Numerical Simulation of the Transitional Flow on Airfoil

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Abstract

This paper considers to design and to validate the transitional method of two-dimensional flow on airfoils. This method involves the combination of empirical terms to determine the position of transition and is used to solve the averaged Navier – Stokes equations closed of k – ω SST turbulence model. Computation is specialized on transition in separated shear layer and it is used in the commercial software FLUENT 6.3.26, which is included of User-Defined-Functions.

Keywords

transition to turbulence, laminar separated bubble, k – ω SST turbulence model, boundary layer, Fluent, UDF

1. Introduction

A problem of the numerical simulation of the transition to turbulence is still actual for applications in internal and external aerodynamics. There are two transitions that take place in the transition laminar to turbulent shear layers: first is transition in attached flow (natural and bypass) and the second is transition in separated flow (so-called laminar separated bubble).

Practical methods are developed from solving a series of integral equations involving the boundary layer (e.g. software XFOIL). Computation is mostly established by solving the averaged Navier – Stokes equations and the turbulence model with transitional bypass model. All methods for computation of transition are based on algebraical or transport equations for an intermittency factor. The position of transition in flow is used to determine the Reynolds number relative to momentum boundary layer thickness or by Reynolds number relative to length of laminar part of the separation bubble in separated flow.

2. Numerical model

The computation is realized for steady and unsteady two-dimensional uncompressible flow by commercial software FLUENT 6.3.26. The flow is described by system of motion equations

$$\frac{D\rho}{Dt} = 0\,, \tag{1}$$

$$\frac{D(\rho u_i)}{Dt} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_i}\left[(\mu + \mu_t)\frac{\partial u_i}{\partial x_j}\right] + F_i\,, \tag{2}$$

where on the right side F_i is vector of external forces. The system of equations is closed with k – ω SST turbulence model by Menter [1] with transport equations pro turbulent energy

$$\frac{D(\rho k)}{Dt} = \mu_t \frac{\partial^2 u_i}{\partial x_j^2} - \beta^* \rho k \omega + \frac{\partial}{\partial x_j}\left[\left(\mu + \sigma_k \mu_t\right)\frac{\partial k}{\partial x_j}\right], \tag{3}$$

and the specific dissipation rate is

$$\frac{D(\rho\omega)}{Dt} = \alpha \frac{\omega}{k}\mu_t \frac{\partial^2 u_i}{\partial x_j^2} - \beta\rho\omega^2 + \frac{\partial}{\partial x_j}\left[\left(\mu + \sigma_\omega \mu_t\right)\frac{\partial \omega}{\partial x_j}\right] + 2\left(1 - F_1\right)\sigma_{\omega 2}\frac{\rho}{\omega}\frac{\partial k}{\partial x_j}\frac{\partial \omega}{\partial x_j}. \tag{4}$$

Turbulent viscosity is defined by term

$$\mu_t = \frac{a_1 \rho k}{\max\left(a_1\omega; F_2 \frac{\partial u}{\partial y}\right)}, \tag{5}$$

where function $F_2 = 1$ in boundary layer and $F_2 = 0$ in free stream. Model's constants a_1, α, β, β^*, σ_k, σ_ω, $\sigma_{\omega 2}$ and connective function F_1 and F_2 are mentioned in [1].

3. Transition in attached flow

By bypass transition to turbulence in attached flow is the start of the transition altogether determined by Reynolds number relative to length around the surface $Re_{st} = u_e s_t / \nu$. This is determined by empirical relations for Reynolds number relative to momentum boundary layer thickness $Re_{2t} = u_e \delta_{2t} / \nu$. There exist many empirical correlations; a few of them are shown on fig. 1. The relation developed by Příhoda [2] was used in the compution. It is of the form

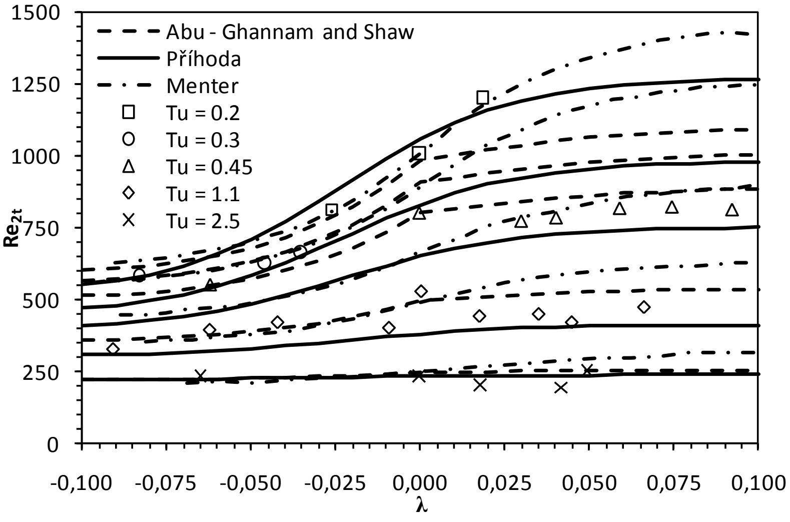

Fig. 1 *$Re_{2t}(\lambda_t)$ for some transition models (assumed and amended from [8])*

$$Re_{2t} = 402 Tu^{-0,6}\left[1 + 0.25\exp\left(-Tu\right)\frac{1 - \exp\left(-40\lambda_t\right)}{1 + 0.4\exp\left(-40\lambda_t\right)}\right], \tag{6}$$

where Tu (%) is turbulence level and λ_t is dimensionless pressure gradient. Index t implies the start of the transition area. Laminar to turbulent transitional flow is established by the intermittency function, suggested by Narasimha [3], in the form

$$\gamma = 1 - \exp\left[-\hat{n}\sigma\left(Re_s - Re_{st}\right)^2\right]. \tag{7}$$

Thus, the intermittency function depends only on coordinate s. The length of the transition is determined by parameters $\hat{n}$ and σ. They are described the generation and the production rate of the turbulent spots. These parameters are determined by relation

$$\hat{n}\sigma = \frac{N}{Re_{2t}^3}, \tag{8}$$

where parameter N is expressing the influence of turbulence level and pressure gradient and it is by Solomon, Walker a Gostelow [4] determined by empirical relation

$$\begin{aligned} N &= 0.86\times10^{-3}\exp\left(2.134\lambda_t \ln Tu - 59.23\lambda_t - 0.564\ln Tu\right) \quad \text{for} \quad \lambda_t < 0 \\ N &= 0.86\times10^{-3} Tu^{-0.564}\exp\left(-10\sqrt{\lambda_t}\right) \quad \text{for} \quad \lambda_t > 0 \end{aligned} \tag{9}$$

Thus, efficient viscosity is

$$\mu_{eff} = \mu + \gamma\mu_t, \tag{10}$$

where productions terms and diffusion terms form transport equations in turbulence model in transition area are affected by intermittency function $\gamma \in \langle 0;\ 1\rangle$. For following condition applies to laminar flow $Re_2 < Re_{2t}$, The effective viscosity is determined only by molecular viscosity $\mu_{eff} = \mu$. The disadvantage is the boundary layer thickness $\delta = y_n(u = 0.99u_e)$ must be known, where y_n is normal wall distance.

4. Transition in separated flow

The laminar separation bubble rises during low Reynolds numbers and flow near maximum of velocity. Then transition is takes place when it is influenced by disturbances and the reattachment of the flow. In relation on pressure coefficient are some types of separation bubble shown on fig. 2. The simply transition model in separated flow is proposed by Roberts [5]. The solution is based on empirical relationship involving the Reynods number Re_{l_1} relative to length of the laminar part of separation bubble in free stream turbulence level. This relation has been specified in a significant number experiments carried out by Jakubec [6]. It is of the form

$$Re_{l_1} = 2\times10^4 \log\left[\operatorname{cotgh}\left(3.5\frac{Tu_e}{100}\right)\right], \tag{11}$$

where Tu_e is local free stream turbulence level, defined by equation

$$Tu_e = \frac{1}{u_e}\sqrt{\frac{2k_e}{3}}. \tag{12}$$

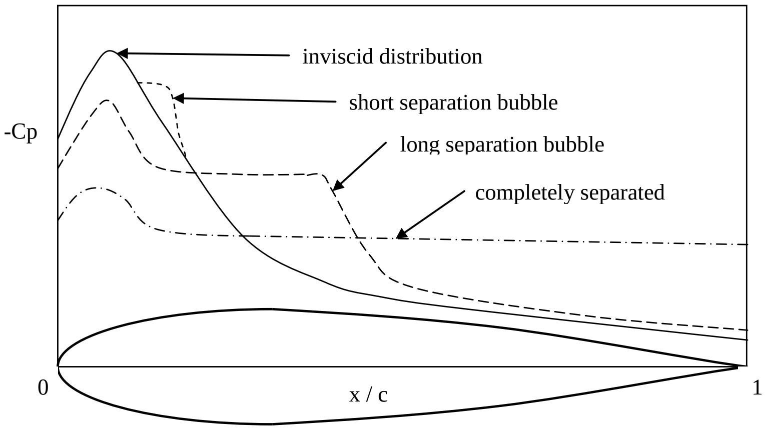

Fig. 2 *Types of laminar separation bubbles*

Reynolds number Re_{l_1} is defined by term

$$Re_{l_1} = \frac{u_e l_1}{\nu}. \tag{13}$$

The position of the transition in separated flow is determined by coordinates $s_t = s_s + l_1$, where s_s is position of the separation on airfoil. By the computation shows a sudden transition to the turbulence region.

5. Results

This method requires a high-quality grid, which is structured in the near of surface. Computational grids are generated with generator of structured grids, see Straka [7]. On fig. 1 is shown the grid on airfoil NACA 0012. In all computations are used discretization schema upwind 2. order.

The first tested case of bypass transition in attached flow was airfoil NACA 0012. The parameters consisted of zero angle of attack, Reynolds number $Re_c = cu_\infty/\nu = 5 \times 10^6$ and a turbulence level $Tu_\infty = 0.2$ %. fig. 2 compares the coefficient of friction with others transitional models a) classic k – ω SST turbulence model; b) transitional model implemented in software XFOIL, which appears from linear theory of stability (so-called e^N method); c) γ - Re_2 transitional model Menter and all [8]; d) computation bases on solution of Prandtl's equations of the boundary layer in curvilinear coordinates with adaptive grid, see Ďuriš [9].

The method was then tested on symmetric airfoil XIS40MOD, according with measurement performed by Würz [10], for a Reynolds number $Re_c = 1.2 \times 10^6$ and a turbulence level $Tu_\infty = 0.0375\%$. All measurements were provided on the suction side of airfoil.

Figures 3 and 4 show the comparison of velocity distribution with angle of attack $\alpha = 1^o$ or $\alpha = 5^o$. More detail measurement of velocity profiles in boundary layer was provided with angle of attack $\alpha = -3^o$. Figure 5 shows the velocity distribution and fig. 6 exemplifies the detail of the area with laminar separation bubble. On fig. 7 are compared integral boundary layer thicknesses - displacement δ_1, momentum δ_2 and energy thickness δ_3.

On fig. 8 shows the distribution of the friction coefficient, where is followed range of separation bubble. Figure 9 shows the comparison of computed velocity profiles and measurement in boundary layer with angle of attack $\alpha = -3^o$ (from laminar during separated up to redeveloped turbulent boundary layer). The figures shown are in agreement between computational results and the measurement and position of laminar separation bubbles.

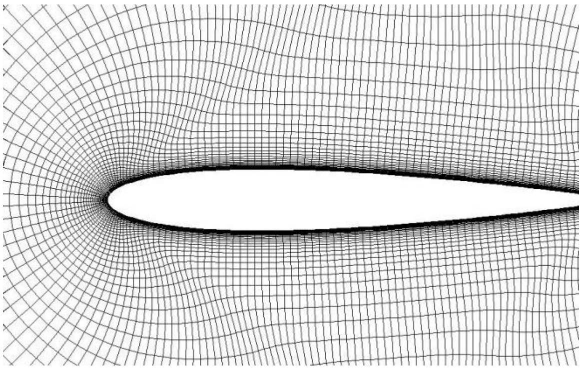

Fig. 3 *Computational grid on NACA 0012*

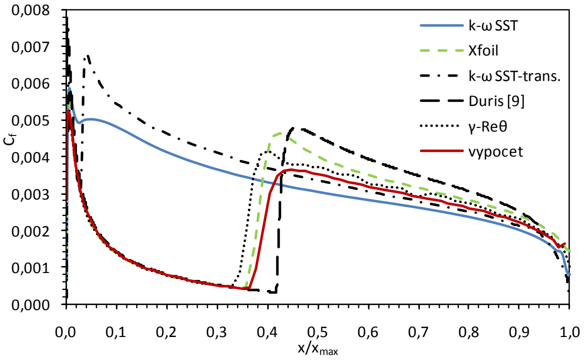

Fig. 4 *Distribution of friction coefficient*

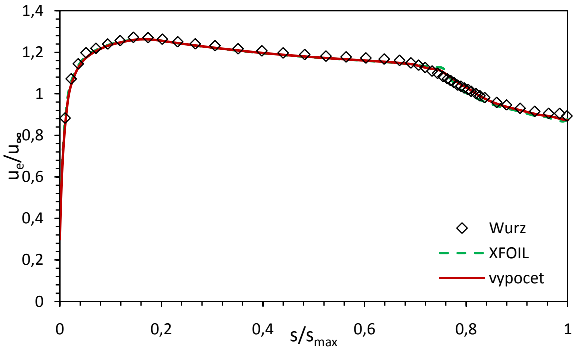

Fig. 5 *Distribution of free stream velocity*
$\alpha = 1°$

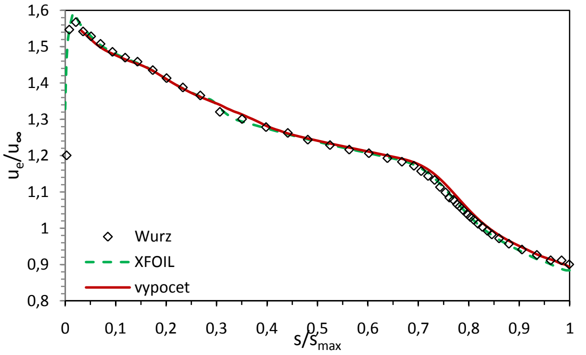

Fig. 6 *Distribution of free stream velocity*
$\alpha = 5°$

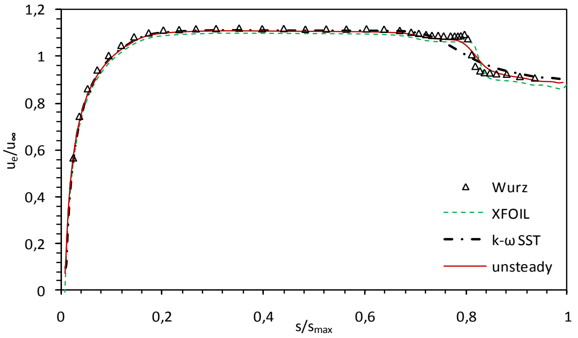

Fig. 7 *Distribution of free stream velocity*
$\alpha = -3°$

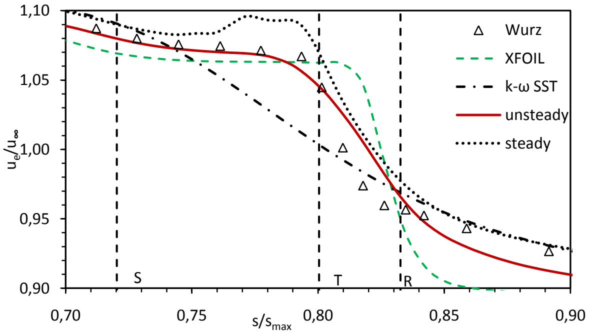

Fig. 8 *Detail in separation*

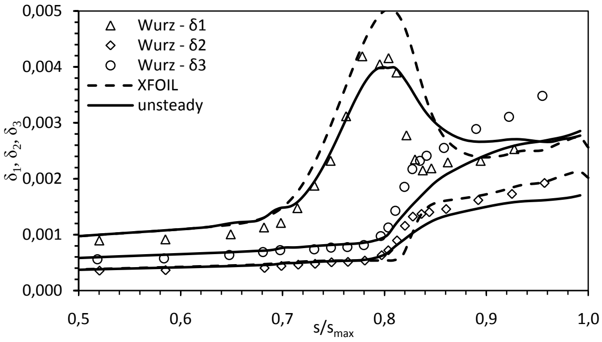

Fig. 9 *Distribution of thicknesses*
of the boundary layer

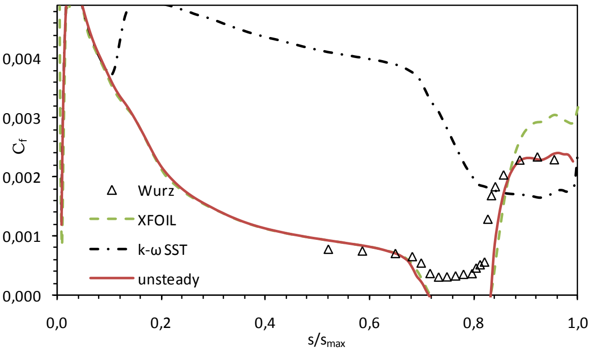

Fig. 10 *Distribution of friction coefficient*

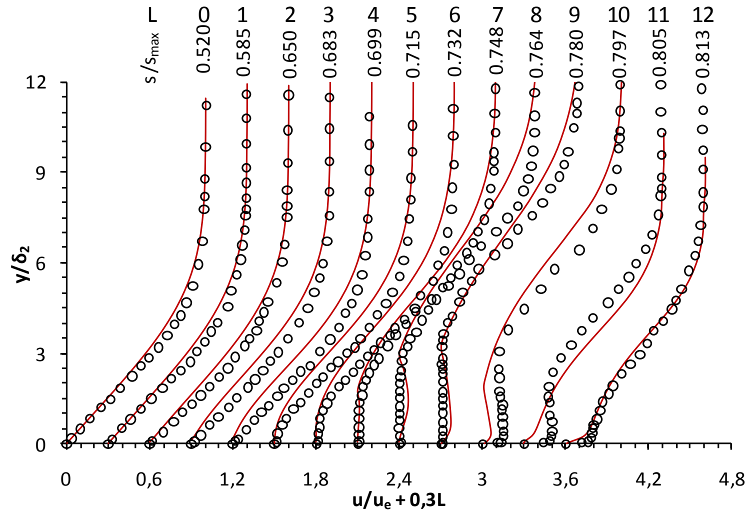

Fig. 11 Velocity profiles

Conclusion

This paper shows the method to solve transition to turbulence in attached and separated flow on airfoils. This method uses empirical relations for determination of position the transition area. The solving motion equations closed by $k - \omega$ SST turbulence model and completed by transition model was provided in commercial software FLUENT and UDF see [11].

Transitional model in attached flow was tested on NACA 0012 (with relative low turbulence level). Results are comparable with the model $\gamma - Re_2$ by Menter at al. [8] and with other transitional models. Good agreement was obtained for transition with laminar separation bubble. This case was tested by airfoil XIS40MOD and the angle of attack $\alpha = 1^o$, 5^o and -3^o and a very low turbulence level. A low turbulence level corresponds to external aerodynamic conditions. Some departures of solution from experiment in location of transition to turbulence can be created by numerical viscosity of used numerical schema in steady solution (pressure correction method). These departures are eliminated by unsteady solution, see fig. 6.

Presented transitional methods arrange quality results and there are applicable for practical computations transition to turbulence not only on airfoil, but on blades of turbines and compressors cascades. With respect to empirical terms, the solution converges very quickly. For the determination of integral quantities with low turbulence level is it profitable to use the software XFOIL, see Příhoda and Popelka [12]. This principle of transitional solution will be used for simulations of boundary layer active control.

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Nomenclature

C_f	friction coefficient, $C_f = \tau_w / \left(0.5 \rho U_\infty^2\right)$	[-]
H_{12}	shape parameter, $H_{12} = \delta_1 / \delta_2$	[-]
H_{32}	shape parameter, $H_{32} = \delta_3 / \delta_2$	[-]

k	kinetic turbulence energy	$[\mathrm{m^2 \cdot s^{-2}}]$
l_1	length of laminar part of separation bubble	[m]
n	spot generation rate	$[\mathrm{m \cdot s^{-1}}]$
p	static pressure	[Pa]
Re_s	Reynolds number	[-]
Re_2	momentum Reynolds number	[-]
x, y	Cartesian coordinate	[m]
s	coordinate along airfoil surface	[m]
Tu	turbulence level	[%]
u,v	velocity	$[\mathrm{m \cdot s^{-1}}]$
δ	boundary layer thickness	[m]
δ_1	displacement boundary layer thickness	[m]
δ_2	momentum boundary layer thickness	[m]
δ_3	energy boundary layer thickness	[m]
γ	intermittency function	[-]
λ	dimensionless pressure gradient, $\lambda = \left(\delta_2^2 / \nu\right)\left(du_e / ds\right)$	[-]
μ	dynamic viscosity	[Pa·s]
μ_T	turbulent dynamic viscosity	[Pa·s]
ν	kinematic viscosity	$[\mathrm{m^2 \cdot s^{-1}}]$
ρ	density	[ρ]
σ	Emmons spot propagation parameter	[-]
ω	specific dissipation	$[\mathrm{s^{-1}}]$

Index

e	local free stream
r	reattachment
s	separation
t	transition
∞	inlet

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