The influence of structural damping on flutter characteristics of a small sport aircraft

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Abstract

Jsou představeny základní matematické modely tlumení a experimentální zjišťování velikosti strukturálního tlumení z frekvenční odezvy systému. Vybraná metodika byla aplikována na měření několika typů letounu. Je uveden příklad výpočtu flutteru ocasních ploch s uvážením vlivu strukturálního tlumení. Je diskutována aplikovatelnost použité metodiky pro potřeby aeroelastického výpočtu.

Author presents the basic mathematical models of damping and principles of experimental determination of structural damping from the system frequency response. The specific method was applied on measurements of few types of small sport aircrafts. The flutter calculation of tail surfaces with consideration of structural damping influence is shown. The applicability of the used method is discussed for needs of flutter calculations.

Keywords

structural damping, logarithmic decrement, flutter analysis, half-power point method

1. Introduction

One of the research topics of Aerospace Research Centre at Department of Aerospace Engineering – Faculty of Mechanical Engineering, CTU in Prague is focused on a flutter characteristics analysis of small sport aircrafts, with maximal take-off weight up to 600 kg. The calculation model, which is used for flutter test, is based on standard “p-k” model where the damping characteristics and frequency are dependent on speed [4]. Thus, the actual real value of damping for each mode of the airplane structure is very desirable physical quantity to know.

Damping as one of the structural property significantly influences the aeroelastic behaviour of airplane structure. This characteristic can be substituted by appropriate mathematical model, when practical calculations are needed, or it can be directly gained from ground frequency tests. The shown method is based on determination of structural damping from FRF (Frequency response function) of the forced structure. The values of damping are collected to obtain an adequate database of how the typical airplane structure behaves under dynamic loading.

2. Structural damping

Damping of mechanical structures can be approximately divided into these three categories

- material damping – related to the molecular structure of the material
- structural damping – usually caused by friction between parts of the structure
- external damping – caused by interaction between the structure and environment

We can see that relations between damping and its causes make it more problematic to measure. As a result the indirect methods are often used, to determine the value of system damping. Considering the influence of structural damping in computation of dynamical structural behaviour is usually accomplished by usage of the simpler model where is expressed by suitable mathematical expression. The damped motion is more frequently described by these mathematical models.

The simplest one is the viscous, where the damping forces are linearly dependent to velocity of the motion. The governing equation of motion is

$$m\ddot{x} + b\dot{x} + kx = F \tag{1}$$

or

$$\ddot{x} + 2\zeta\Omega\dot{x} + \Omega^2 x = \frac{F}{m} \tag{2}$$

where Ω means the undamped natural frequency and ζ is the damping ratio defined by

$$\Omega^2 = \frac{k}{m}; \quad 2\zeta\Omega = \frac{b}{m} \tag{4}$$

Hysteresis mathematical model is used for cases, where damping force is proportional to elastic force. The phase shift between forces is π/2. The motion of system is described with equation

$$m\ddot{x} + (1 + j\eta)kx = F \tag{5}$$

Using the expression for Ω leads to

$$\ddot{x} + (1 + j\eta)\Omega^2 x = \frac{F}{m} \tag{6}$$

The term in brackets $(1 + j\eta)$ represents the complex stiffness.

If the value of damping is proportional to stiffness and mass of the structure, the reasonable mathematical model to use is a proportional model, characterized by governing equation

$$m + (a_1 m + a_2 k)\dot{x} + kx = F$$

$$\ddot{x} + (a_1 + a_2\Omega^2)\dot{x} + \Omega^2 x = F \tag{7}$$

The terms a1 and a2 represents the coefficients of proportional damping.

Mathematical models are described in more details in [1].

3. Experimental evaluation of structural damping

The classical approach, how to evaluate the structural damping of the system, is to determine the decay of amplitudes, from time flow of the oscillating motion. Assuming the linear behaviour of the system applies to ratio of two successive amplitudes

$$x_{(t+T)} / x_t = e^{-\zeta\,\Omega T} \tag{8}$$

Structural damping can be simply expressed in form of logarithmic decrement ϑ – shown in the Fig.1.

$$\ln\left(\frac{x_2}{x_1}\right) = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} = \vartheta' = const \tag{10}$$

In most cases the value of logarithmic decrement is assumed to be $\vartheta << 1$, so than we can write

$$\vartheta \doteq 2\pi\zeta \tag{11}$$

The practical evaluation of damping is very tough and therefore this method can be recommended only as a control tool.

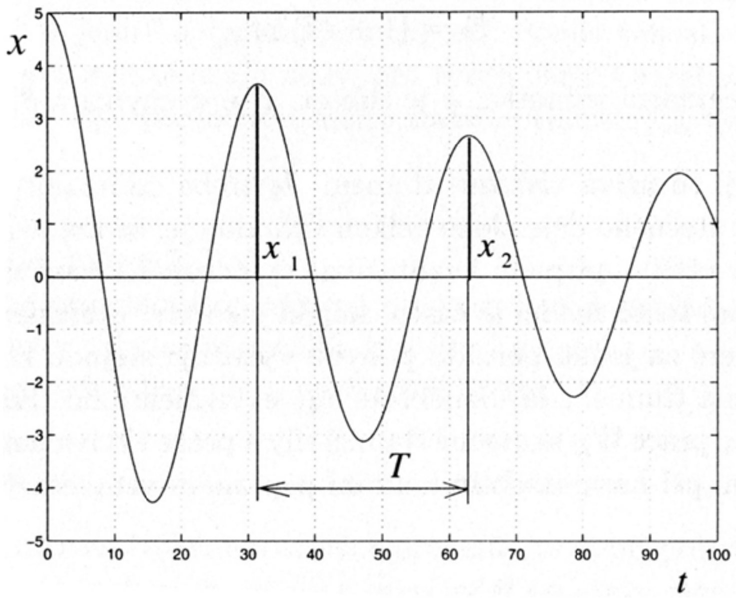

Fig. 1. *Amplitude decay of free harmonic damped motion*

The half-power point method is reasonable, when FRF (Frequency response function) of structure is measured. Method is based on relationship between the damping ratio and position of half-power points. The half-power points are those in a FRF diagram in which the amplitude decays to $\frac{1}{\sqrt{2}}$ of a peak value (Fig. 2). This results to be the half peak value in power spectra diagram, so that why the half-power points. Assuming a linear behaviour and small damping the relation is

$$\omega_2 - \omega_1 = 2\Omega\zeta$$

$$\frac{\omega_2 - \omega_1}{2\Omega} = \zeta \tag{12}$$

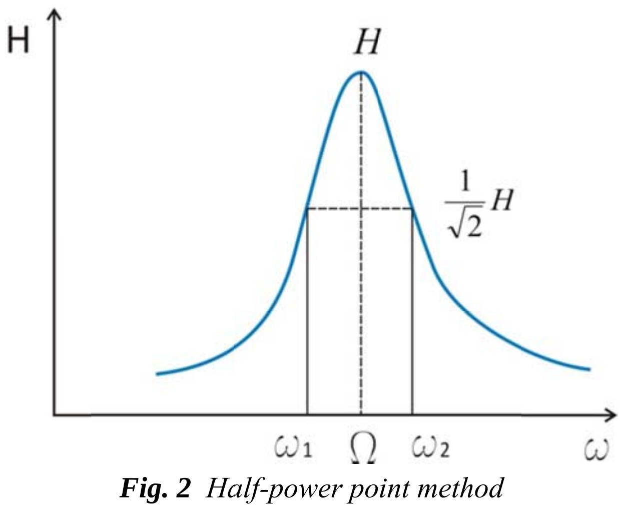

Fig. 2 *Half-power point method*

The main advantage of this method is the possibility to determine the value of structural damping directly from measured frequency response function. The results are valid only for modes with linear behaviour and sufficient frequency shift differing them from other modes. The method is discussed in more details in [3].

4. Data acquisition

Data were gathered during the ground frequency tests of several small aircrafts. The measurement system consisted of an 11 channel analyser TL-5412_CDD, 2 electrodynamics exciters, 2 channel signal generator and power amplifiers. The piezoelectric accelerometers of IEPE standards were used. The measured data were analysed in ME scope software which analyses frequency characteristics by means of Fast Fourier Transformation of signal data from a time domain in to a frequency plane.

The airplane with fixed control surfaces was softly hung on adjustable frame, when natural frequencies of body were measured. Vice versa the modal parameters of the control surfaces were measured, when body structure was fixed.

The structure was excited by two shakers. These were powered by a sweep sine signal with corresponding or opposite phase, to obtain symmetric or antisymmetric excitation. The frequency range was set from 2 to 100 Hz. Natural frequencies were specified by means of amplitude values in real and imaginary components.

5. Results

The values of structural damping were measured on six types of sport aircrafts. These were all-metal low-wing monoplanes, composite low-wing monoplanes and strut braced high-wing monoplanes. Frequency test were performed with a complete equipped airplanes at the light-mass test configuration, so it means that only one light pilot and empty fuel tanks were simulated. The half-power point method was used to determine the logarithmic decrement of typical modes of both structure and control surfaces. Modes were chosen according to conditions of used method, to obtain significant results. These are shown in Table 1. For each mode the frequency and negative logarithmic decrement are given.

	mode type	type of airplane											
		SD - 4 Viper		PiperSport		VL-3 Sprint		Samba		TL-3000 Sirius		Skylane	
		f [Hz]	d [-]	f [Hz]	d [-]	f [Hz]	d [-]	f [Hz]	d [-]	f [Hz]	d [-]	f [Hz]	d [-]
wing	1. sym. bending	11,2	0,072	11,3	0,055	8	0,040	5,8	0,075	10,8	0,115	12,1	0,104
	1. sym. torsion	49,4	0,092	40,8	0,114	31,1	0,195	31,6	0,069	21,6	0,109	34,1	0,184
	1. antisym. bending	13,45	0,113	23,7	0,172	19,7	0,103	15,6	0,212	17,3	0,089	19,4	0,210
	1. antisym. torsion	48,3	0,084	39,6	0,043	30,5	0,206	31,9	0,057	30,4	0,151	34,8	0,253
HT	1. sym. bending	18,8	0,167	22,9	0,217	17,7	0,125	16,1	0,176	13,3	0,083	33,6	0,178
	1. antisym. bending	15,9	0,100	13,1	0,144	13,5	0,183	14,2	0,121	12,2	0,129	11,8	0,295
Contr. surf.	FRE*	30,6	0,386	20,1	0,328	25,2	0,536	6,06	0,476	19,2	0,278	15,6	0,550
	FRA**	20,4	0,482	14,4	0,486	25,6	0,508	7,25	0,597	11,4	0,579	14,9	0,591

*FRE – fundamental rotation of elevator

**FRA – fundamental rotation of ailerons

Tab. 1. *Frequencies and values of negative logarithmic decrement*

The methodology FAR 23.629 allows to use the damping value of $g = +0{,}03$ as an inherent structural damping. This is adequate to value of negative logarithmic decrement $d = 0{,}1$. However, this value should be used with caution if the damping of the mode decreases very rapidly with an increase in airspeed.

On the Figure 3 the relationship between frequency and value of logarithmic decrement is shown. As we can see, there are four areas corresponding to modes of the structure or control surfaces. As a benchmark the value of damping according to FAR 23.629 is given. Modes of the structure more or less correspond to the value given by FAR23.629, with average value of d=0,122 for wing modes and d=0,160 for horizontal tail modes. The modes of control surfaces show considerably higher values of damping d=0,483 in average, which is nearly five times higher than FAR 23.629 value.

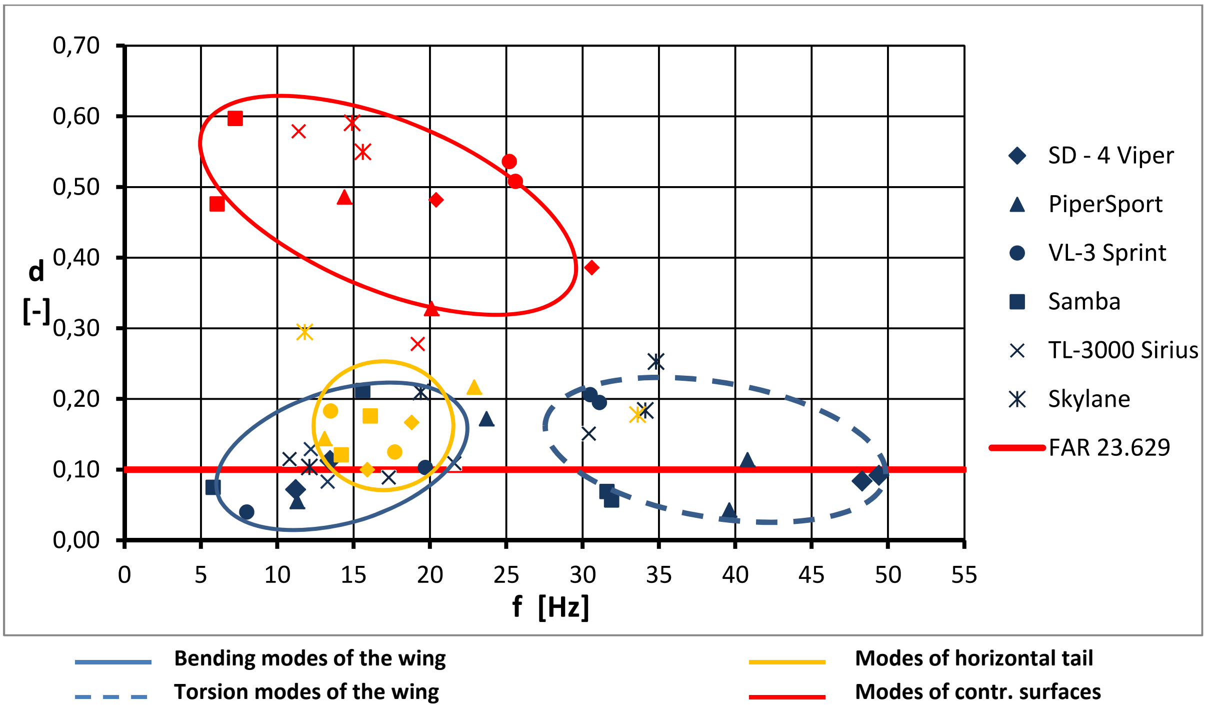

Fig. 3. *Map of logarithmic decrement vs. frequency for modes of structure and control surfaces*

6. Example of calculation

The example calculation was performed on all-metal down-wing airplane of foreign manufacturer with design speed of V_D = 275km/h see Fig. 4 – 5. The modes of tail structure, rudder and elevator were calculated. Force effects of a pilot were simulated by 1kg of mass, placed on a control stick. Flutter equations were based on Lagrange's energy equations of a structure elastic system with control surfaces. The "p-k" computational model was used to find eigenvalues of complex flutter matrix. The structural damping values were added into generalized stiffness matrix [4]. The results are presented in Fig. 6 - 8 which show the dependence of damping (in form of negative logarithmic decrement) and frequency versus equivalent air speed. The degrees of freedom stand with five eigenmodes of primary structure with fixed elevator and rudder and three eigenmodes of elevator with fixed stabilizer and fin.
In Fig. 6 the structural damping was not used in calculation. The second mode of elevator is supposed to be critical. The value of negative logarithmic decrement falls to zero at speed of V_{FL}=170,6km/h, which is the critical flutter speed and gives the speed ratio V_{FL}/V_D=0,620. Considering the inherent structural damping of d=0,1 , given by FAR 23.629, the flutter speed goes up to approximately V_{FL}=273,6km/h and the speed ratio raises to V_{FL}/V_D=0,995. According to FAR 23.629 the airplane structure should be flutter free up to 1,2V_D, Therefore it is supposed that some undamped dynamical effects can be observed.

In the second case (see Fig. 7), respecting FAR 23.629, the value d=0,1 was used for all eigenmodes of control surfaces and primary structure. The speed V_{FL}=267,1km/h and speed ratio V_{FL}/V_D=0,971 lead to the same conclusion as in the previous case.
In the last figure (Fig. 8) the real measured structural damping of critical mode d=0,512 was added to the computation. The critical second mode of elevator still shows a decline of damping at speeds over 170km/h but the flutter speed goes above 1,2V_D. The result is, that the tail structure was said to be flutter free, respecting the real structural damping. In addition to improve the inherent flutter stability, the manufacturer was advised to rebalance the elevator.

Fig. 4. *Ground vibration test*

Fig. 5. *Forcing of tail structure*

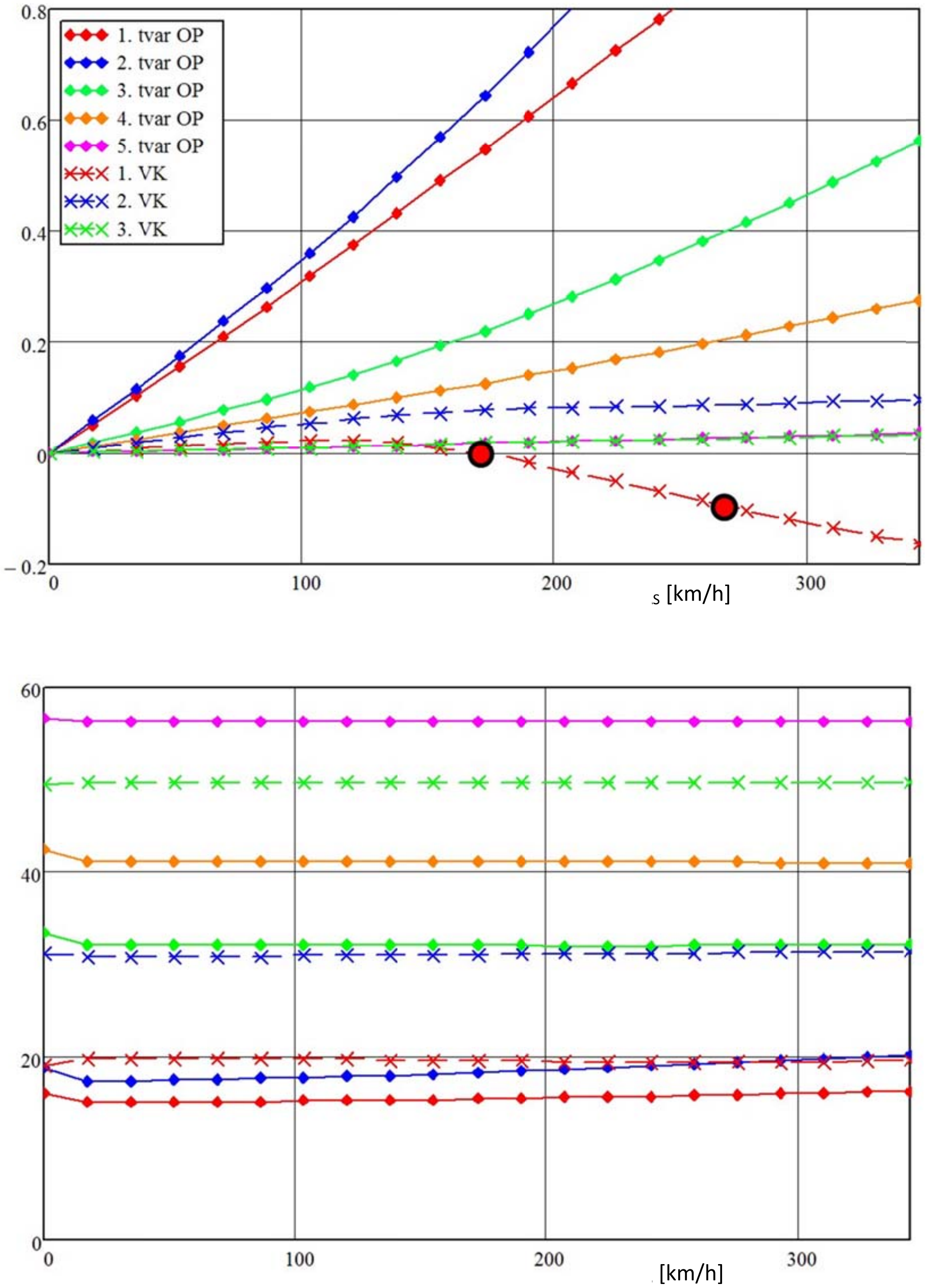

Fig. 6. *V-d and v-f diagrams (Without structural damping)*

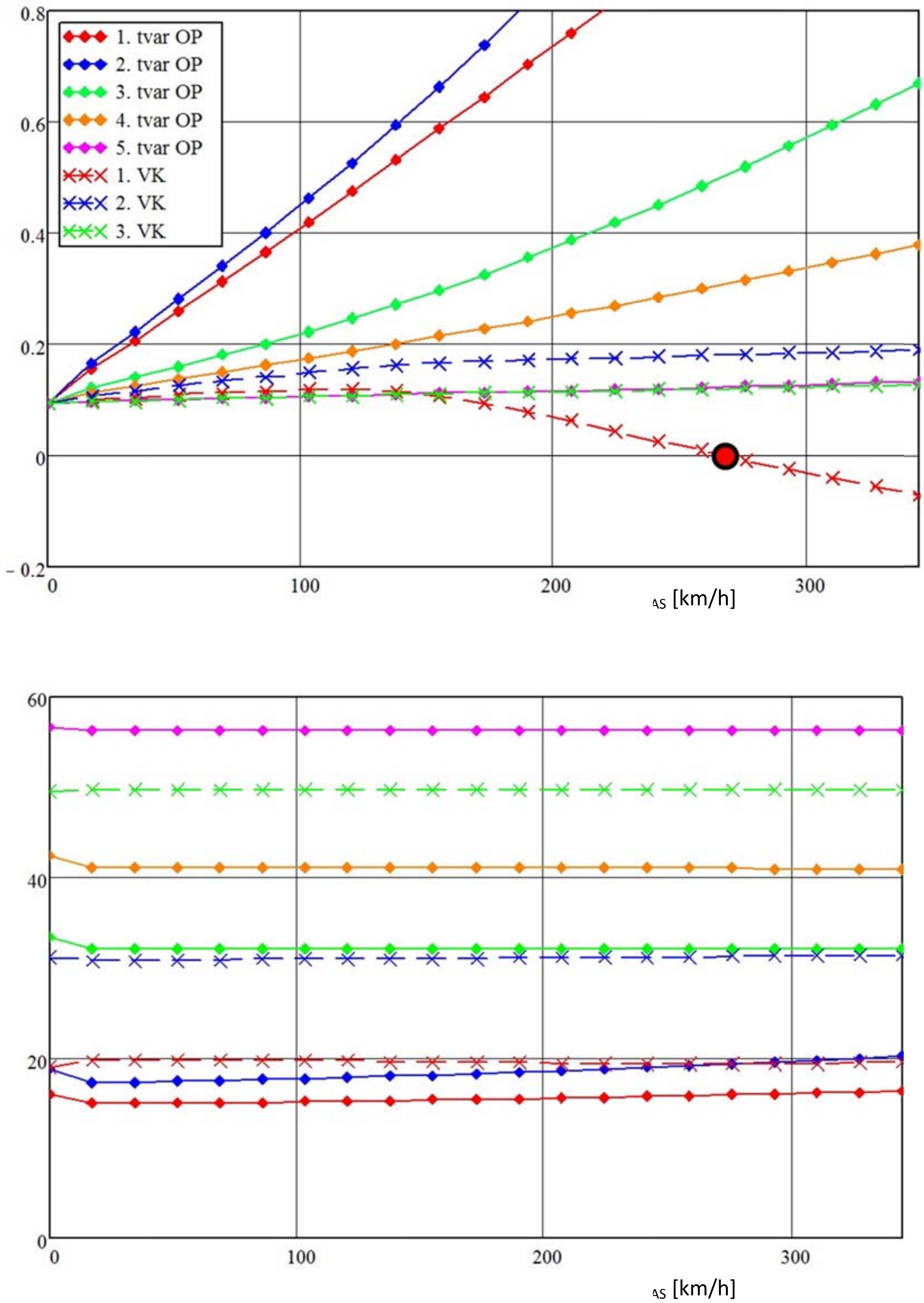

***Fig. 7.** V-d and v-f diagrams (Structural damping according to FAR 23.629)*

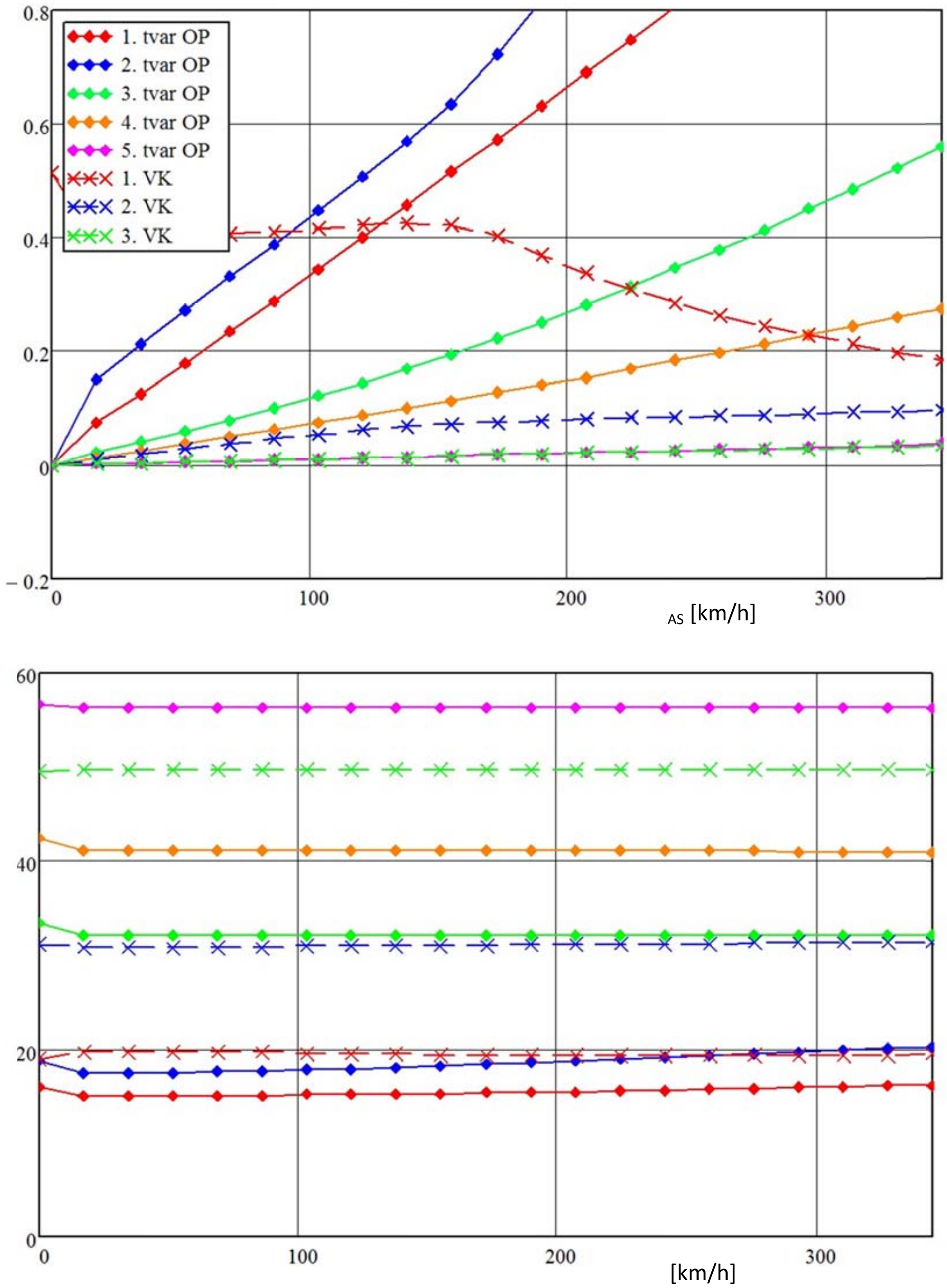

Fig. 8. *V-d and v-f diagrams (Measured structural damping)*

7. Conclusion

The values of structural damping were evaluated for typical eigenmodes of six aircraft structures. The half-power point method was used. The results show that modes of control surfaces are always remarkably higher damped than modes of primary structure. The possible causes of this phenomenon could be the effect of control system path especially its stiffness and friction between rotating and moving elements. The question is how the structure behaves, when effects of real pilot are considered. This should be the topic of subsequent research. Although the half-power point method seems to be very useful for determination of structural damping, it is valid only for linear eigenmodes with sufficient frequency shift from other modes, otherwise it gives incorrect results. This is a main disadvantage, since the airplane structure modes can be generally complex with characteristics of nonlinearity.

The effects of structural damping were shown on example flutter calculation of tail structure. It was proved, that knowledge of real damping values is very reasonable, especially for control surfaces modes, which are supposed to be the most critical.

List of symbols

m	mass	[kg]
b	viscous damping	[N/ms^{-1}]
k	stiffness	[N/m]
Ω	natural frequency	[rad/s]
T	period	[s]
ζ	damping ratio	[-]
ϑ	logarithmic decrement	[-]
ω	frequency	[rad/s]

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