Optimization of logistic routes using mathematical models

Ing. Josef Košťálek

Supervisor: Kavan Michal, doc. Ing. CSc.

Abstrakt
Logistika je oborem nacházející silné uplatnění v průmyslové výrobě i návazných oblastech. Logistika jako obor využívá poznatky celé řady jiných disciplín. Předmětem mého příspěvku je ukázat využití některých speciálních matematických a výpočetních aplikací v oblasti stanovení nejvhodnější logistické trasy. Představuji zde mnou vytvořený model ve snadno dostupném prostředí MS Excel, který je schopen naplánovat nejkratší možnou spojnici pro 14 libovolných bodů s návratem do bodu výchozího. Model, kterému jsem dal označení Edita 3, řeší úlohu známou pod termínem ,,Úloha obchodního cestujícího" ovšem zobecněnou pro libovolný počet míst 1 až 14.

Key words: Logistics, optimization of routes, mathematical models, linear programming

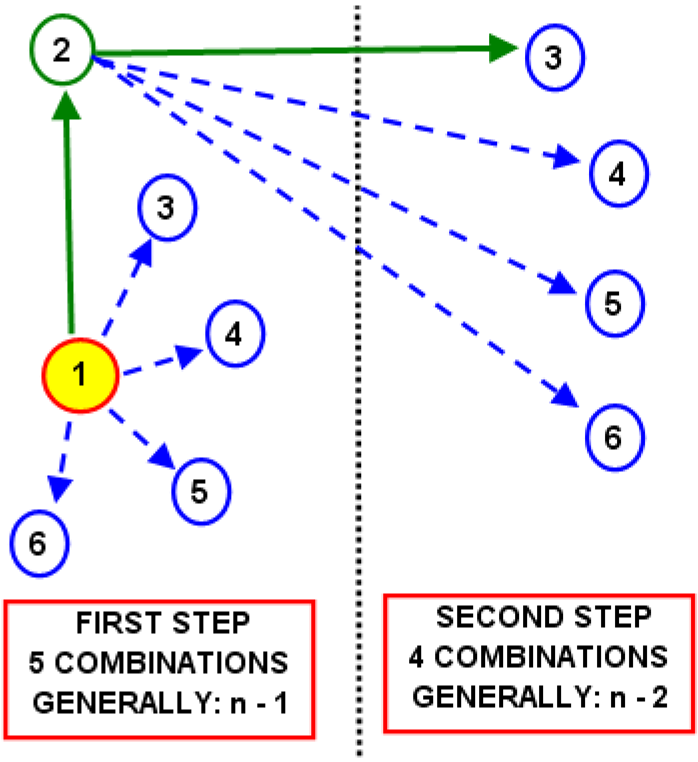
1. Introduction

Today industrial production uses a lot of cooperative structures. One car contains about 20 000 different components from many suppliers from different cities and countries. This situation puts high demands on areas related to ordering, shipping and storage. And so started the discipline logistics. Any production can work when logistics doesn´t work properly. Transport and distribution of components and finished products is becoming a very important part of the management and organization of production.
There´s a known problem in route planning called ,,travelling salesman problem". My paper describes a mathematical model solving this problem for 14 points. What´s travelling salesman problem? There is a set of points and my objective is planning the route among these points. The route must intersect all points and a finish of the route is at the same point like a start of the route. Well, for example a salesman goes from a town number one, he comes to all other towns but to every town comes just only once and there is the finish of his route back at the town number one. Certainly the route must be the shortest. There is the problem to find an optimum route among all points. This problem often arises in a number of logistical situations (for example transport components to factories, transportation of goods to the shops, check branches, distribution of mail, but also supply workplaces in assembly hall or manufacturing plant etc.).

2. To solution of „travelling salesman problem"

2.1. To describe situation with mathematical form

Solution of ,,travelling salesman problem" is more complicated. There is a lot of combinations of routes.



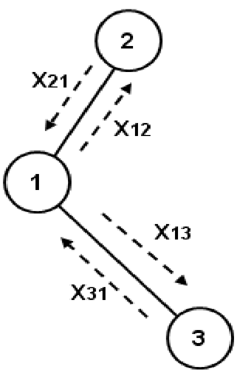
Picture 1: The number of combinations

c … the number of combinations
n … the number of points (towns, stations, workplaces)

$$c = (n-1) \bullet (n-2) \bullet (n-3) \ldots = (n-1)!$$

Formula 1: The number of combinations

When the number of points grows, the number of combinations increases very rapidly. How to find the shortest (optimum) route? The problem will have to be transformed to mathematical form and then it will be mathematical solved. The result is made up values of variables. Each variable symbolizes way between two points. When is value zero, there is not way and when is value one there is way.



Picture 2: Transformation using mathematical variables

The solution contains four mathematical formulas. I´m describing the principle using for 14 points.

$$F = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} \cdot x_{ij} = \min.$$

$F = 10•x_{12} + 16•x_{13} + 77•x_{14} + \dots + 20•x_{1\_10} + 35•x_{1\_11} + 88•x_{1\_12} + 89•x_{1\_13} + 55•x_{1\_14}$

$+ 10•x_{21} + 50•x_{23} + 60•x_{24} + \dots + 180•x_{2\_10} + 89•x_{2\_11} + 100•x_{2\_12} + 89•x_{2\_13} + 10•x_{2\_14}$

…..

$\dots •x_{13\_1} + \dots$

$\dots •x_{14\_1} + \dots\dots = \min.$

Formula 2: Total length of the route

$$\sum_{i=1}^{n} x_{ij} = 1, \quad i = 1,2,\dots n$$

$x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} + x_{1\_10} + x_{1\_11} + x_{1\_12} + x_{1\_13} + x_{1\_14} = 1$

$x_{21} + x_{23} + x_{24} + x_{25} + x_{26} + x_{27} + x_{28} + x_{29} + x_{2\_10} + x_{2\_11} + x_{2\_12} + x_{2\_13} + x_{2\_14} = 1$

. . . . .

$x_{14\_1} + x_{14\_2} + x_{14\_3} + x_{14\_4} + \dots + x_{14\_8} + x_{14\_9} + x_{14\_10} + x_{14\_11} + x_{14\_12} + x_{14\_13} = 1$

Formula 3: Condition – Every point may be leaved just once

$$\sum_{j=1}^{n} x_{ij} = 1, \quad j = 1,2,\dots n$$

$x_{21} + x_{31} + x_{41} + x_{51} + x_{61} + x_{71} + x_{81} + x_{91} + x_{10\_1} + x_{11\_1} + x_{12\_1} + x_{13\_1} + x_{14\_1} = 1$

. . . . .

$x_{1\_14} + x_{2\_14} + x_{3\_14} + x_{4\_14} + x_{5\_14} + \dots + x_{8\_14} + x_{9\_14} + x_{10\_14} + x_{11\_14} + x_{12\_14} + x_{13\_14} = 1$

Formula 4: Condition – Every point may be visited just once

$$u_i - u_j + n \cdot x_{ij} \le n - 1,$$

$$i = 1,2,\dots n$$

$$j = 1,2,\dots n$$

$u_2 – u_3 + 14x_{23} \le 13$

$u_3 – u_2 + 10x_{32} \le 13$

. . . . .

$u_{13} – u_{14} + 14x_{13\_14} \le 13$

$u_{14} – u_{13} + 14x_{14\_13} \le 13$

Formula 5: Condition – Route forms continuous circuit

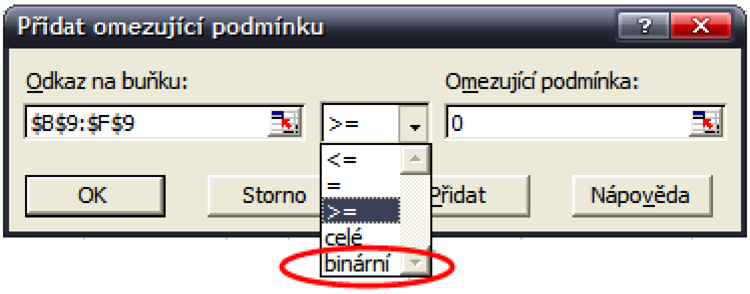
$$x_{ij} = 0\,;1$$

Formula 6: Condition – variable is only one or zero

I´m looking for values of variables to total length of the route will be minimum (Formula 2) and simultaneously conditions 3, 4, 5 and 6 will have to valid. For 14 points there are 195 variables, 184 conditions and 6 227 020 800 combinations. This problem is with another and another point more complicated. I have been described to transformation to mathematical notation, but I need to get a result and the optimal route.

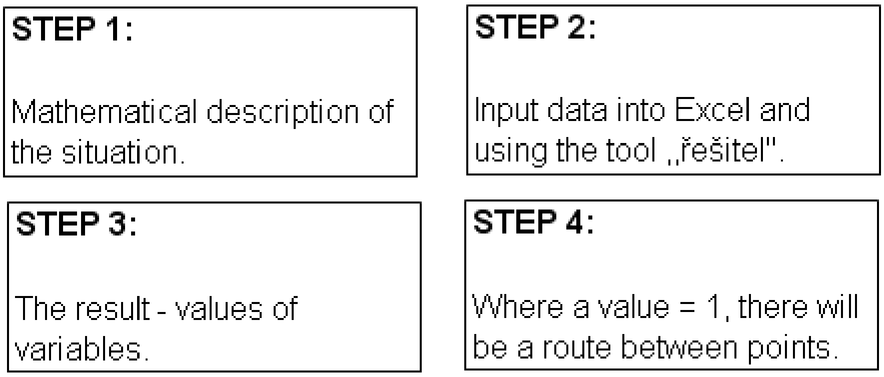
2.2. To calculation mathematical notation

I´m calculating values of variables and I´m using a special mathematical tool called „Linear Programming" and because I need only values one or zero I will use modifications of Linear Programming and it is „Binary Linear Programming".



Picture 3: Using Binary Linear Programming in Excel

This tool is in program Excel its name is „řešitel". Mathematical form will be written to form for Excel and Excel will give solution.



Picture 4: The procedure of finding optimal route

Excel can calculation Linear Programming only for maximum 200 unknown variables. It is very important, because I sad in my paper, that: for 14 points there are 195 variables. 14 points is the upper limit for solution in Excel.

The procedure of finding the optimal route is so difficult and lengthy. I´m describing in the next part of this paper my model for easy solution this problem (travelling salesman problem) with maximum 14 points. I named this model EDITA 3.

3. My model for finding optimal routes named EDITA 3 by me

The principle, which I described in the first part of my paper, was used in this model. All calculations were programmed. Operation of the program is very simple just input distances and the names of the points (for example towns) and then run the tool „řešitel". The model automatically finds the result, which is using functions of Excel transformed from values of „0 and 1" on the listing of points (towns) forming the optimal route. This model can work quite generally with a number of points 1 to 14. There is a difference to conventional formulas. User does not need a static model, but the model must be able to work with changing number of cities.

DISTANCES POINTS

		1	2	3	4	5	6	7	8	9	10	11	12
		Chrudim	Havl. Brod	Hradec Kr.	Ml. Boleslav	Pardubice	Písek	ychnov n. K	Trutnov	Tábor	Vlašim		
1	Chrudim		54	31	96	10	165	51	77	121	83		
2	Havl. Brod			85	112	63	123	96	130	77	62		
3	Hradec Kr.				81	21	186	38	47	142	104		
4	Ml. Boleslav					86	160	118	84	130	101		
5	Pardubice						170	46	67	126	88		
6	Písek							216	228	47	85		
7	Rychnov n. K.								59	171	133		
8	Trutnov									183	144		
9	Tábor										41		
10	Vlašim												
11													
12													
13													
14													

Picture 5: Input date to model EDITA 3
(part of picture)

OPTIMAL ROUTE

ROUTE IS PLANNING FOR: **10** POINTS

LENGHT OF OPTIMAL ROUTE **576** Km

POINTS - TOWNS FORMING THE ROUTE

Chrudim
Pardubice
Hradec Kr.
Rychnov n. K.
Trutnov
Ml. Boleslav
Vlašim

Picture 6: Output date from model EDITA 3
(part of picture)

4. Conclusion

Travelling salesman problem is a known task in logistics. There are a lot of books with known formulas but my model is original, because it is able to transform these static formulas in a dynamic working model in the range 1 to 14 points.
Typical applications for this model is finding the shortest route, if specified distances between the points. You can also specify the times of journeys between the points and the result would be not the shortest, but the fastest route.
Another task for future work is to look for ways to increase the maximum number of points.

Literature:

1. Fábry, J.: Matematické modelování. Praha: VŠE, 2010. 143 s.
   ISBN 978-80-245-1266-2
2. Jablonský, J.: Programy pro matematické modelování. Praha: VŠE, 2007. 259 s.
   ISBN 978-80-245-1178-8
3. Kožíšek, J., Stieberová B.: Statistická a rozhodovací analýza. Praha: ČVUT, 2008. 252 s.
   ISBN 978-80-01-04209-0
4. Schels, I.: Excel 2007 vzorce a funkce. Praha: Grada Publishing, 2008. 506 s.
   ISBN 978-80-247-1965-8
5. Štůsek, J.: Řízení provozu v logistických řetězcích. Praha: C. H. Beck, 2007. 227 s.
   ISBN 978-80-7179-534-6