Aeroelasticity analysis of wing UL-39

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Abstract in Czech

Tato práce se zabývá výpočtem modálních a flatrových charakteristik, vyšetření účinnosti řízení a stanovení mezní rychlosti torzní divergence pravé poloviny křídla letounu UL-39. Řešení je prováděno pomocí MKP softwaru MSC.Nastran.

Abstract in Czech

This paper deals with computation of modal and flutter characteristic, investigating ailerons effectiveness and determine torsion divergence critical velocity at right half-wing of the aircraft UL-39. The problems are solved in FEM software MSC.Nastran

Key words

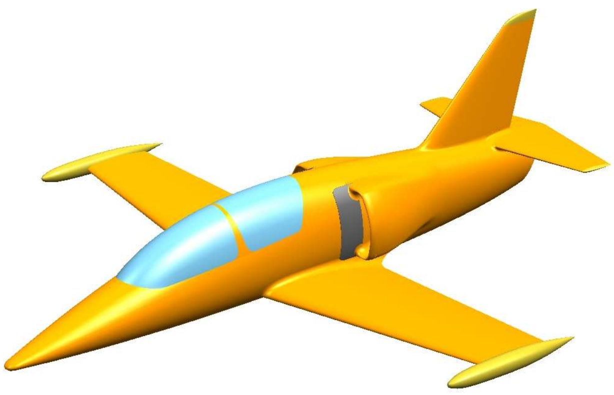
Aeroelasticity analysis, normal modes, flutter, ailerons reversal, wing torsion divergence, MSC.Nastran

1. Introduction

This paper is focused on providing the first view on aeroelasticity behavior of wing aircraft UL-39. And exploring the possibility of solution static aeroelasticity problems by using FEM software. Used software for solution is MSC.Nastran 2005.

1.1 UL-39

UL-39 is ultra-light all-composite plane for two person, with retractable landing gear. The propeller is compose of input channel and low pressure blower. The blower is drive via motorcycle engine. The wing is trapezium shape with primary and secondary beam. On the end of wing is placed external wing-tip fuel tank. The tail surfaces are classical configuration with floating elevator.



Pic. 1. *UL-39*

***Tab 1.** – Basic characteristic*

Stall speed	V_{min}	65 [km/h]
Design speed	Vd	340 [km/h]
Wing span	l_{KR}	7,2 [m]
Aspect ratio	λ	5,6
Wing surface	S	8,504 [m^2]

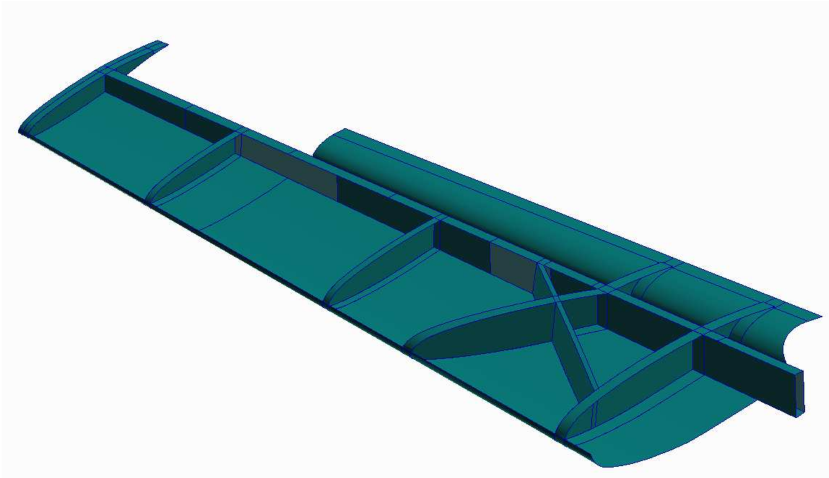
Fuel mass	m_p	38,8 [kg]
Ceiling	H	3000 [m]
High	l	3,025 [m]
Length	l_{TR}	7,33 [m]
Aerodynamic chodor	b_{SAT}	1,275 [m]

2. FEM Model

FEM (Finite element model) is consist from two part. First one is called structural model, it is a geometrical model of wing, with finite element mesh and defined material characteristic. There are also defined boundary conditions and local mass. Second model is called aerodynamic model. It was created for purpose of calculating aerodynamic loads. This model is without any material characteristics. Instead of finite element mesh is aerodynamic model form by aero-boxes. Those two models are independent on each other, so for connection was used mathematical function called Spline which transferring loads and deformations.

2.1 Structural Model

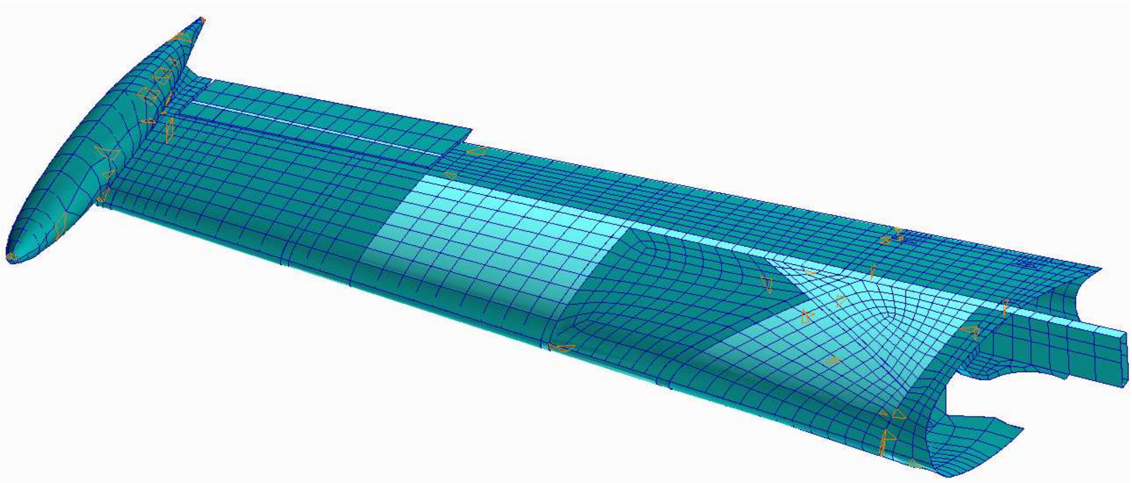
The structural was completely created in preprocesor Patran. Model is composed from 185 surfaces. Laminate modeler was used for defining material properties. Used materials are Divinicel foam, Carbon composite Biaxial Carbon 200, Roving TORAYA T700SX, Carbon fabric, epoxy, resin and Chrome-manganese steel.
Total weight of structural model with fuel is 77,2 kg.



***Pic. 2.** Structural model of wing.*

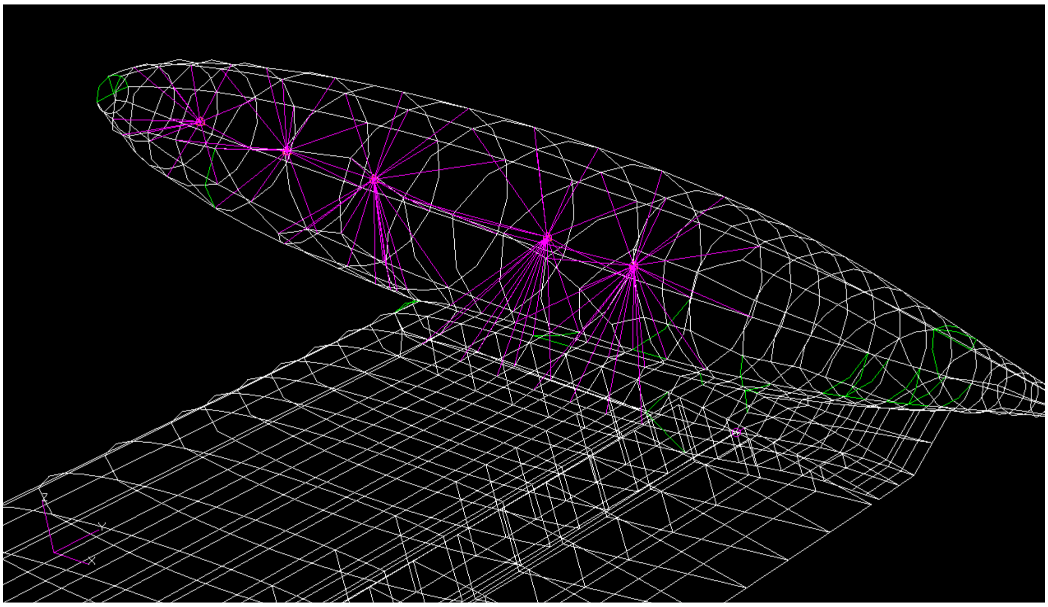
2.2 Mesh Model

Model contains 8322 nods and 2988 elements. The primary type of elements used on model is square type called QUAD 97,5%. The rest of elements are triangular and point type.



Pic. 3. *Structural model with mesh.*

Influence of path control was simulating by add moment of inertia on the aileron like POINT element and redistributed to aileron by MPC element. Calculation of add moment of inertia was done according [1]. The fuel was simulating as a local mass and redistributed to surrounding nodes by MPC element. The same was also done for simulating of landing gear retractable mechanism and flaps. Table 2. summarize weight of local mass and add moment of inertia.



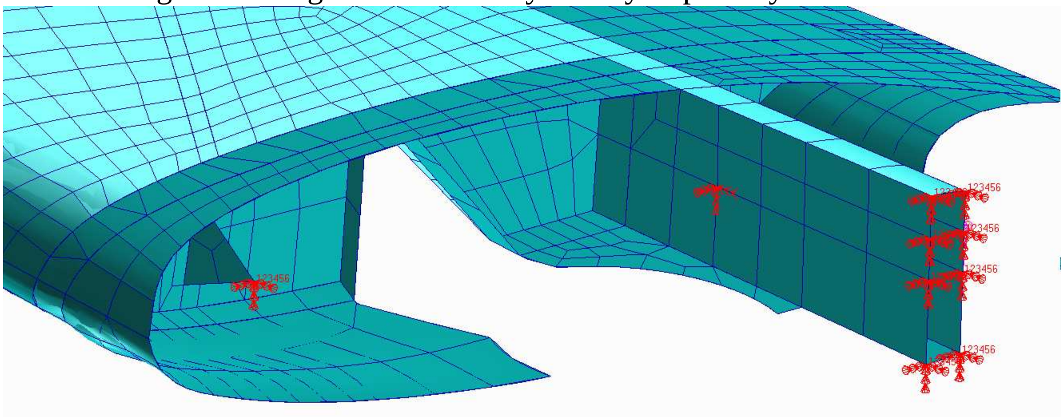
Pic. 4. *MPC element of fuel used in wing-tip*

Tab. 2. *–Local mass*

Name	Mass	Moment of inertia	Number of MPC
Fuel	38,8 kg		13
Landing gear	11,8 kg		1
Retractable mechanism	1,25 kg		1
Flaps	5 kg		2
Add moment of inertia	0,001 kg	0,107 kg.m^2	1

2.2 Boundary conditions

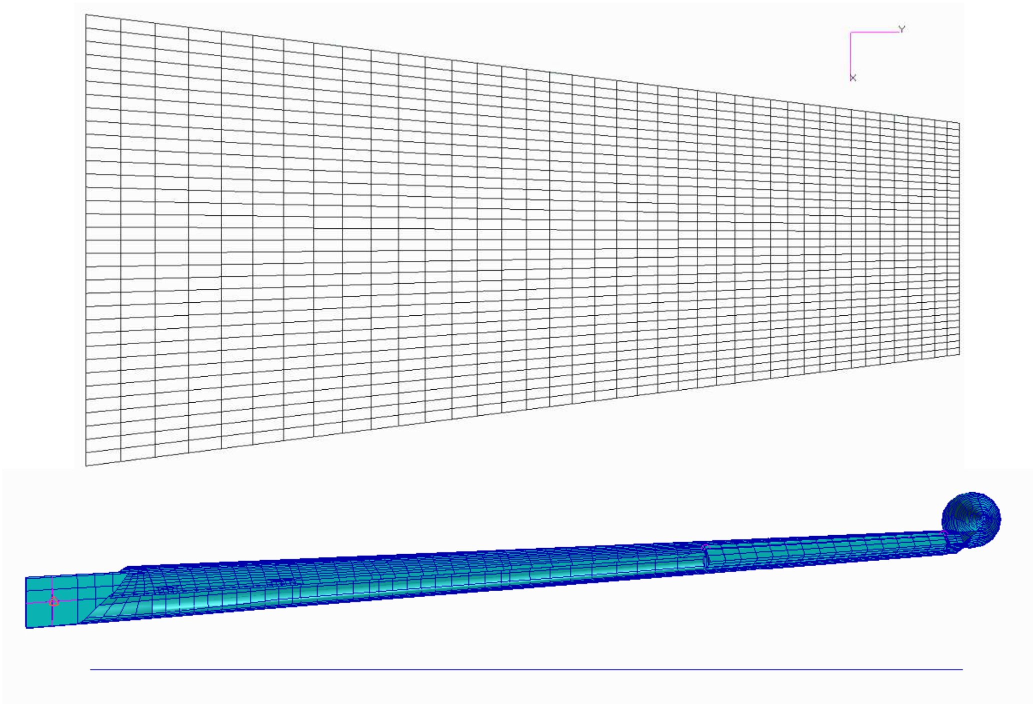
Boundary conditions was done by restriction all six DOF in nodes corresponding to connection fuselage with wing and in axis of symmetry of primary beam.



Pic. 5. *Boundary conditions.*

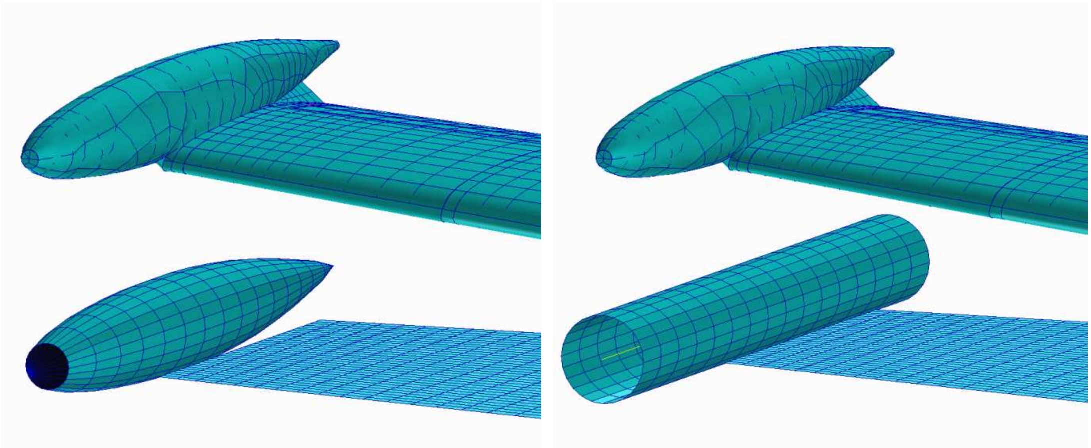
2.3 Aerodynamic model

For the calculations of aerodynamic loads on wing was defined „Lifting surface“ which used double lattice method. DLM calculate the lift on behalf of aerodynamic linearizated potential theory.



Pic. 6. *Lifting surface - wing*

For simulating of aerodynamic motion and loads on external wing tip was used YZ-Body. The Body is composed from two parts. First one is Slender body for simulating motion own body and aerodynamic forces on behalf Slender Body Theory. The theory gives the lift proportional to the rate of change of cross-section area. Second part of the body is Interference body which is used for simulation interaction body with other body and/or lifting surfaces. Part of lifting surface was defined as Control device for purpose of simulations the aileron.



Pic. 7 .Slender body (left) and Interference body (right)

3. Normal modes

The normal modes is structural analysis only so no aerodynamic model was needed. The normal modes was used for compution of natural frequencies and mode shape of structure. Which are one of the input to flutter analysis. If is structure vibrating on frequency same or very close to natural frequency it can lead to structural damage or failure. Operation structure on frequency close to natural frequency decreases fatigue life. For obtain the natural frequencies Nastran solution SOL103 was used. This solutions use reduced form of the equation of motion (1) where no damping and no applied loading are considered.

$$[\mathbf{M}]\{\ddot{\mathbf{u}}\} + [\mathbf{K}]\{\mathbf{u}\} = \mathbf{0} \tag{1}$$

Where: $[\mathbf{M}]$ mass matrix
$[\mathbf{K}]$ stiffness matrix
$\{\boldsymbol{u}\}$ assume a harmonic solution $\{\boldsymbol{u}\} = \{\boldsymbol{\phi}\}\,\mathbf{sin}\,\boldsymbol{\omega t}$
$\{\phi\}$ the eigenvector or mode shape
ω is the circular natural frequency

Solutions of reduced form of the equation of motion is :

$$\left([\mathbf{K}] - \boldsymbol{\omega}_{\mathbf{i}}^{\mathbf{2}}[\mathbf{M}]\right)\{\boldsymbol{\phi}_{\mathbf{i}}\} = \mathbf{0}, \quad \mathbf{i} = \mathbf{1}, \mathbf{2}, \mathbf{3}, \ldots \tag{2}$$

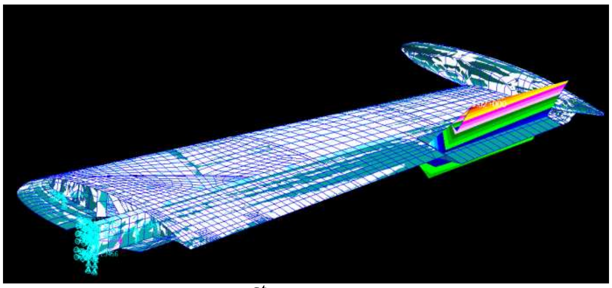
The results of equation (2) are eigenvalues i=1,2,3,… and eigenvector which define mode shape of structure and are in relation with natural frequency for certain mode:

$$\mathbf{f}_{\mathbf{i}} = \frac{\boldsymbol{\omega}_{\mathbf{i}}}{\mathbf{2\pi}} \tag{3}$$

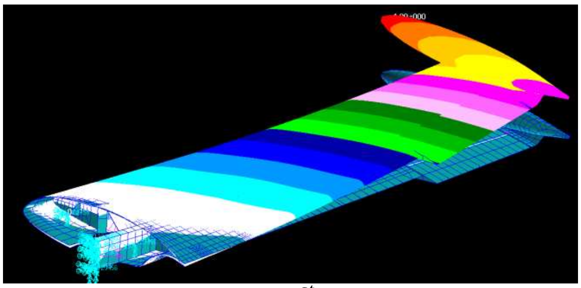
where f is natural frequency. For obtaining eigenvalues, eigenvcetor and natural frequencies from (2) The Lanczos algorithm was used. The analysis was done on model with full fuel tank, and it’s presented in Tab 3.

Tab 3. *– Results of normal modes analysis*

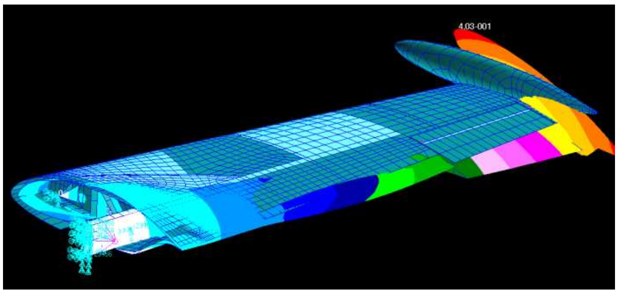
Mode	Mode shape	Natural frequencies
1^{st} mode	1^{st} shape of aileron	0,32 Hz
2^{nd} mode	1^{st} bending	3,33 Hz
3^{rd} mode	combination torsion and front-rear motion	13,1 Hz
4^{th} mode	1^{st} torsion	16,5 Hz
5^{th} mode	2^{nd} bending	21,4 Hz
6^{th} mode	1^{st} combination of torsion and bending	31,3 Hz
7^{th} mode	2^{nd} combination of torsion and bending	43,6 Hz
8^{th} mode	isolated vibration on trailing edge	54,6 Hz



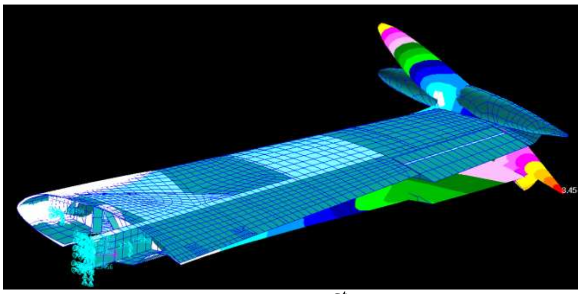
Pic. 8. *1^{st} shape of aileron*



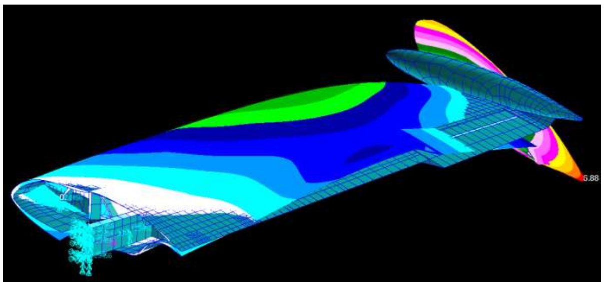
Pic. 9. *1^{st} bending.*



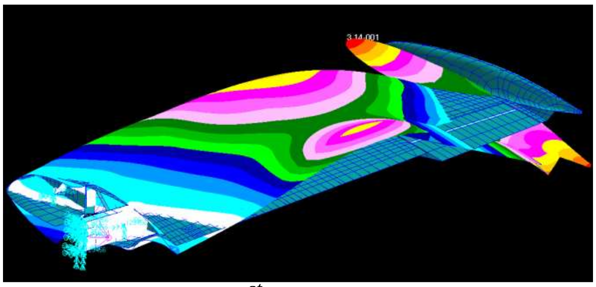
Pic. 10. *torsion and front-rare motion*



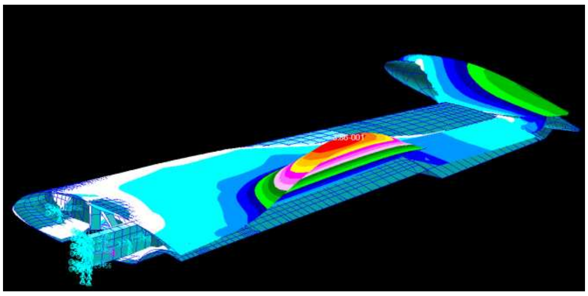
Pic. 11. *1^{st} torsion*



Pic. 12. *2^{nd} bending*



Pic. 13. *1^{st} torsion and bending.*



Pic. 14. *isolated vibration on trailing edge*

3. Flutter analysis

Flutter is dynamic aeroelasticity stability problem. It is self-excited and potentially destructive vibration where aerodynamic forces on an object couple with a structure's natural mode of vibration to produce rapid periodic motion. Flutter can occur in any object within a strong fluid flow under the conditions that a positive feedback occurs between the structure's natural vibration and the aerodynamic forces. That is when the vibration movement of the object increases an aerodynamic load which in turn drives the object to move further. If the energy during the period of aerodynamic excitation is larger than the natural damping of the system the level of vibration will increase, resulting in self-exciting oscillation. The vibration levels can thus build up and are only limited when the aerodynamic or mechanical damping of the object match the energy input, this often results in large amplitudes and can lead to rapid failure.

In process of flutter certification is numerical solutions first step which can give us a critical modes. Second steps are vibrations test aimed on critical modes. This test can more precisely determine natural frequency important for flatter calculations. Last step of flutter certification process are flight test. FAA regulations required that airplane must be flutter free to $1{,}2.V_D$. In our case is V_D=340 km/h, so $1{,}2V_D$= 408 km/h.

For flutter analysis was used Nastran solutions SOL145 „Dynamic Flutter Analysis", for analysis was chosen British PK-Method. This method was developed in 1928 by Mr. Frazer&Duncan. They were attempting to solve the flutter problem using aerodynamic stability derivatives of rigid aircraft. This approach introduce the aerodynamic loads into the equations of motion as frequency dependent stiffness and damping terms. In 1971 this method was developed by Mr.Hassing by introduction aerodynamic loads as complex springs. Advantage of PK-metod is also that results are plotted directly for given velocities, and damping is a more realistic estimated of the physical damping.

Input for solutions flutter solutions are dynamic characteristic which are represented by natural frequencies, material characteristic geometric characteristic of structure and flight conditions (density, velocity). The PK-Method of flutter solution is using equation (4).

$$\left[M_{hh}p^2 + \left(B_{hh} - \frac{1}{4}\rho\bar{c}VQ^I_{hh}/k\right)p + \left(k_{hh} - \frac{1}{2}\rho V^2 Q^R_{hh}\right)\right]\{u_h\} = 0 \tag{4}$$

Where:

M_{hh}	mass matrix
p	eigenvalue
B_{hh}	damping matrix
ρ	fluid density
c	reference length
V	velocity
Q^I_{hh}	modal aerodynamic damping matrix,function of Mach number and reduced frequency
Q^R_{hh}	modal aerodynamic damping matrix,function of Mach number and reduced frequency
k	reduced frequency
k_{hh}	modal stiffness matrix
$\{u_h\}$	modal amplitude vector

[Q_{hh}] is aerodynamic matrix which comes from „Double Lattice subsonic lifting surface theory" or DLM. On this matrix is applied spline function and is also reduced to obtaining the matrix in generalized form. The equations (4) has to be rewritten to matrix form for solutions in Nastran (5).

$$[A - pI]\{u_h\} = 0$$

where $[A]$ is the real matrix

$$[A] = \left[\begin{array}{c|c} 0 & I \\ \hline -M_{hh}^{-1}\left[K_{hh} - \frac{1}{2}\rho V^2 Q_{hh}^R\right] & -M_{hh}^{-1}\left[B_{hh} - \frac{1}{4}\rho \bar{c} V Q_{hh}^I / k\right] \end{array}\right] \qquad (5)$$

And for real roots of (5) is the damping expressed as (6). Obtaining the roots from equations (5) is iteration process.

$$g = 2\gamma = {}^{2pc}/_{(ln2)V} \qquad (6)$$

Via damping we can determine when the flutter occurs. The computed damping is aerodynamic damping, in this case we do not know structural one so the FAA regulations „FAR 23.629 Flutter" estimate as critical damping value 0,03. But if the curve slop of damping is too high, critical velocity is on line of zero damping. The flutter calculations was done for the same model as in Normal modes solutions. Results are summarize in Tab.4 & Tab.5 and critical modes are plotted in Pic.15 & Pic.16.

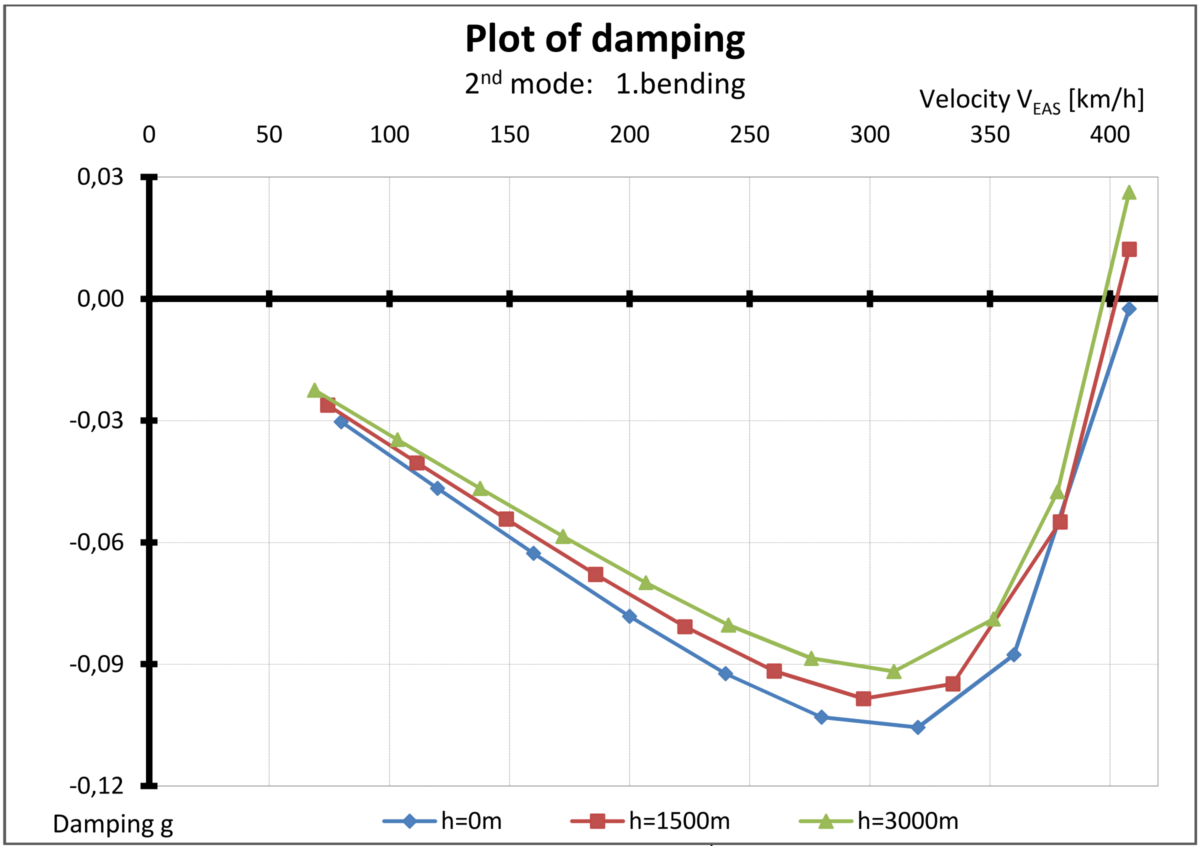
Tab 4. *– Results of flutter analysis*

Mode	**1st mode**	**2nd mode**	**3rd mode**	**4th mode**
H=0m	OK	g= - 0,02	g= - 0,02	OK
H=1500m	OK	V_{FL}=402 km/h	g= - 0,00015	OK
H=3000m	OK	V_{FL}=397 km/h	g= - 0,0001	OK

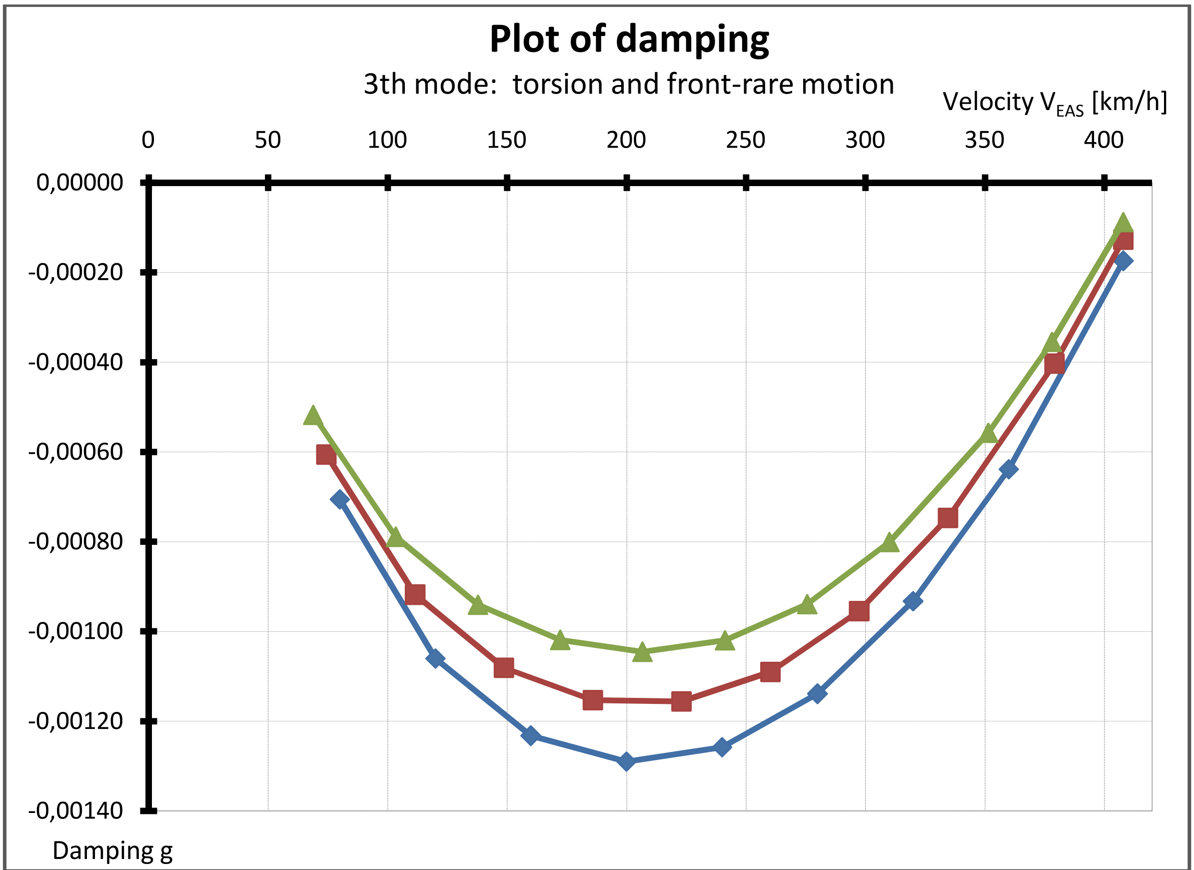
Tab 5. *– Results of flutter analysis-continue*

Mode	**5th mode**	**6th mode**	**7th mode**	**8th mode**
H=0m	OK	OK	OK	OK
H=1500m	OK	OK	OK	OK
H=3000m	OK	OK	OK	OK

OK Flutter free to 1,2V_D
V_{FL} Velocity of flutter
g maximal damping between computed velocity V=0 km/h and 1,2V_D=408 km/h, only for this modes that are too close of line zero damping.



Pic. 15. *V-g plot for 2nd mode*

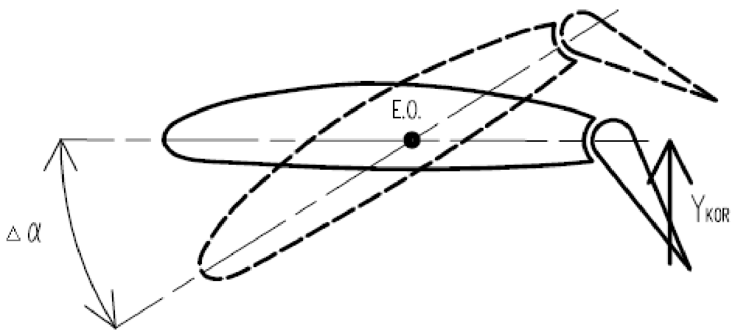


Pic. 16. *V-g plot for 3rd mode*

From results we can see that critical modes are two 2nd & 3rd. The 2nd mode crosses the stability axis and the slope is steep . In actual flight may be a 20 kilometers an hour between completely stable and extremely unstable plane. Flutter occurs at velocity close under $1,2V_D$. The 3rd mode have trend going to unstable area but the slope of curve is not steep. In investigate s velocities flutter will not occur, but it is sure that at the speed little bit higher than $1,2V_d$ flutter will occur. Rest of the investigate modes have no trends of instability.

4. Control reversal

Control reversal is static aeroelasticity problem, thus it's without time depending and do not have oscillation character of deformation. We consider aerodynamic and elastic forces only, in solution of static aeroelasticity. Control reversal lead to loss of controllability of the plane, but do not lead to destruction of the structure. A limiting reversal. speed is reached when the change in lift due to control surface rotation is nullified by the change in lift due to twist of the lifting surface.



Pic. 17. *Principle of control reversal*

For control reversal problem was used solution SOL 144 „Static Aeroelastic Analysis" where is possible to define certain flight parameters of the model (such as angle of attack, deflection of control surface, flight speed and so on.) and watch the final movement of the model (via non-dimensional stability and control derivative coefficients, trim parameters and so on..).
For solutions of control reversal problem was used this setting of model:
Definition of constant deflection of aileron δ=0,3 [rad] and released model for rotation about axis of symmetry (x-axis). The setting was done in source code of input Nastran file *.bdf as:

Boundary condition:	Front and rear hinge: SCP 123 56
Support Rigid body DOF:	NODE 7895 DOF:4
Rigid Body Motion Trim Variables:	ROLL; URDD4
Trim Parameters for Subcase:	URDD4=0.0; AILE= 0.3; M=0.0
Aeroelastic Model Parameters:	PARAM AUNITS 1.0
	PARAM BAILOUT -1
Symmetry of aerodynamic motion:	SYMXZ -1; SYMXY 0 (Default)

Note: „SCP 123 56" mean that was restricted all motion in DOF 12356 except 4DOF thus rotation about x-axis.
Monitoring of model response is via parameter ROOL which is one of the printed output and is defined by equations (7):

$$ROLL = \frac{p(l/2)}{2v}[-] \qquad (7)$$

Where: l………..…wing span [m]
p……… „Rool rate“ angular velocity of rotation about x-axis [rad/sec]
v……… velocity[m/s]

If is equations (7) divide by deflection of aileron in [rad] we can obtain ailerons effectiveness (8)

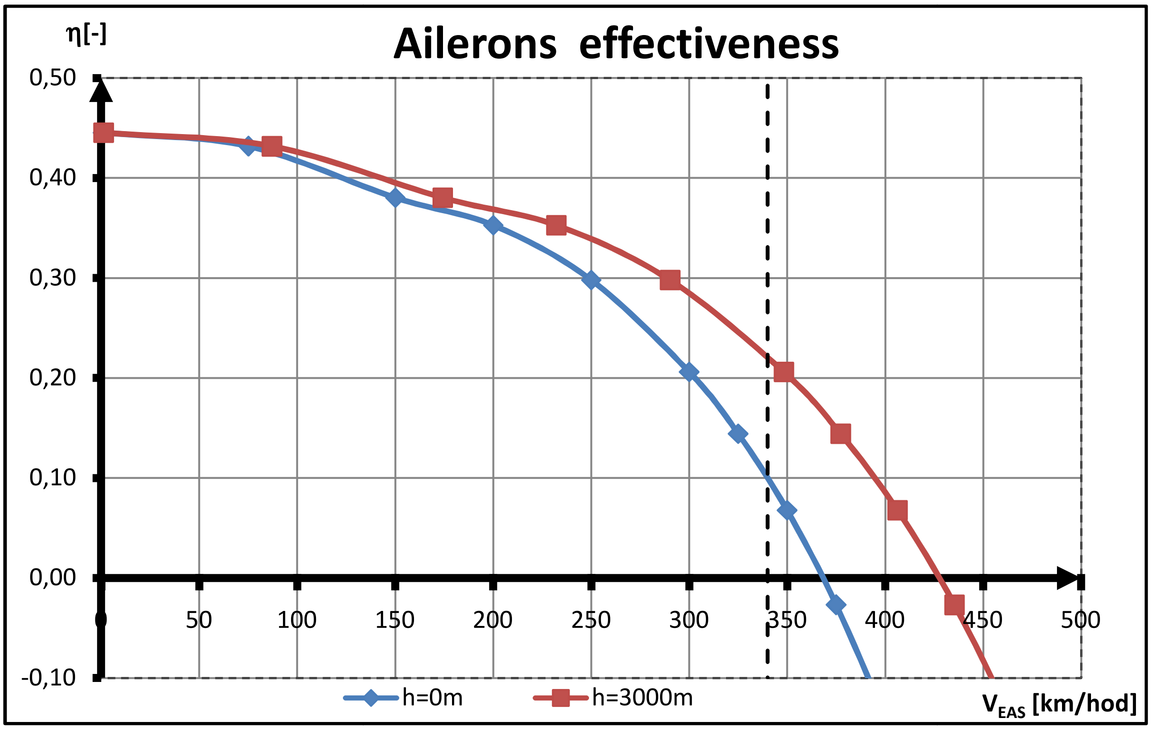
$$\eta_x = \frac{p(l/2)}{2v\delta}[-] \quad (8)$$

Where: η_x…….. airelons effectivnes [-]
δ……… deflection of aileron [rad]

The calculations was done for dynamic pressures corresponding to speeds form 0 km/h to 450 km/h and altitude 0m and 3000m. The results are summarized in Tab.6 and Pic.18.

Tab 6. *– Results of control reversal analysis*

Dynamic pressure Q [MPa]	Effectiveness η_x[-]	H=0m V_{TAS} [km/h]	H=3000m V_{TAS} [km/h]
4,73E-08	0,44	1	1
2,66E-04	0,43	75	87
1,06E-03	0,38	150	174
1,89E-03	0,35	200	232
2,95E-03	0,29	250	290
4,25E-03	0,20	300	348
4,99E-03	0,14	325	377
5,79E-03	0,06	350	406
6,65E-03	-0,02	375	435
7,56E-03	-0,14	400	464
8,54E-03	-0,28	425	493
9,57E-03	-0,44	450	522



Pic. 18. *Aileron effectiveness plot*

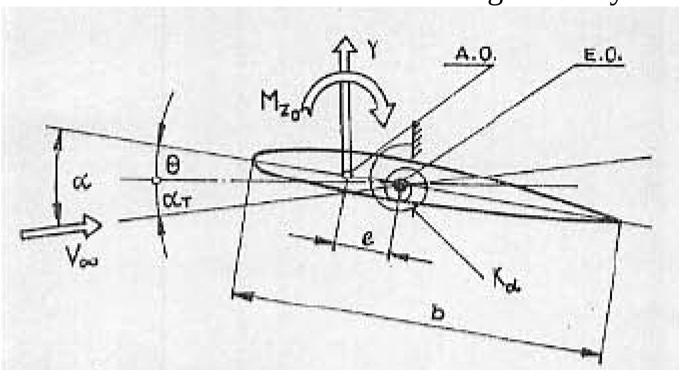
Velocity of control reversal V_{REV} is when aileron effectiveness decrease on zero $\eta_x = 0$. The critical speed was determine by linear interpolation and results are in Tab.7

***Tab 7.** – Control reversal speeds*

Altitude	V_{REV}
0 m	386 km/h
3000 m	427 km/h

4. Wing torsion divergence

Torsion divergence is also problem of static aeroelasticity as control reversal. But it is problem which leads do destruction of the structure. Divergence may occur without warning.



***Pic. 19.** Principle of torsion divergence*

For understanding the problem, assume a wing in horizontal flight with small angle of attack α. The aerodynamic lift force Y acting in aerodynamic center (A.O.) creates a torque M_{z0} to elastic axis (E.O.). This torque causes a torsion deformation of a wing, and increasing angle of attack Θ. This is flowed by increasing aerodynamic lift forces. With increasing speed the torsion deformation is also increasing. In the moment when a structure is not capable to damp difference of torque, the torsion divergence of wing occurs. This critical speed is called $V_{DIV.}$

For the calculations was used SOL 144 as in chapter 3. The process for obtain V_{DIV} was following. The model of wing was released in rotation about y-axis, thus in mean of change angle of attack. Also was defined condition of the flight at constant flight level. Investigate will the motion of model with increasing dynamic pressure. The critical speed V_{DIV} can be obtain from aerodynamic derivation $C_{Z\alpha}$ also know as C_y^{α}. This derivation show change of aerodynamic lift force witch changing angle of attack. In area of V_{DIV} the $C_{Z\alpha}$ will grow to extreme high values. It is given by torsion deformation of wing and great difference of lift in small difference of angle of attack. The setting was done in source code of input Nastran file *.bdf as:

Boundary condition:	Rear hinge: NODE 7853 SCP 26
	Rear hinge: NODE 7905 SCP 126
	Front hinge: NODE 7650 SCP 26
Support Rigid body DOF:	NODE 7905 35
	NODE 849 5 (ailerons)
Rigid Body Motion Trim Variables:	ANGLEA; PITCH; URDD3; URDD5
	ROLL; URDD4
Trim Parameters for Subcase:	ANGLEA=FREE; PITCH=0.0; URDD3=-1.0;
	URDD5=FREE; ROLL=0.0; URDD4=0.0
	AILE= FREE
Aeroelastic Model Parameters:	PARAM AUNITS 1.0193E-04

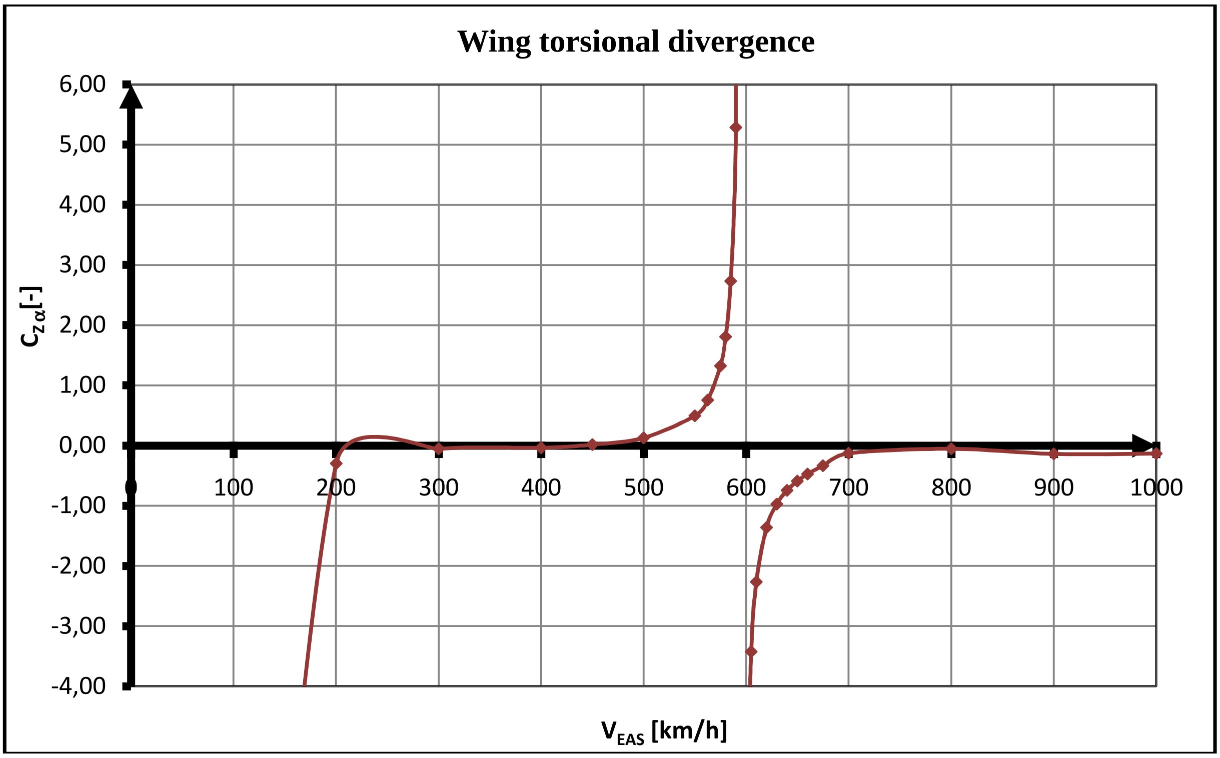
PARAM BAILOUT -1

Symmetry of aerodynamic motion: SYMXZ 1; SYMXY 0 (Default)

***Tab 8.** – Results of divergence analysis*

Velocity V_{EAS} [km/h]	Derivation $C_{Z\alpha}$ [-]
100	-15,8488
200	-0,2956
300	-0,0550
400	-0,0358
450	0,0177
500	0,1295
550	0,4936
562	0,7548
575	1,3260
580	1,8061
585	2,7324
590	5,2853
595	46,1169

Velocity V_{EAS} [km/h]	Derivation $C_{Z\alpha}$ [-]
600	-7,2340
605	-3,4215
610	-2,2630
620	-1,3616
630	-0,9716
640	-0,7452
650	-0,5910
660	-0,4743
675	-0,3359
700	-0,1288
900	-0,0530
800	-0,1402
1000	-0,1341



***Pic. 20.** Dependent of $C_{Z\alpha}$ at velocity plot for H=0m*

Velocity of wing torsion divergence was determine by analysis as V_{DIV}=601 km/h for H=0m and V_{DIV}=695 km/h for H=3000m

5. Conclusions

This paper deals with aeroelastic analysis in FEM software MSC.Nastran. The analysis determine that flutter may occur at speed V_{FL}=397km/h, control reversal V_{REV}=368 km/h and torsion divergence V_{DIV}=601 km/h.

This analysis will be useful for investigation aeroelastic phenomenon. And for determination of which structure parameters have significant influence on those aeroelastic phenomenon. Also this method will compare with experimental investigation of flutter phenomena on a real aircraft structure.

List of symbols

$[B_{hh}]$	damping matrix
C [m]	reference length
$C_{Z\alpha}$[-]	Aerodynamic derivation C_y^{α}
f [Hz]	Natural frequency
g[-]	damping
h [m]	altitude
k	Reduced frequency
[K]	Stiffness matrix
k_{hh}	Modal stiffness matrix
l [m]	Wing span
[M], $[M_{hh}]$	Mass matrix
M_{z0}[N.m]	torque
p [-]	eigenvalue
p_r [rad/sec]	Rool rate
Q [Mpa]	Dynamic pressure
Q^I_{hh}	modal aerodynamic damping matrix, imaginary part
Q^R_{hh}	modal aerodynamic damping matrix, real part
S [m^2]	Wing surface
{u}	Harmonic solution
$\{u_h\}$	Modal amplitude vector
V[km/h]	Velocity
V_D [km/h]	Design speed
V_{DIV} [km/h]	Flutter critical velocity
V_{ESA} [km/h]	Equivalent air speed
V_{FL} [km/h]	Flutter critical velocity
V_{MIN} [km/h]	Stall speed
V_{REV} [km/h]	Flutter critical velocity
Y [N]	Lift force
α [rad]	Angle of attack
δ [rad]	deflection of aileron
η_x[-]	Ailerons effectiveness
Θ [rad]	Increment angle of attack
λ [-]	Aspect ration
P [kg/m^3]	Fluid density
$\{\phi\}$	The eigenvector or mode shape
ω	Tthe circular natural frequency

List of abbreviations

A.O.	Aerodynamic axis
DOF	Degree of freedom
DLM	Double lattice method

E.O.	Elastic axis
FAA	Federal Aviation Administration
FAR	Federal Aviation Regulations
FEM	Finite elements method
MPC	Multi constraint points
SOL103	Normal modes analysis
SOL144	Static Aeroelastic Analysis
SOL145	Dynamic Flutter Analysis

References

[1] Stender, W., Kiessling, F: Aeroelastic Flutter Prevention in Gliders and Small Aircraft, DLR-Mitteilung 91-03, 1991

[2] Doc. Ing.Daňek. V. CSc.; Aeroelasticita, VUT Brno, 1986

[3]Ajgl,V.:Diplomová práce- Modální analýza a Flutterové vlastnosti ocasních ploch, ČVUT FS, 2009

[4] Žoldák, M.: Diplomová práce- Modální analýza a Flutterové vlastnosti křídla, ČVUT FS, 2009

Internet sources

[I1] Advisory Circular, Means of Compliance with Title 14 CFR Part 23 § 23.629, Flutter; 9/28/2004
http://rgl.faa.gov/Regulatory_and_Guidance_Library/rgAdvisoryCircular.nsf/0/371D5EA900EE1C1786256F2D0048ED41?OpenDocument

[I2] FAR 23.629 Flutter
http://www.flightsimaviation.com/data/FARS/part_23-629.html

[I3] MSC.Software Discussion Forums.

MSC.Software manuals for MSC.Nastran/Patran

[M1] MSC.Patran User's Guide

[M2] MSC.Patran Reference Manual, Part 1-7

[M3] MSC Advanced Dynamic Analysis

[M4] MSC.Patran User's Guide

[M5] MSC.Nastran Aeroelastic Analysis User's Guide

[M6] MSC.Patran FlightLoads and Dynamics User's Guide

[M7] MSC.Nastran Quick Reference Guide

[M8] MSC.Nastran Basic Dynamic Analysis Users Guide

Used software:

FindText
FlutterPlotter
MSC.Patran 2005
MSC.Nastran 2005