Numerical modelling of fatigue damage in fiber reinforced composites and the proposal of experimental verification

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Abstrakt

Vzhledem k rostoucímu množství aplikací kompozitních materiálů je nezbytné provádět stále detailnější výpočty. Nelze se vyhnout ani použití kompozitních materiálů v konstrukcích, které jsou vystaveny cyklickému zatěžování. V tomto příspěvku jsou popsány a testovány možnosti často diskutované metodiky – predikce únavového chování na základě zbytkové tuhosti. Modely zbytkové tuhosti byly identifikovány pomocí experimentálních dat a poté implementovány do MKP kódu. Neopomenutelným krokem je ověření získaných výsledků výpočtů. Vzhledem k tomu bylo dále nutné navrhnout vhodnou metodiku experimentální verifikace. V příspěvku je proto popsána také konstrukce navrženého testovacího zařízení.

Klíčová slova

composite, fatigue, residual stiffness, finite element method, experimental measurement

1. Introduction

The issue of fatigue in composites is generally more complicated and less described then in the case of metals. The basic reasons are as follows. The propagation of the crack is limited by the inhomogeneous character of material, thus the description of crack growth is quite complicated. Another important fact is that there are more expressions of fatigue damage. The most important ones are the decrease of stiffness and strength. Especially the stiffness of structure influences the modal characteristics of structure or resistance to buckling.

The fatigue in composites became a real phenomenon in 1970's thanks to well-known authors Broutman [1], Howe [2] and Owen [3]. The models formed into three basic categories according to used approach: fatigue life models, progressive damage models and phenomenologically based models. The possibilities of the first group are limited due to huge variance of stacking sequences and properties of specific constituents. Progressive damage models differ from the fatigue life models in introducing of some damage variable which describes the damage of composite. These models are introducing the physically based modelling of some damage mechanism.

The principle of phenomenological modelling is to observe some outer sign of fatigue damage [4]. Very frequently used ones are residual stiffness and residual strength. Both residual stiffness and residual strength are very important variables. The reasons are as follows. It is obvious, that the strength reduction is also accompanied by the reduction of static safety factor. The stiffness reduction has a negative impact on resistance to buckling of thin walled structures and then it influences e.g. the modal characteristics of the structure.

Some authors have been suggesting algorithms for modelling of progress fatigue damage in laminated composites structures. These modern algorithms, such as FADAS (Fatigue Damage Simulator) algorithm [5], combine both types of mentioned phenomenological models.

Residual stiffness model is used for stress redistribution mapping and residual strength model for determination of failure point using an appropriate criterion.

During last four decades of research, failure criteria which are based only on residual stiffness have also been developed. The best known ones were proposed by O'Brien and Reifsnider [6] and by Hwang and Han [7]. These criteria allow determination of failure point only on the base of residual stiffness.

2. Description and basic review of residual stiffness models

Residual stiffness degradation models describe the damage of the material as a function of stress level and number of cycles. Generally speaking these models are mathematically formulated curves which are usually identified on experimental dates. A typical shape of residual stiffness (modulus) curve is shown in figure 1.

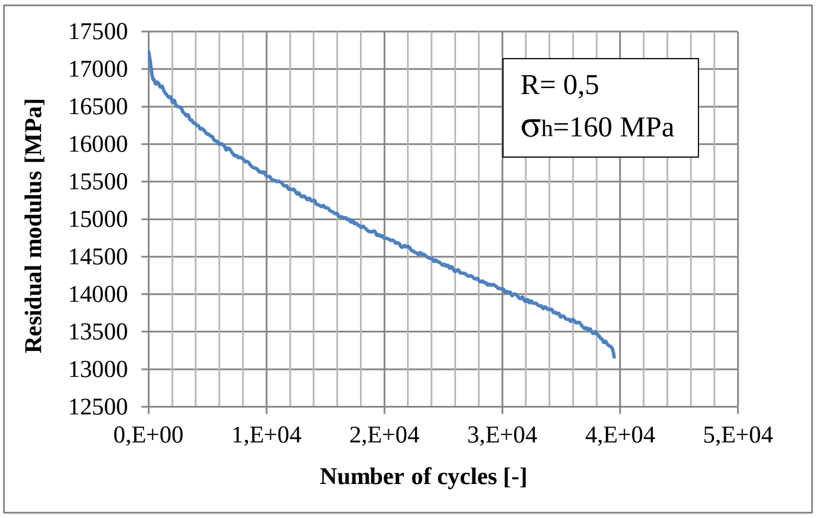

Figure 1. Residual stiffness curve of glass/epoxy laminate **-** $[(0/90)_8]$. Reinforcement was realized by fabric 280 g/m^2 with plain weave embedded in epoxy resin [8].

Considering the shape of residual stiffness curve, it is obvious, that as a satisfactory approximation can be used some form of power relation. This fact is evident in many stiffness degradation model formulations. In the following summary are shown two of basic models, which are frequently mentioned by other authors, e.g. in [9].

- Paepegem – Degrieck: $$\frac{dD}{dN} = \frac{A \cdot (\Delta\sigma)^C}{(1-D)^B} \quad (1)$$
- Liu – Lessard: $$\frac{dD}{dN} = \frac{A \cdot (\sigma_{\max})^C}{B \cdot D^{B-1}} \quad (2)$$

Stiffness degradation models are the evolution laws and they have usually been formulated in differential form, see equation (1) and (2). Symbols *A*, *B* and *C* are the model parameters and they have to be identified using experimental dates. Parameters σ_{max} and $\Delta\sigma$ are peak stress and the range of stress of loading cycle. Symbol *D* expresses the damage parameter and can be formulated using equation (3), where E_0 is a virgin modulus and $E(n)$ is the residual modulus after *n* cycles.

$$D = 1 - \frac{E(n)}{E_0} \quad (3)$$

3. The proposal of use of models for different material system types

As a satisfactory approximation of experimental data was found degradation model Liu – Lessard (2). After substituting from equation (3) to equation (2) and integration of the model, the following relation (4) will arise. Degradation model in this form has already been implementable to FE code.

$$E(n) = E(0) \cdot \left\{1 - \left[A \cdot (\sigma_{\max})^{C} \cdot n\right]^{1/B}\right\} \tag{4}$$

It is necessary to mention one important fact. All above mentioned models were developed on the base of one dimensional experiment. In real structures laminate layers are usually subjected to general plane state of stress, see figure 2.

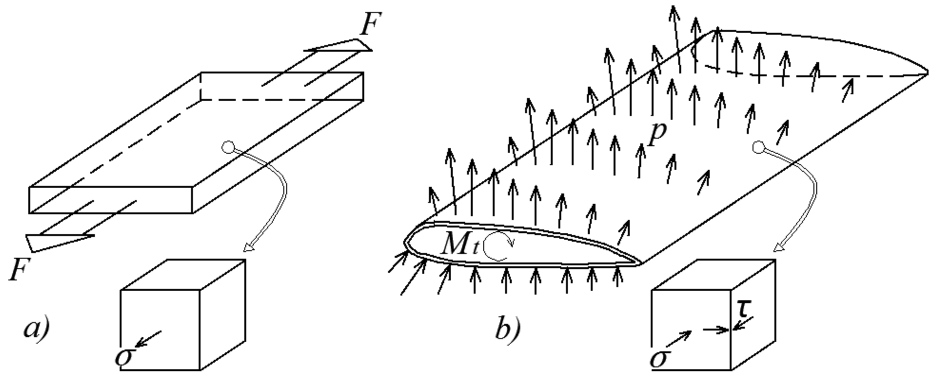

Figure 2. Mechanical model of unidirectional tensile test – part *a)* and a mechanical model of thin walled composite wing, which is loaded with a pressure field and torque – part *b)*. The difference between stress states of both loading modes is obvious in shown infinitesimal volumes.

A significant piece of work was to suggest how to take into account a general state of stress to degradation models. Two different methodologies were proposed. Each of them is applicable on specific material systems. Generally, fiber composites are orthotropic material systems, but in some cases they can be regarded as in-plane isotropic. This is dependent on the used reinforcement and stacking sequence. As an in-plane isotropic reinforcement it is possible to mention e.g. CSM (Continuous Strand Mat) reinforcement. CSM reinforcement is a fibrous system with randomly oriented fibres. On the other hand, reinforcement such as fabrics or unidirectional layers, have to be regarded as orthotropic and calculation of equivalent stress is not possible.

3.1 Model of degradation of in-plane isotropic materials

In the case of in-plane isotropic material systems, it is possible to calculate equivalent stress with the use of an appropriate hypothesis. Based on information from literature [9], in the case of CSM/Epoxy composites Mohr's hypothesis for brittle materials is in a good agreement with static experiments. Mohr's hypothesis can be expressed using equation (5).

$$\sigma^{ekv} = \sigma_1 - \frac{X_t}{X_c}\sigma_3, \tag{5}$$

Symbols X_t and X_c are tensile and compression strength, σ_1 and σ_3 are maximal and minimal principles and σ^{ekv} is equivalent stress. The model which was implemented to FE code was modified to the following form (6). It is clear that it is similar to model (4).

$$E(n) = E(0) \cdot \left\{1 - \left[A \cdot \left(\sigma_{\max}^{ekv}\right)^{C} \cdot n\right]^{1/B}\right\} \tag{6}$$

3.2 Model of degradation of orthotropic laminates

In the case of classical laminates, which are reinforced by unidirectional layers or by fabrics, the situation of damage calculation is more complicated. As already mentioned, the calculation and following use of equivalent stress is not possible. The stress state of each layer is generally defined by three components of stress tensor (plane stress is expected), see figure 3.

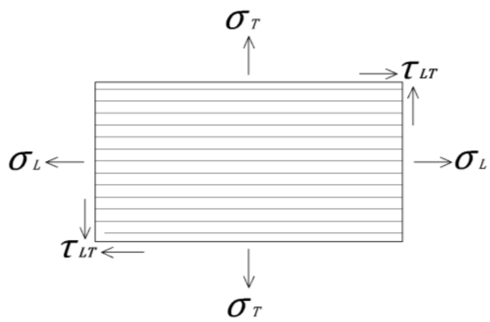

Figure 3. General stress state of UD layer**.**

The stiffness of each layer can be expressed using stiffness matrix C for two-dimensional case (7).

$$C = \begin{bmatrix} \frac{E_L}{1-\upsilon_{LT}\upsilon_{TL}} & \frac{\upsilon_{LT}E_T}{1-\upsilon_{LT}\upsilon_{TL}} & 0 \\ \frac{\upsilon_{LT}E_T}{1-\upsilon_{LT}\upsilon_{TL}} & \frac{E_T}{1-\upsilon_{LT}\upsilon_{TL}} & 0 \\ 0 & 0 & G_{LT} \end{bmatrix} \tag{7}$$

Symbols E_L and E_T are Young's moduli in longitudinal and transversal direction, G_{LT} is sheer modulus and υ_{LT} and υ_{TL} are major and minor Poisson's constants. It's expectable, that multiaxial cyclic loading will influence all material constants, which are used in definition of elastic constants in matrix C. In other ideas, the decrease only of moduli E_L, E_T and G_{LT} will be expected. Degradation model of stiffness matrix of thin-walled orthotropic laminates was suggested in the following form (8).

$$\begin{aligned} &E_L(n) = E_L(0) \cdot \left\{1 - \left[A \cdot (\sigma_L)^{C} \cdot n\right]^{1/B}\right\}, && \sigma_L \succ 0 \\ &E_L(n) = E_L(0), && \sigma_L \prec 0 \\ &E_T(n) = E_T(0) \cdot \left\{1 - \left[Q \cdot (\sigma_T)^{M} \cdot n\right]^{1/V}\right\}, && \sigma_T \succ 0 \\ &E_T(n) = E_T(0), && \sigma_T \prec 0 \\ &G_{LT}(n) = G_{LT}(0) \cdot \left\{1 - \left[P \cdot \left(\left|\tau_{LT}\right|\right)^{O} \cdot n\right]^{1/U}\right\} && \end{aligned} \tag{8}$$

Parameters A, C, B, M, Q, V, P, O, and U are material parameters and have to be identified from experiments. Model (8) is usable only for specific loading cycles. This model won't probably work in the case of application of alternating cycles, because the expectation of zero damage in compression isn't true in the case, that tensile loading was previously applied. The reason stems from the change of damage mechanisms [10].

4. Implementation of models to FE code

The need of implementation of model to FE code is caused by the following fact. Of course, that in case of some simple structures (tensioned rod), it's possible to calculate stress state using analytical methods. From the point of view of laminates, a very frequently used method is classical lamination theory (CLT). In practical applications it is necessary to analyse complicated structures with stress concentrations and difficult loading modes. In these cases, the use of FEM is unavoidable.

Due to the fact, that in these structures, the stress field isn't uniform, it is obvious, that stress redistribution will occur and it is necessary to map a whole loading history. The methodology which is introduced in this paper is based on discretization of loading history – the incremental solution. New moduli values and stress fields are calculated after previously defined number of cycles (cycles step). FE analysis is performed after each cycle step. FE pre-processor also allows defining of material for each finite element separately and updating of material parameters (moduli) on the base of stress state of this finite element.

FE analysis is performed in order to get stress field of the structure. In the case of thin walled composite structures, there is a serious chance of non-linear behaviour of these structures. In figure 4, there is shown the comparison of linear and non-linear FE analysis of simple fixed beam. The difference of stress field is clear. Generally, it's impossible to exclude the possibility of nonlinearities and due to this fact, in all calculations non-linear analysis was performed.

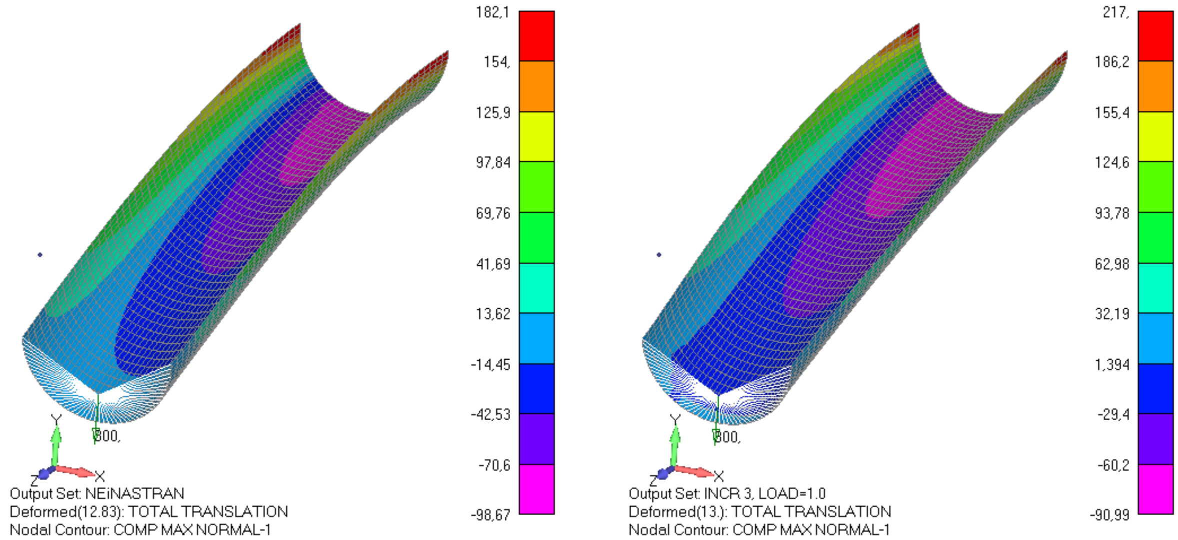

Figure 4. The comparison of the distribution of stress σ_L from linear and nonlinear analysis of thin-walled structure. Deformation scale factor is 5.

4.1 Implementation of degradation model of in-plane isotropic laminates

Discussed model was described in chapter 3.1, see relation (6). The algorithm of damage calculation is shown in the following figure 5. Model was implemented to FEMAP v10.3 pre-processor and analysis was performed using NEi Nastran 10.1 solver. The calculation is controlled using a script which was written in Visual Basic.

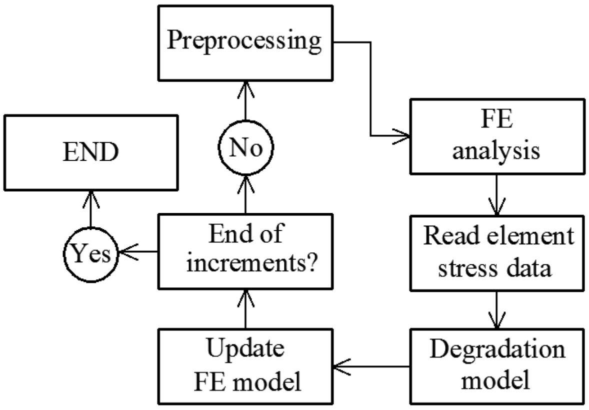

Figure 5. Block scheme of computational algorithm for calculations of stiffness reduction in in-plane isotropic laminates.

4.2 Implementation of degradation model of orthotropic laminates

The computational algorithm for implementation of model (8), which was described in chapter 3.2, is shown in figure 6. It is obvious, that this algorithm evaluates progressively layer by layer and updates moduli values in FEM model.

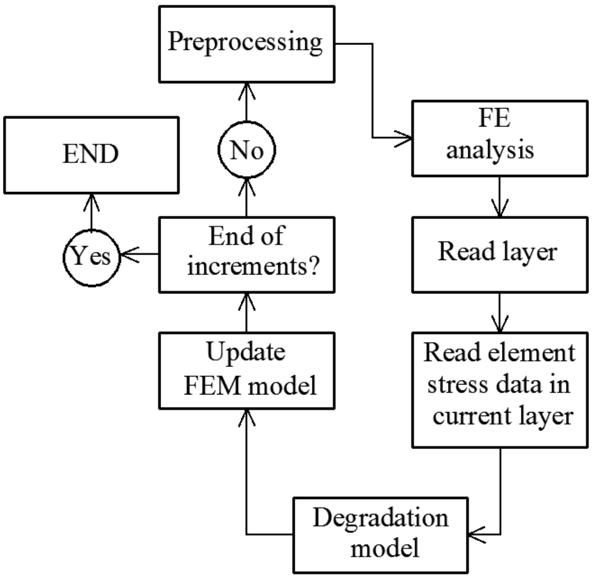

Figure 6. Block scheme of computational algorithm for calculations of stiffness reduction of orthotropic laminates.

5. Identification of models and testing of computational algorithms

As already mentioned, degradation models have to be identified using experimental data. Identification consists in calculation of model's coefficient using methods of numerical mathematics (least squares method).

5.1 Identification and testing of degradation model for in-plane isotropic laminates

Suggested degradation model for damage calculations of in-plane isotropic laminates was introduced in chapter 3.1. This model was mathematically explained using relation (6).In the case of discussed material systems model is identifiable using only one residual stiffness curve, which was measured during tension fatigue test. In the case of 1D stress state, the normal stress is equal to equivalent stress.

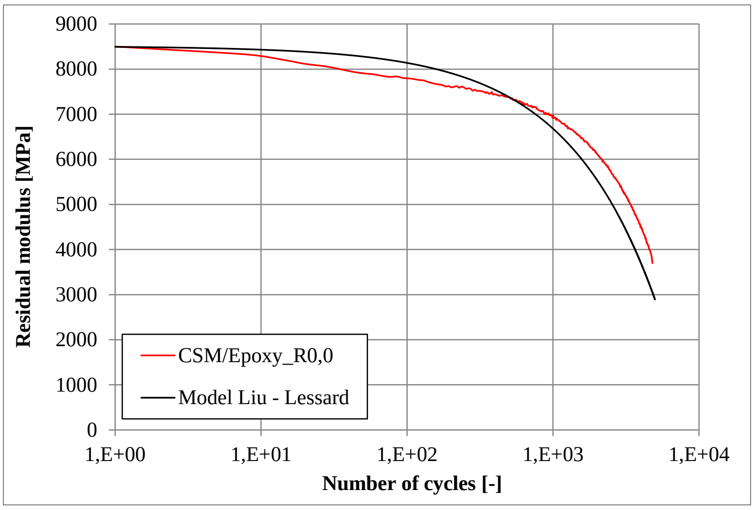

Figure 7. Residual stiffness curve of CSM/Epoxy composite measured during 1D tension fatigue test and the curve, which is representing identified degradation model Liu – Lessard.

In figure 7, there is shown experimentally measured residual modulus curve (red one). This curve was measured during 1D fatigue test on the base of testing machine's upper crossbeam position. This position was changing due to decrease of stiffness of testing specimen. Loading of specimen was realized with constant force (force loading) with frequency of 10 Hz. Using this curve, model (6) was identified. The curve, which is representing a model, is also shown in figure 7 (black one). The final form of identified model is defined using equation (9).

$$E(n) = E(0)\cdot\left\{1-\left[4{,}2\cdot 10^{-15}\cdot\left(\sigma_{max}^{ekv}\right)^{5{,}88}\cdot n\right]^{\frac{1}{1{,}43}}\right\} \quad (9)$$

To show the possibility of this approach to fatigue calculations a simple example of fixed beam is presented. The mechanical scheme of the beam is shown in figure 8. The loading cycle had the following parameters. The cycle asymmetry value was zero and upper cycle force was 200N. The parameters of virgin material, which were defined in pre-processor, are as follows: laminate thickness = 2 mm, virgin Young's modulus = 8500 MPa.

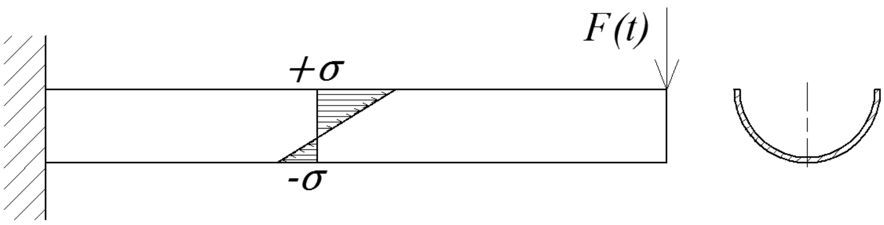

Figure 8. Scheme of fixed beam, which was modelled using FEM in order to test degradation models.

Results of calculations are shown in the following figures. The algorithm was set to calculate residual stiffness after 5000 cycles in 100 increments (50 cycles in one cycle step). In figure 9, there is shown the distribution of residual modulus after a whole loading block of 5000 cycles. The comparison of displacements in the direction of loading force is shown in figure 10. The increase of displacements of approximately 0,5 mm is clear.

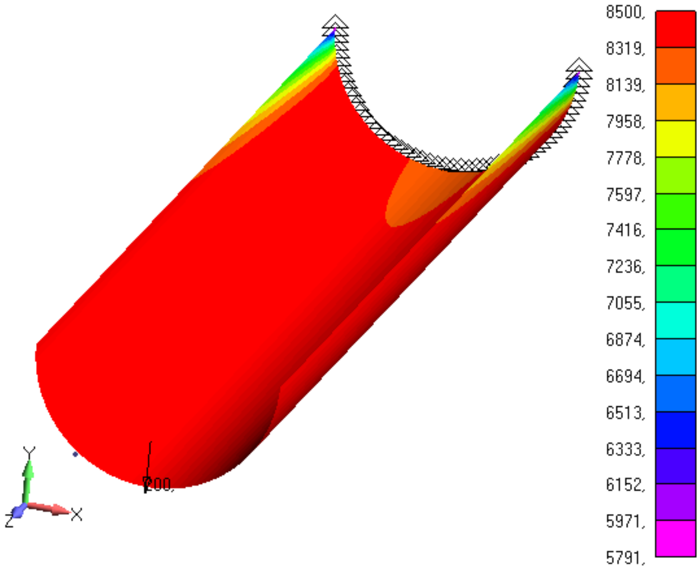

Figure 9. The distribution of residual modulus after 5000 cycles. A virgin modulus value was set to 8500 MPa and the lowest value of residual modulus is 5791 MPa.

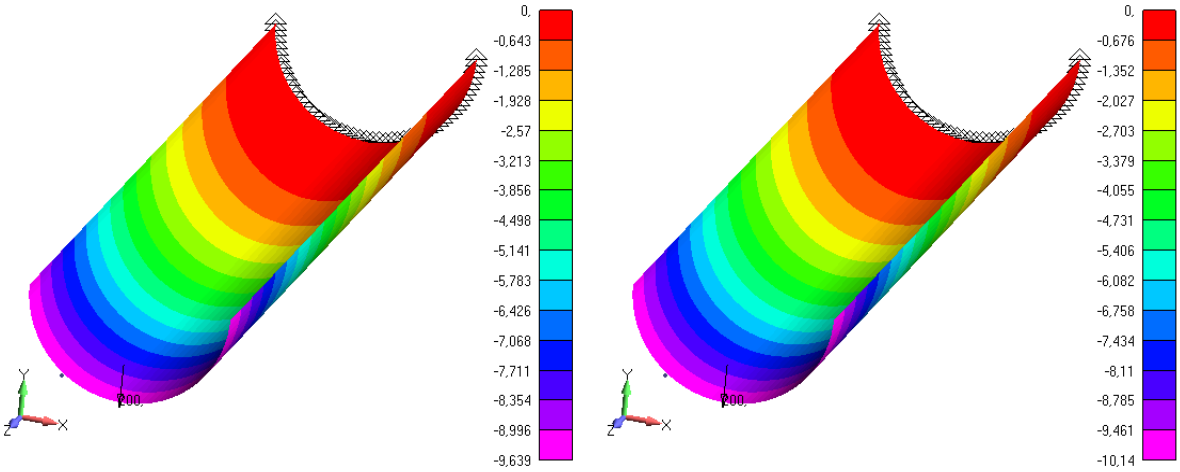

Figure 10. The comparison of displacements in the direction of loading force (y direction). Displacements increased by 0,5 mm.

5.2 Identification and testing of degradation model for orthotropic laminates

Discussed model was tested using the same FE model, thus using fixed beam, which was described in previous chapter 5.1. As already mentioned in chapter 3.2, proposed model (8) hasn't been completely universal and has been usable only for some specific types of cycles. The most important fact is, that model will probably not work for loading, which causes alternating stress in some part of structure. As illustrated in figure 8, the normal stress, which is caused by the bending moment, is tensile on one side of neutral axis and compressive on the other side of neutral axis. Due to value of cycle asymmetry coefficient of loading force, the stress won't alternate between tension a compression. Based on this fact, for analysing of discussed beam, proposed model is useable.

Identification of model (8) is complicated. There are more coefficients and especially the identification of shear modulus damage model is questionable. Due to the fact, that model for orthotropic laminates was tested using laminate, which was reinforced by balanced fabric and based on this fact had the same parameters in direction *L* and *T*, the part model (8) for decrease of Young's moduli in these directions was equal. This model was identified using residual stiffness curve, which is shown in figure 11. The methodology of experimental measurement of this curve was described in chapter 5.1.

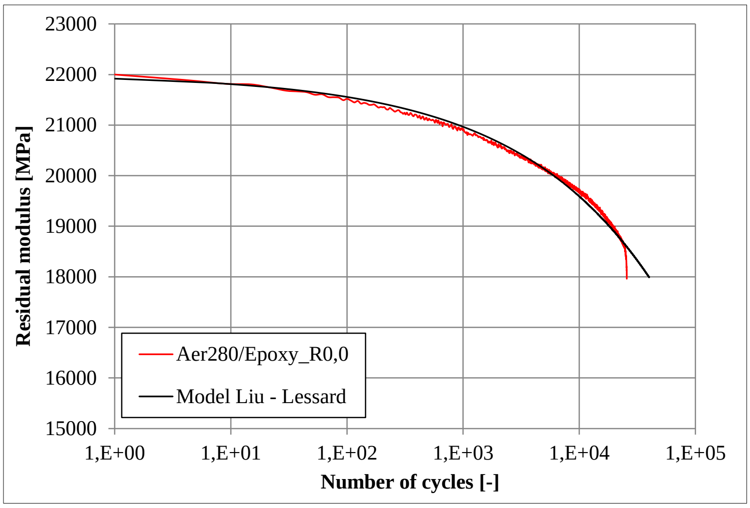

Figure 11. Residual modulus E_L curve of laminate, which was reinforced by balanced fabric $[(0/90)_3]$ and identified model Liu – Lessard.

Identification of shear modulus degradation model was different in the following fact. The residual shear modulus curve wasn't measured on the base of position of upper crossbeam of testing machine. The increase of displacement was too big and the measurement didn't have a satisfactory quality. Due to this fact, the measurement of modulus was realized for eight times during fatigue test using Petit - Rosen standard method (fatigue test was stopped and a static tensile test of specimen was performed), see figure 12. In table 1, there are shown the values of model (8) coefficients.

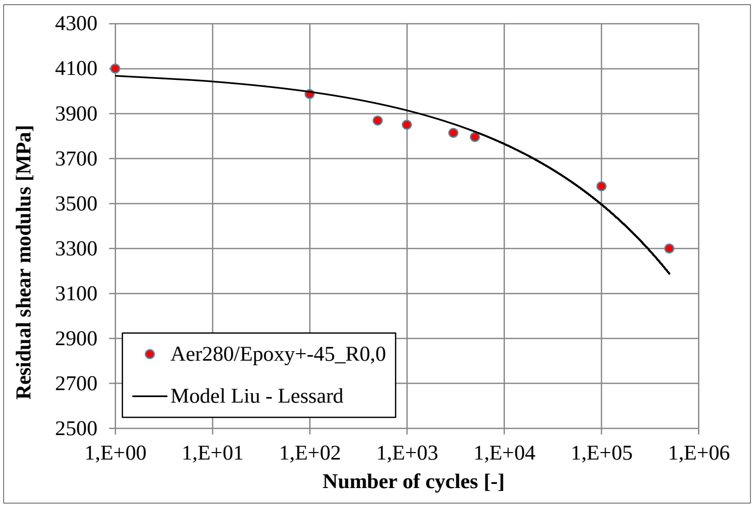

Figure 12. Residual shear modulus G_{LT} of laminate, which was reinforced by balanced fabric $[(0/90)_3]$ and identified model Liu – Lessard.

Table 1. Coefficients of identified model

Coefficient	A	B	C	Q	M	V	P	O	U
Value	$4,3.10^{-15}$	2,72	3,70	$4,3.10^{-15}$	2,72	3,70	$4,25.10^{-15}$	3,90	5,21

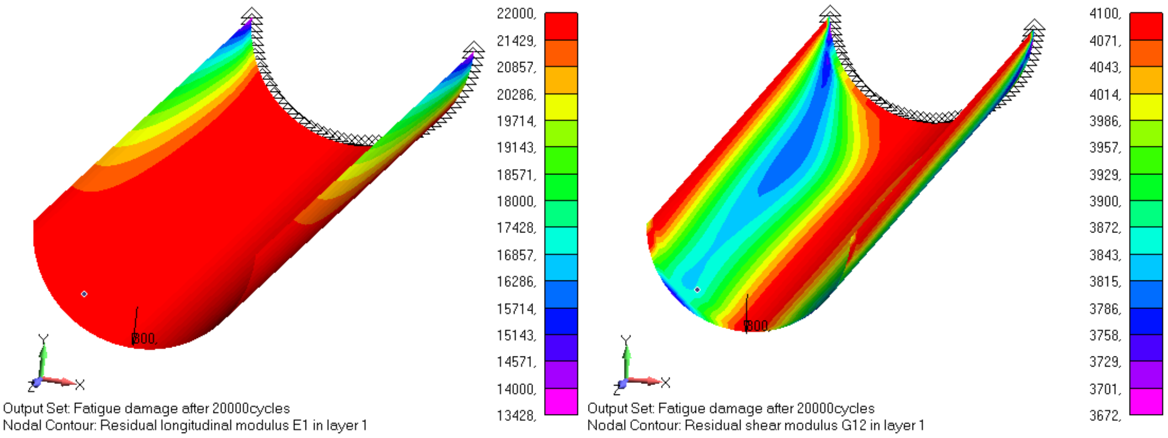

Figure 13. The results of FE calculations of residual moduli E_L (on the left) and G_{LT} (on the right) in the first layer after 2.10^4 cycles.

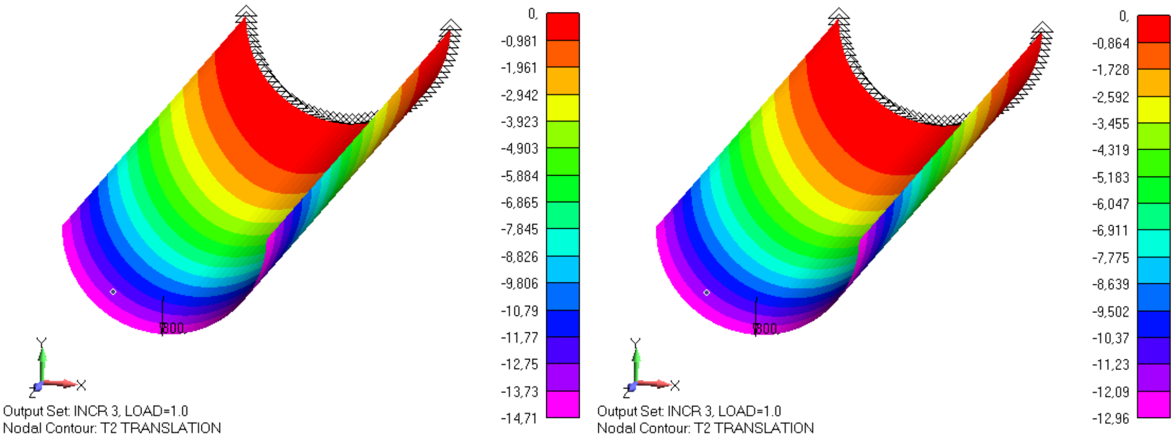

Figure 14. The comparisonof displacements of virgin structure (on the right) and after 2.10^4 cycles (on the left). The increase of displacements is 1,75 mm.

As a testing material a glass fiber laminate with three layers and fiber content of 40% was chosen and then modelled using FEM. All layers were oriented with zero angles toward to longitudinal axis of the beam. In figure 13, there are shown distributions of longitudinal modulus E_L and shear modulus G_{LT} after 2.10^4 cycles in the first layer (calculated in 200 increments). It is obvious, that damaged areas follows areas with high values of stress. The increase of displacements is shown in figure 14.

6. The proposal of experimental verification of FE calculations

Based on results of calculations, which are shown in previous chapters, it's clear, that an appropriate method for verification of calculations may be monitoring of displacements. In figure 15 is shown the design concept of testing machine assembly. A testing specimen (marked by green colour) has the same geometry as modelled beam described in previous chapters. A beam is loaded by an electrical actuator (marked by yellow colour). To reach an optimal ration of force and displacements in the case of use of different testing specimen, the testing machine is equipped with lever mechanism which has the function of simple transmission. Displacements will be measured using position sensors. Measured values of displacements will be compared with calculations. Loading frequency is limited by the value of 8 Hz.

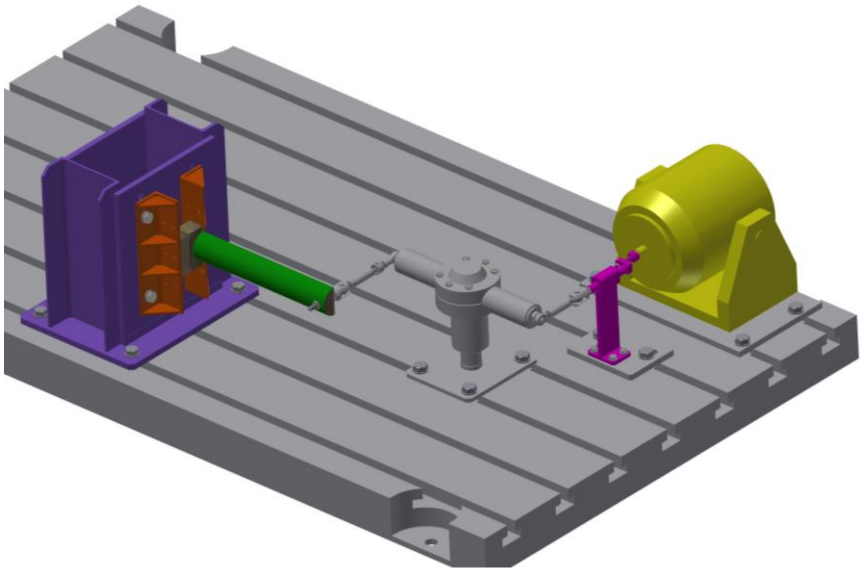

Figure 15. The assembly of bending testing machine. The main aim of this testing machine is to verify FE calculations.

7. Conclusion

The issue of fatigue in composites is very live and has been offering many possibilities for development of new approaches and methodologies. A satisfactory tool for dimensioning of cyclic loaded composite structures hasn’t been found yet. The above described approach offers some possibilities, but the experimental verification has to be performed to show accuracy of prediction. Generally, composite structures are highly fatigue resistant. But the dimensioning of these structures can be very shifty. Although the fracture will not occur during the life of structure, decrease of strength or stiffness can reduce the design values of safety factors.

The following development of presented issue will be based on improvement of stiffness degradation model and implementation of residual strength models for prediction of failure. Of course, that criteria based on residual stiffness will be tested.

List of symbols

D	damage parameter	(1)
$\Delta\sigma$	peak to peak amplitude of the stress	(MPa)
σ_{max}	peak stress	(MPa)
E_0	virgin Young's modulus	(MPa)
E_n	Young's modulus after n cycles	(MPa)
σ_L	longitudinal normal stress	(MPa)
σ_T	transversal normal stress	(MPa)
σ^{eqv}	equivalent stress	(MPa)
σ_1	major principle stress	(MPa)
σ_3	minor principle stress	(MPa)
X_t	tensile strength	(MPa)
X_c	compressive strength	(MPa)
E_L	longitudinal modulus	(MPa)
E_T	transversal modulus	(MPa)
υ_{LT}	major Poison's constant	(1)
υ_{TL}	minor Poison's constant	(1)
G_{LT}	shear modulus in direction LT	(MPa)
CLT	classical lamination theory	
FEM	finite elements method	

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