent used for a geometry creation and a grid generation
- **MATLAB**: Numerical computing environment and fourth-generation programming language (matrix laboratory)
- **RBF**: Radial Basis Function

**References**