Aerodynamic Characteristics of Wing Having Spanwise Chamber

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Abstrakt

Příspěvek se zabývá problematice křídel mající prohnutí křídel ve směru rozpětí. V první části příspěvku je odvozena numerická metoda využívající principu Prandtlovy rovnice vztlakové čáry. Tato metoda dovoluje řešit prostorově prohnutá křídla. V další části práce je diskutován vliv prohnutí křídla na aerodynamické charakteristiky křídla. V poslední části se autor věnuje plánovanému rozšíření používané metody.

Abstract

This paper deals with problematics of wings having spanwise chamber. The numerical method based on philosophy of Prandtl's lifting lines theory is derived in first part of this paper. This method allows analysis of spatial chambered wings. Influence of spanwise chamber on aerodynamic characteristics is discussed in following chapter. In the last part of this paper an author focus to possible extension of presented method.

Klíčová slova

Aerodynamika, Teorie vztlakové čáry, Vztlak, Indukovaný odpor.

Key words

Aerodynamics, Lifting line theory, Lift, Induced drag.

1. Introduction

Lifting line theory was developed by Ludwig Prandtl at the beginning of last century. This theory can be used for prediction of lift distribution over finite wing. Prandtl's theory is unique with its simplicity and it is still in use today for preliminary calculations of finite wing aerodynamics characteristics.

1.1 Concept of lifting line theory

In the lifting line theory the wing is replaced by infinite number of horseshoe vortices, as shown in fig. 1. Bound vortices passes aerodynamic centers of airfoils, this creates the lifting line. Trailing vortices starts at lifting line and continue downstream to infinity. These vortices induce velocities at the lifting line and change the local angles of attack. The 2D biot-savart law is used to calculate this velocities.

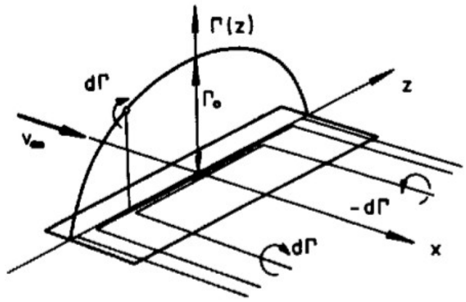

Fig. 1. Horseshoe votices [1]

Circulation distribution over lifting line is calculated with Kutta-Joukowski theorem. This approach needs value of local lift coefficients. Local lift coefficient is expressed as relation of lift curve slope of an airfoil and local angle of attack, which is function of circulation distribution, through the biot-savart law. This leads to integro-differential equation of wing. Glauert and Multhopp solved this equation analytically [1], theirs solutions are widely used for preliminary design of finite wings.

1.2 Usage of lifting line theory

Theory gives good results for thin (AR > 5) straight wings with negligible dihedral. Classical lifting line theory uses only linear relation between lift and angle of attack. Main advantages and disadvantages are summarized in following table.

Table 1. *– Summary of classical lifting line theory*

advantages	disadvantages
• Simplicity • Fast computation	• No dihedral and swept • Linear relation between airfoil lift and angle of attack

Many authors [2], [3], [4] have tried to eliminate the disadvantages of lifting line theory. They used nonlinear lift curves and they analyzed thin swept wings. Each of those methods are based on the assumptions described in previous paragraph, but solution is obtained by numerical solution of nonlinear equations. The numerical lifting line theory allows solution spatial chambered wing is derived in next section.

2. Numerical lifting line theory for spatial chambered wing

Wing is discretized in to finite number of horseshoe votricies. Each of this vortices has different strength (circulation) and it consists of bound filament and two semi-infinite trailing filaments. Bound vorticies represents the lifting line of the wing (connection of aerodynamic centers of the wing).

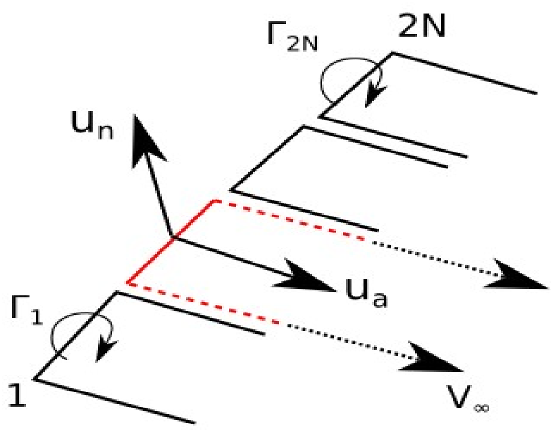

Fig. 2. Discretized wing

2.1 Derivation of numerical lifting line theory

Numerical lifting line is derived with following assumption. Magnitude of force acting on one bound segment can be calculated by eq. 1.

$$\vec{d}\,F_i = \Gamma_i \rho \vec{V}_i \times d\vec{l}_i \tag{1}$$

This is general three dimensional lifting law [3]. Local velocity vector V can be expressed as sum of velocity vector in infinity and induced velocity v_{ind} which is sum of velocities induced by other vortices. Induced velocity by one horseshoe vortex element can be calculated by eq. 2. [4]. This induced velocity is consists of contribution from bounded part of vortex and from two trailing vortices. Induced velocity for one segment is function of circulation of all vorticies of discretized wing.

$$\vec{V}_P = \frac{\Gamma_i}{4\pi}\left[\frac{\vec{u}_\infty \times \vec{r}_2}{r_2\left(r_2 - \vec{u}_\infty \cdot \vec{r}_2\right)} + \frac{\left(r_1 + r_2\right)\left(\vec{r}_1 \times \vec{r}_2\right)}{r_1 r_2\left(r_1 r_2 + \vec{r}_1 \cdot \vec{r}_2\right)} - \frac{\vec{u}_\infty \times \vec{r}_1}{r_1\left(r_1 - \vec{u}_\infty \cdot \vec{r}_1\right)}\right] \tag{2}$$

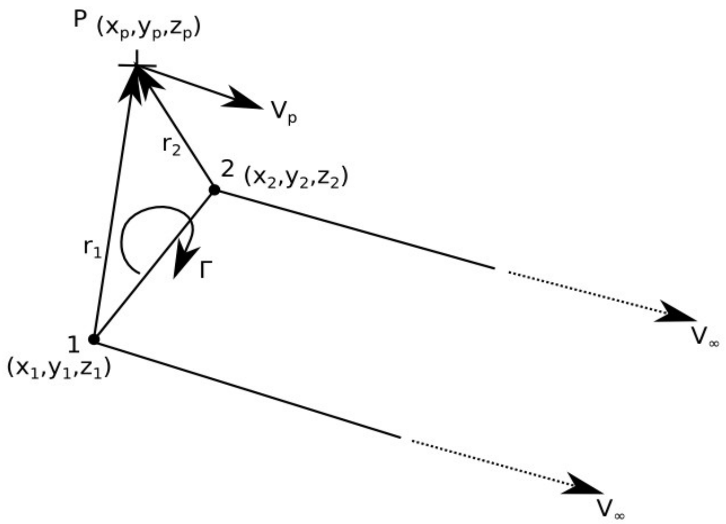

Fig. 3. Induced velocity by horseshoe element

System created by eq. 1. can be closed by well known equation for lift force acting on segment (eq. 3)

$$dF_i = \frac{1}{2}\rho v_\infty^2 C_{Li}\, dA_i \tag{3}$$

Lift coefficient in eq. 3. is function of local angle of attack. Local angle of attack depends on induced velocity and thus on circulation of each vortices. Absolute value of differential force computed by eq. 1. must be equal to differential force computed by eq. 3. This assumption lead to system of 2N nonlinear equation for 2N unknown circulation associated with 2N vortices.

$$|\Gamma_i \rho \left(\vec{V}_\infty + \sum_{j=1}^{2N} \vec{v}_{ind\,j} \right) \times \vec{d}\, l_i| = \frac{1}{2} \rho v_\infty^2 C_{Li} dA_i \tag{4}$$

Local angle of attack for computation of local lift coefficient at section can be expressed by following relation.

$$\alpha_i = \arctan\left(\frac{\vec{V}_i \cdot \vec{u}_{ni}}{\vec{V}_i \cdot \vec{u}_{ai}} \right) \tag{5}$$

Software in GNU Octave was created for solving this system of nonlinear equation. GNU Octave is high-level interpreted language, intended for numerical computation.

2.2 Testing of numerical lifting line theory

Present method was tested against well known analytical solution of lifting line theory. Solution for planar elliptical wing can be found in [2] this solution is covered by eq. 6 and 7. When lift and induced drag are known it is possible to calculate the efficiency factor (eq. 8.) For planar elliptical wing platform the efficiency factor is equal to 1.

$$\frac{dC_L}{d\alpha} = \frac{a_0}{1 + a_0 / \pi\lambda} \tag{6}$$

$$C_{Di} = \frac{C_{L^2}}{\pi\lambda} \tag{7}$$

$$e = \frac{C_L^2}{C_{Di} \pi\lambda} \tag{8}$$

Lift and drag coefficients for results from numerical method is calculated with following expressions. Integration variable dz denotes integration along lifting line.

$$C_L = \frac{2}{v_\infty A} \int_{-b/2}^{b/2} \Gamma_{(z)} dz \tag{9}$$

$$C_{Di} = \frac{2}{v_\infty A} \int_{-b/2}^{b/2} \Gamma_{(z)} \alpha_{ind(z)} dz \tag{10}$$

As we can see in the fig. 4. accuracy of numerical method depends on the number of used vortices. The more vortices is used the more accurate results are. Error is less than 1% with 35 vortices per semi-span. This number of vortices was used in following computations.

Lift curve and drag polar can be seen in the fig. 5. We can observe great agreement between methods.

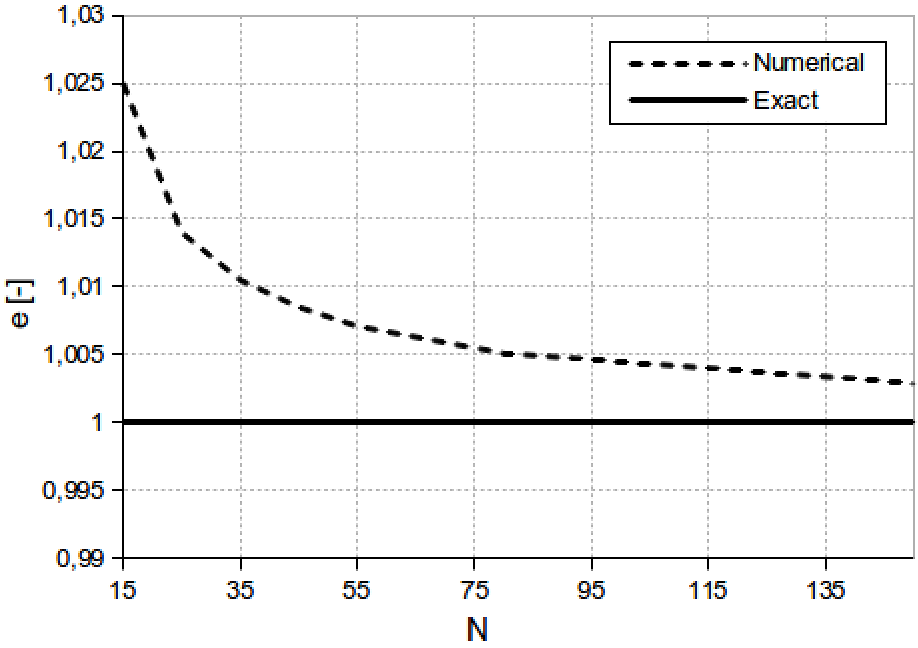

Fig. 4. Influence of number of used horseshoes vortices

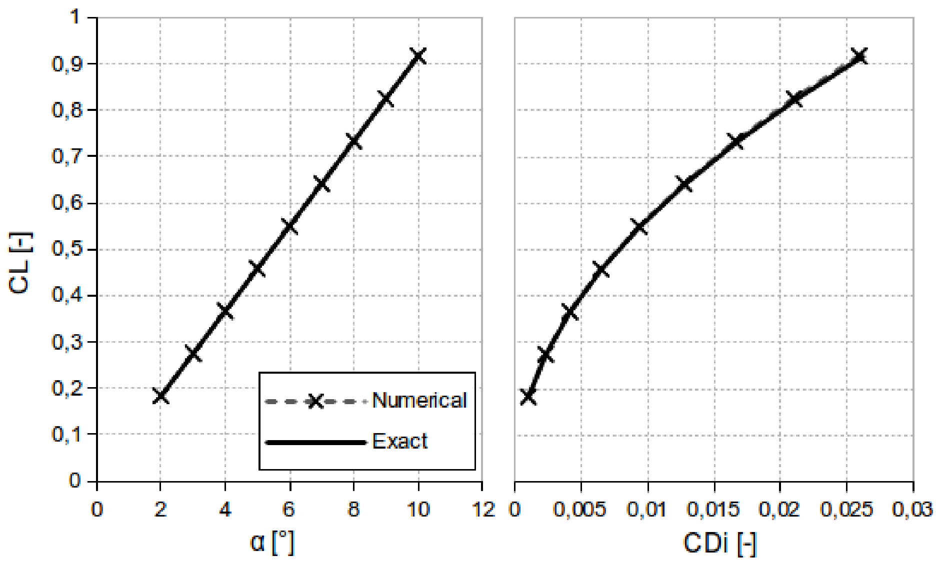

Fig. 5. Lift curve and drag polar

3. Aerodynamic characteristics of wing having chambered span

Fig. 6. Typical paraglider having spanwise chamber

3.1 Motivation

Figure above shows us typical wing having spanwise chamber. Theory derived above can be used for fast, preliminary analysis of chambered wing for paragliders or some unconventional wings.

3.2 Influence of spanwise chamber

Wing with curved lifting line is analyzed in this paper. Lifting line of wings has shape of circular arc and part of ellipse. All of tested wing has same unfolded platform. Geometrical characteristics are summarized in tab. 2. Four wings were analyzed. The largest value of d correspond to wing A, the smallest value of this parameter correspond to wing D. Wing has no twist and slope of lift curve is assumed equal to 2π.

Table 2. *– wing geometry*

parameter	value	unit
λ	10.18	[-]
b	8	[m]
d	30,10,5,3	[m]

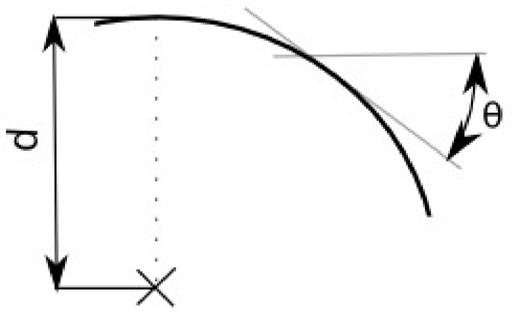

Fig. 7. Geometry of circular arc wings

$$C_L = \frac{2}{v_\infty A} \int_{-b/2}^{b/2} \Gamma_{(z)} \cos(\theta)\, dz \tag{11}$$

Influence of circular chambered lifting line is shown in fig. 8. There you can observe large and adverse effect of spanwise chamber on efficiency factor. Induced drag coefficient is calculated according equation 10. Lift coefficient is calculated with respect of local dihedral according to eg. 11. where definition of angle θ is shown in fig . 7.

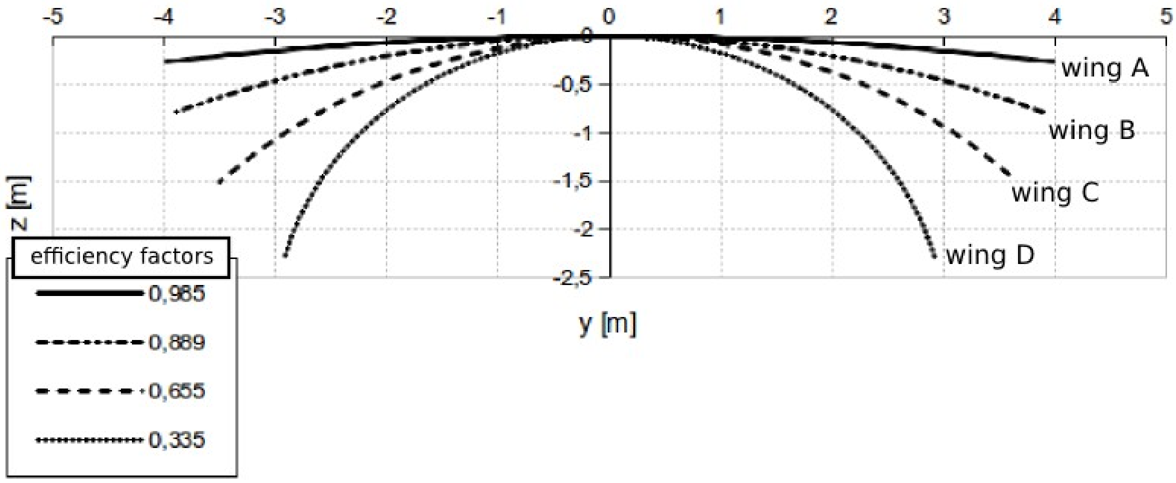

Fig. 8. Circular arc lifting line and their efficiency factors

Figure 9 shows us lift curves and drag polar of four analyzed wings. The adverse effect of influence can be seen on both graphs.

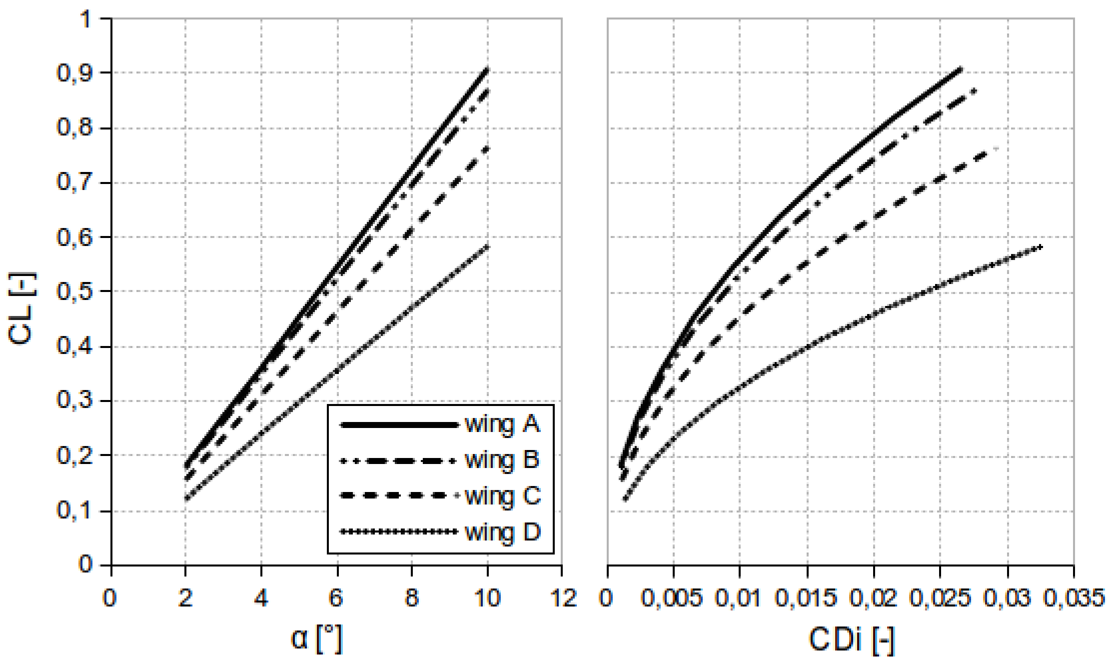

Fig. 9. Lift curve and drag polar of chambered wings

4. Future work in field of chambered wing aerodynamic modeling

First step will be verification of this concept which was used for analysis of chambered wings. It is planned to use CFD methods to verify these results. Next step will be study of effect of nonlinear lift characteristics.

5. Conclusion

Method based on Prantl's lifting line principle was introduced and used for analysis of wings having spanwise chamber. Result was discussed and it is clearly seen that spanwise chamber has large adverse effect on aerodynamic characteristics. For performance improving of wings having spanwise chamber is necessary to study this effect.

List of used symbols

Γ	circulation	($m^2 \cdot s^{-1}$)
CL	lift coefficient	(1)
CDi	induced drag coefficient	(1)
ρ	density	($kg \cdot m^{-3}$)
A	area	(m^2)
v,V	velocity	(1)
b	span	(m)
λ	aspect ratio (b^2/A)	(1)
dl	vector from point 1 to 2 of bound part of horseshoe vortex	(1)
d	radius of lifting line	(m)
N	number of vortex per semispan	(1)
u	unit vector	(1)
e	efficiency factor	(1)
α	angle of attack	(deg)
θ	local dihedral angle	(deg)

Subscripts

∞	denotes conditions in infinity
n	normal
a	aligned with local chord
ind	denotes induced variable

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