Dynamic and static bending properties of hybrid carbon tube

Ing. Zdeněk Pošvář

prof. Ing. Milan Růžička, CSc.

Abstrakt

Tato práce se zabývá porovnáním dvou hybridních kompozitových trubek vyrobených metodou navíjení se stejnou základní skladbou stěny. Druhá trubka navíc obsahuje jednu tlumící vrstvu. Zásadní otázkou je, jak velký vliv má tato vrstva na statické a dynamické vlastnosti. Cílem této práce je srovnání experimentálně získaných vlastností s analyticky vypočtenými a prezentace výsledků plynoucích z tohoto srovnání.

Klíčová slova

Tlumení, ohybová tuhost, kompozitová trubka, vlastní frekvence

1. Introduction

This paper is focused on the effect of damping layers in a composite structure. Motivation for implementation of the damping layers is mainly in milling machine tools. Several technologies are available to made carbon-composite profiles, one of the best for tubes or beams is the filament winding technology. These works consist of two tubes comparison, the first one is just carbon-epoxy and the second one is a carbon-epoxy with an extra damping layer from additional material. One of the basic questions is: Could analytical model of dynamic properties be used for composite materials? Could be Timoshenko or Bernoulli model used for „long“ carbon beams?

2. Static properties

Basic parameters for composite profiles are bending and shear stiffness. Several methods are available to compute these parameters for a wound carbon-epoxy profile. The classical laminar theory was used for each layer see *Figure 1* and parameters of unidirectional layer were computed by rule of mixture. Stiffness of the profile is just sum of the stiffness of the layers.

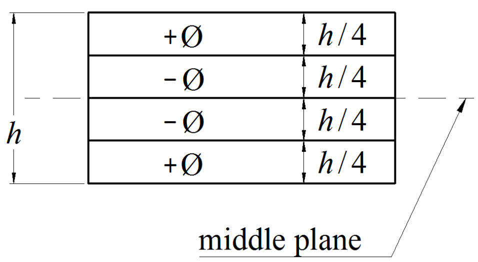

Figure 1. *Computation model of wound layer*

Shear stiffness of layer

$$A_T = \int_{(A)} E_A \, dA \quad [N] \tag{1}$$

and bending stiffness of layer

$$D_T = \int_{(A)} E_D \cdot w^2 \, dA \quad [N \cdot mm^2] \,. \tag{2}$$

Table 1. *– Layup of the tube*

Layup			
material	**volume of fibers**	**thickness *h* [mm]**	**angle Ø [°]**
HSC	60%	0,28	28,2
HSC	60%	0,19	87,7
DC	xx	0,51	xx
HSC	60%	0,19	87,8
HSC	60%	0,71	0,0
HSC	60%	0,33	32,7

Table 2. *– Computed properties*

Computed prop.	**D_T**	**A_T**
with damping	$1767 \cdot 10^6$	$1387 \cdot 10^3$
without damping	$1676 \cdot 10^6$	$1375 \cdot 10^3$

2.1 Effect of the beam length

Short or long beam? It is one of the most important things during the composite beams designing, when we talk about the static bending properties. The Bernoulli theory calculates deflection just as the bending loading. But by the Timoshenko theory, the deflection has two parts, the first is from the bending load and the second is from the shearing load.

The deflection of free end for cantilever beam with force F_Z at the free end by Timoshenko is

$$w_T = \frac{5 \cdot F_z \cdot l^3}{6 \cdot D_T} + \frac{F_z \cdot l}{A_T} \tag{3}$$

and by Bernoulli is

$$w_B = \frac{F_z \cdot l^3}{3 \cdot D_T} \,. \tag{4}$$

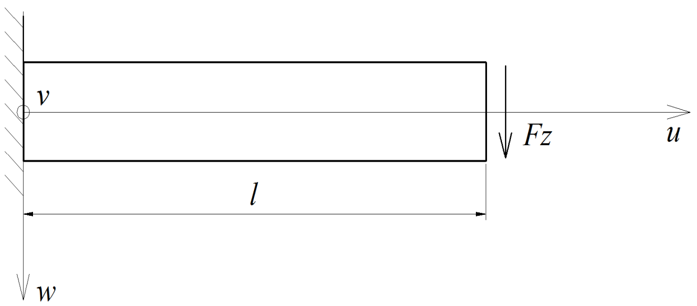

Figure 2. *Coordinate system and loading of beam*

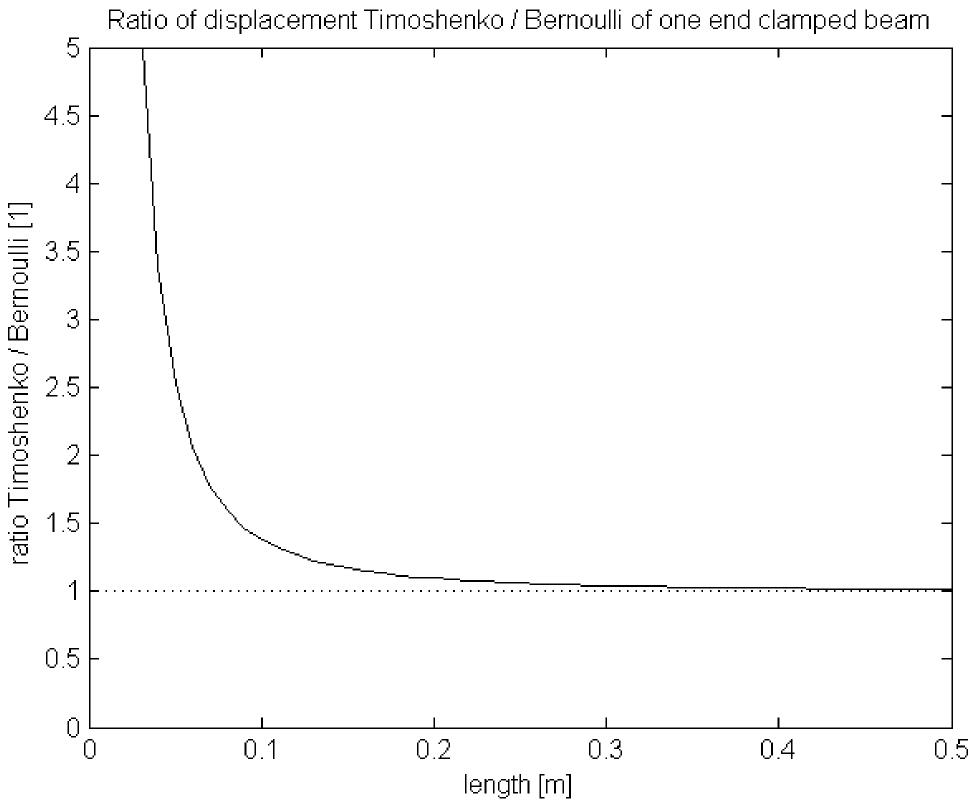

Figure 3. *Ratio of displacement Timoshenko / Bernoulli of one end clamped beam*

As it is shown at the Figure 3, for both tubes the deflection starts to be equal around 0.3 meters. For the tube 1.2 meters, the difference between the Bernoulli and Timoshenko theory is just 1.5% (0.6 meters for cantilever beam). Therefore, the Bernoulli theorem can be used without any problems as static approximation of deflection

2.2 Experimental bending test

The bending stiffness and bending strength were measured (with Bernoulli theorem) in a 3-point bending test. Drawing with parameters of the test is in *Figure 4*. For the 3-point bending test with 1 meter between supports, the error between Bernoulli and Timoshenko is just 1.5% (refer to *Figure 3*). It could be looked at this case as a long beam.

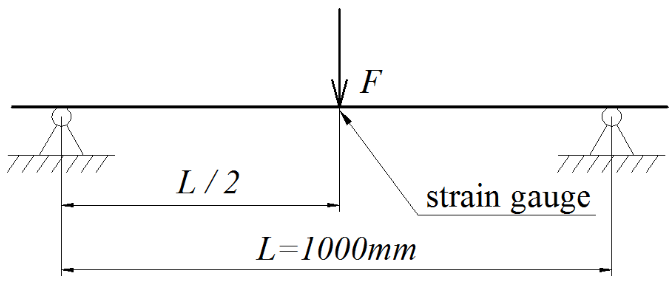

Figure 4. *Bending 3-point test*

Figure 7 shows the linear dependence between stress and strain up to strength. Therefore the bending stiffness is constant through the whole range of loading. During the measurement of the strength of the tube without damping layer (DL), the strain gauge signal run out of the range (bounds ±5500μm/m). Therefore, for this coupon only the force values were captured during the failure. For experiments, *HBM Spider-8* strain gauge panel was used.

Table 3. *– Parameters of the tubes from measurement*

Measured prop.	**Ø d_1 [mm]**	**Ø d_2 [mm]**	**weight [kg/m]**	**length [mm]**	**$(E*J)_m$**
with damping (vz2)	29,9	34,2	0,27	1200	$1788 \cdot 10^6$
without damping (vz1)	29,9	33,3	0,25	1200	$1752 \cdot 10^6$

Damping layer has changed the mechanism of failure, as it is shown in *Figure 5* (delamination between tows) and *Figure 6* (failure across the tows).

Figure 5. *Broken tube without damping layer*

Figure 6. *Broken tube with damping layer*

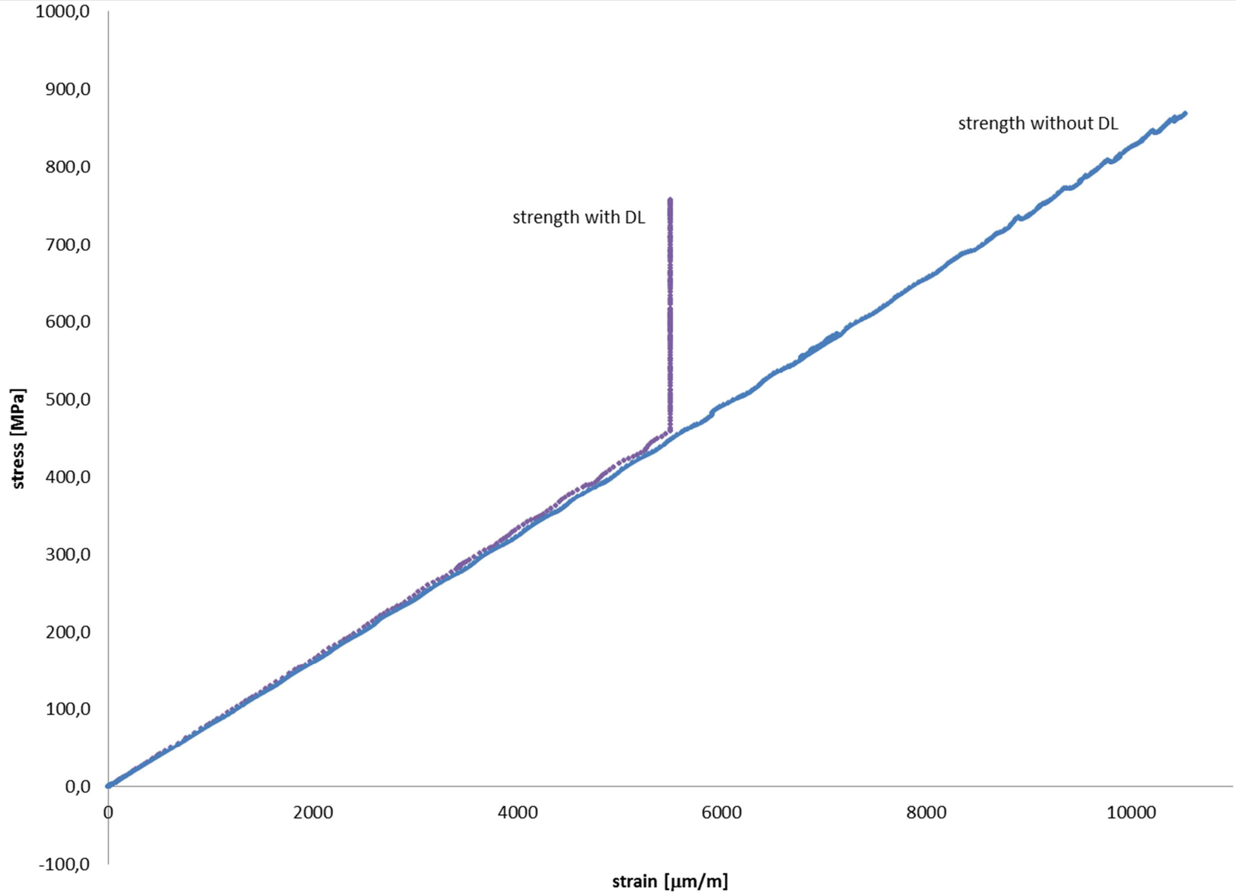

Figure 7. *Stress – Strain chart*

3. Dynamic properties

Dynamic properties usually mean eigen-modes with corresponding natural frequencies and damping of each mode. For comparison of the tube's properties, frequencies and damping defined by a relative damping ratio were selected. Both tubes were measured by experimental modal analysis (EMA).

Table 4. *– Natural frequencies of the tube without damping layer*

without damping - natural frequency [Hz]				
computation		**experiment**		
Timoshenko	**Bernoulli**	**Bernoulli**	**EMA**	
31	31	33	frequency	b_r [%]
193	196	204	203	0,172
530	549	571	535	0,181
1010	1075	1119	1026	0,252
xx	xx	xx	1568	0,155
1615	1777	1849	1649	0,291
xx	xx	xx	1666	0,299
xx	xx	xx	2054	0,161
2324	2655	2763	2348	0,421

Natural frequencies are compared with analytical method, as is shown in *Table **4*** and *Table 5*. For relative damping ratio was used viscose model of damping, but the measuring error is about 10-15%, so in damping properties is the differences between tubes negligible. It could be just a measuring error. More frequencies were measured by EMA in comparison with the computation. One of the reasons is that several mode-shapes obtained from the experiment, were not the bending modes. It could be torsion, wall vibrations or mixed modes.

Table 5. *– Natural frequencies of the tube with damping layer*

with damping - natural frequency [Hz]				
computation		**experiment**		
Timoshenko	**Bernoulli**	**Bernoulli**	**EMA**	
31	31	32	frequency	b_r [%]
194	197	198	200	0,228
532	551	555	533	0,099
1013	1081	1087	1009	0,229
xx	xx	xx	1541	0,081
xx	xx	xx	1617	0,362
1618	1786	1797	1632	0,300
xx	xx	xx	2017	0,211
2324	2669	2685	2304	0,477

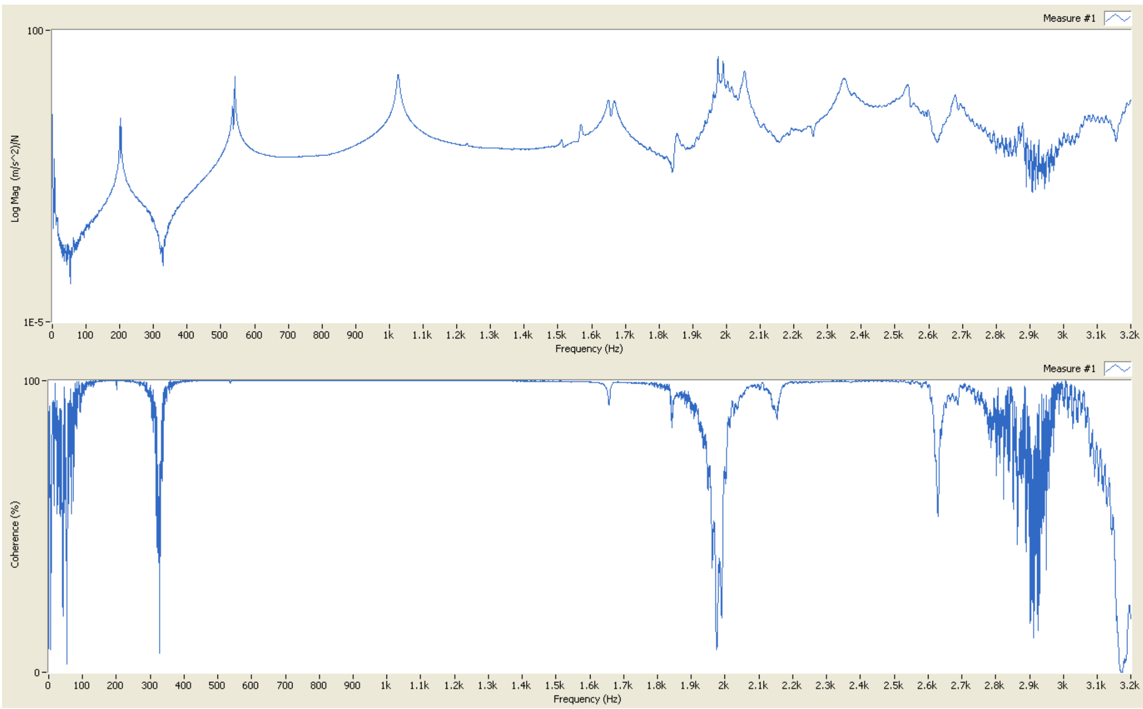

Figure 8. *Example of Transfer function and Coherence for one measuring point*

Under 100Hz, there is a problem with low coherence (not at all measuring points as huge as it shows in *Figure 8*). It is the purpose, why natural frequencies were not measured in that area.

$$\omega = \frac{1}{2 \cdot \pi} \cdot \sqrt{\frac{k}{m}} \qquad (5)$$

It could be problem with natural frequency of supports or exciter. The first approximation of modal properties of supports is written as (5), but with very low stiffness the displacement could be larger than measuring range.

4. Conclusion

Bernoulli theorem suits well for static properties of long composite tube without demand on damping layer and Timoshenko theorem suits very well for modal properties. It is necessary to change the method of experimental modal analysis because main demand is for lower frequencies, where is problem with natural frequencies more often.

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List of symbols

HSC	High Strength Carbon	
DC	Damping Cork	
d_1	internal diameter	(mm)
d_2	outer diameter	(mm)
D_T	bending stiffness	(N·mm^2)
$(E*J)_m$	measured bending stiffness (3-point test)	(N·mm^2)
A_T	shear stiffness	(N)
A	area of layer	(mm^2)
F_Z	loading force	(N)
l	length of cantilever beam	(mm)
L	length of supported beam	(mm)
w_T	cantilever beam deflection by Timoshenko	(mm)
w_B	cantilever beam deflection by Bernoulli	(mm)
ω	natural frequency of support	(Hz)
k	stiffness of support	(N·m^{-1})
b_r	relative damping ratio	(%)
m	part of weight of beam for the support	(kg)
Ø	winding angle of layer (each ± Ø)	(°)
h	thickness of layer	(mm)
E_A	shear elastic parameter of layer	(MPa)
E_D	tensile elastic parameter of layer	(MPa)

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