Aeroelasticity analysis of wing UL-39

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Abstract in Czech
Tato práce se zabývá výpočtem modálních a flatrových charakteristik, vyšetření účinnosti řízení a stanovení mezní rychlosti torzní divergence pravé poloviny křídla letounu UL-39. Řešení je prováděno pomocí MKP softwaru MSC.Nastran.

Abstract in English
This paper deals with computation of modal and flutter characteristic, investigating ailerons effectiveness and determine torsion divergence critical velocity at right half-wing of the aircraft UL-39. The problems are solved in FEM software MSC.Nastran

Key words
Aeroelasticity analysis, normal modes, flutter, ailerons reversal, wing torsion divergence, MSC.Nastran

1. Introduction
This paper is focused on providing the first view on aeroelasticity behavior of wing aircraft UL-39. And exploring the possibility of solution static aeroelasticity problems by using FEM software. Used software for solution is MSC.Nastran 2005.

1.1 UL-39
UL-39 is ultra-light all-composite plane for two person, with retractable landing gear. The propeller is compose of input channel and low pressure blower. The blower is drive via motorcycle engine. The wing is trapezium shape with primary and secondary beam. On the end of wing is placed external wing-tip fuel tank. The tail surfaces are classical configuration with floating elevator.

Pic. 1. UL-39
## Tab 1. – Basic characteristic

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stall speed $V_{\text{min}}$</td>
<td>65 [km/h]</td>
</tr>
<tr>
<td>Design speed $V_d$</td>
<td>340 [km/h]</td>
</tr>
<tr>
<td>Wing span $l_{KR}$</td>
<td>7,2 [m]</td>
</tr>
<tr>
<td>Aspect ratio $\lambda$</td>
<td>5,6</td>
</tr>
<tr>
<td>Wing surface $S$</td>
<td>8,504 [m²]</td>
</tr>
<tr>
<td>Fuel mass $m_p$</td>
<td>38,8 [kg]</td>
</tr>
<tr>
<td>Ceiling $H$</td>
<td>3000 [m]</td>
</tr>
<tr>
<td>High $l$</td>
<td>3,025 [m]</td>
</tr>
<tr>
<td>Length $l_{TR}$</td>
<td>7,33 [m]</td>
</tr>
<tr>
<td>Aerodynamic chodor $b_{SAT}$</td>
<td>1,275 [m]</td>
</tr>
</tbody>
</table>

### 2. FEM Model

FEM (Finite element model) is consist from two part. First one is called structural model, it is a geometrical model of wing, with finite element mesh and defined material characteristic. There are also defined boundary conditions and local mass. Second model is called aerodynamic model. It was created for purpose of calculating aerodynamic loads. This model is without any material characteristics. Instead of finite element mesh is aerodynamic model form by aero-boxes. Those two models are independent on each other, so for connection was used mathematical function called Spline which transferring loads and deformations.

#### 2.1 Structural Model

The structural was completely created in preprocessor Patran. Model is composed from 185 surfaces. Laminate modeller was used for defining material properties. Used materials are Divinicel foam, Carbon composite Biaxial Carbon 200, Roving TORAYA T700SX, Carbon fabric, epoxy, resin and Chrome-manganese steel. Total weight of structural model with fuel is 77,2 kg.

![Pic. 2. Structural model of wing.](image)

#### 2.2 Mesh Model

Model contains 8322 nodes and 2988 elements. The primary type of elements used on model is square type called QUAD 97,5%. The rest of elements are triangular and point type.
Influence of path control was simulating by add moment of inertia on the aileron like POINT element and redistributed to aileron by MPC element. Calculation of add moment of inertia was done according [1]. The fuel was simulating as a local mass and redistributed to surrounding nodes by MPC element. The same was also done for simulating of landing gear retractable mechanism and flaps. Table 2. summarize weight of local mass and add moment of inertia.

<table>
<thead>
<tr>
<th>Name</th>
<th>Mass</th>
<th>Moment of Inertia</th>
<th>Number of MPC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel</td>
<td>38.8 kg</td>
<td></td>
<td>13</td>
</tr>
<tr>
<td>Landing gear</td>
<td>11.8 kg</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Retractable mechanism</td>
<td>1.25 kg</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Flaps</td>
<td>5 kg</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>Add moment of inertia</td>
<td>0.001 kg</td>
<td>0.107 kg.m(^2)</td>
<td>1</td>
</tr>
</tbody>
</table>
2.2 Boundary conditions
Boundary conditions was done by restriction all six DOF in nodes corresponding to connection fuselage with wing and in axis of symmetry of primary beam.

![Pic. 5. Boundary conditions.](image)

2.3 Aerodynamic model
For the calculations of aerodynamic loads on wing was defined „Lifting surface“ which used double lattice method. DLM calculate the lift on behalf of aerodynamic linearized potential theory.

![Pic. 6. Lifting surface - wing](image)

For simulating of aerodynamic motion and loads on external wing tip was used YZ-Body. The Body is composed from two parts. First one is Slender body for simulating motion own body and aerodynamic forces on behalf Slender Body Theory. The theory gives the lift proportional to the rate of change of cross-section area. Second part of the body is Interference body which is used for simulation interaction body with other body and/or lifting surfaces. Part of lifting surface was defined as Control device for purpose of simulations the aileron.
3. Normal modes

The normal modes is structural analysis only so no aerodynamic model was needed. The normal modes was used for computation of natural frequencies and mode shape of structure. Which are one of the input to flutter analysis. If is structure vibrating on frequency same or very close to natural frequency it can lead to structural damage or failure. Operation structure on frequency close to natural frequency decreases fatigue life. For obtain the natural frequencies Nastran solution SOL103 was used. This solutions use reduced form of the equation of motion (1) where no damping and no applied loading are considered.

\[ [M]\ddot{u} + [K]u = 0 \quad (1) \]

Where:
- \([M]\) mass matrix
- \([K]\) stiffness matrix
- \(\{u\}\) assume a harmonic solution \(\{u\} = \{\phi\} \sin \omega t\)
- \(\{\phi\}\) the eigenvector or mode shape
- \(\omega\) is the circular natural frequency

Solutions of reduced form of the equation of motion is:

\[ ([K] - \omega_i^2[M])\{\phi_i\} = 0, \quad i = 1, 2, 3, \ldots \quad (2) \]

The results of equation (2) are eigenvalues \(i=1,2,3,\ldots\) and eigenvctor which define mode shape of structure and are in relation with natural frequency for certain mode:

\[ f_i = \frac{\omega_i}{2\pi} \quad (3) \]

where \(f\) is natural frequency. For obtaining eigenvalues, eigenvctor and natural frequencies from (2) The Lanczos algorithm was used. The analysis was done on model with full fuel tank, and it’s presented in Tab 3.
### Tab 3. – Results of normal modes analysis

<table>
<thead>
<tr>
<th>Mode</th>
<th>Mode shape</th>
<th>Natural frequencies</th>
</tr>
</thead>
<tbody>
<tr>
<td>1\textsuperscript{st} mode</td>
<td>1\textsuperscript{st} shape of aileron</td>
<td>0.32 Hz</td>
</tr>
<tr>
<td>2\textsuperscript{nd} mode</td>
<td>1\textsuperscript{st} bending</td>
<td>3.33 Hz</td>
</tr>
<tr>
<td>3\textsuperscript{rd} mode</td>
<td>combination torsion and front-rear motion</td>
<td>13.1 Hz</td>
</tr>
<tr>
<td>4\textsuperscript{th} mode</td>
<td>1\textsuperscript{st} torsion</td>
<td>16.5 Hz</td>
</tr>
<tr>
<td>5\textsuperscript{th} mode</td>
<td>2\textsuperscript{nd} bending</td>
<td>21.4 Hz</td>
</tr>
<tr>
<td>6\textsuperscript{th} mode</td>
<td>1\textsuperscript{st} combination of torsion and bending</td>
<td>31.3 Hz</td>
</tr>
<tr>
<td>7\textsuperscript{th} mode</td>
<td>2\textsuperscript{nd} combination of torsion and bending</td>
<td>43.6 Hz</td>
</tr>
<tr>
<td>8\textsuperscript{th} mode</td>
<td>isolated vibration on trailing edge</td>
<td>54.6 Hz</td>
</tr>
</tbody>
</table>

**Pic. 8.** 1\textsuperscript{st} shape of aileron

**Pic. 9.** 1\textsuperscript{st} bending.

**Pic. 10.** torsion and front-rear motion

**Pic. 11.** 1\textsuperscript{st} torsion

**Pic. 12.** 2\textsuperscript{nd} bending

**Pic. 13.** 1\textsuperscript{st} torsion and bending.

**Pic. 14.** isolated vibration on trailing edge
3. Flutter analysis

Flutter is a dynamic aeroelasticity stability problem. It is self-excited and potentially destructive vibration where aerodynamic forces on an object couple with a structure’s natural mode of vibration to produce rapid periodic motion. Flutter can occur in any object within a strong fluid flow under the conditions that a positive feedback occurs between the structure’s natural vibration and the aerodynamic forces. That is when the vibration movement of the object increases an aerodynamic load which in turn drives the object to move further. If the energy during the period of aerodynamic excitation is larger than the natural damping of the system the level of vibration will increase, resulting in self-exciting oscillation. The vibration levels can thus build up and are only limited when the aerodynamic or mechanical damping of the object match the energy input, this often results in large amplitudes and can lead to rapid failure.

In process of flutter certification is numerical solutions first step which can give us a critical modes. Second steps are vibrations test aimed on critical modes. This test can more precisely determine natural frequency important for flutter calculations. Last step of flutter certification process are flight test. FAA regulations required that airplane must be flutter free to $1.2V_D$. In our case is $V_D=340$ km/h, so $1.2V_D= 408$ km/h.

For flutter analysis was used Nastran solutions SOL145 „Dynamic Flutter Analysis“, for analysis was chosen British PK-Method. This method was developed in 1928 by Mr. Frazer&Duncan. They were attempting to solve the flutter problem using aerodynamic stability derivatives of rigid aircraft. This approach introduce the aerodynamic loads into the equations of motion as frequency dependent stiffness and damping terms. In 1971 this method was developed by Mr.Hassing by introduction aerodynamic loads as complex springs. Advantage of PK-metod is also that results are plotted directly for given velocities, and damping is a more realistic estimated of the physical damping.

Input for solutions flutter solutions are dynamic characteristic which are represented by natural frequencies, material characteristic geometric characteristic of structure and flight conditions (density, velocity). The PK-Method of flutter solution is using equation (4).

$$
\begin{align*}
\left[ M_{hh}p^2 + \left( B_{hh} - \frac{1}{4} \rho ^2 c V Q^I_{hh}/k \right) \right] p + \left( k_{hh} - \frac{1}{2} \rho V^2 Q^R_{hh} \right) \{ u_h \} & = 0
\end{align*}
$$

(4)

Where:

- $M_{hh}$ mass matrix
- $p$ eigenvalue
- $B_{hh}$ damping matrix
- $\rho$ fluid density
- $c$ reference length
- $V$ velocity
- $Q^I_{hh}$ modal aerodynamic damping matrix, function of Mach number and reduced frequency
- $Q^R_{hh}$ modal aerodynamic damping matrix, function of Mach number and reduced frequency
- $k$ reduced frequency
- $k_{hh}$ modal stiffness matrix
- $\{ u_h \}$ modal amplitude vector
[Q_{hh}] is aerodynamic matrix which comes from „Double Lattice subsonic lifting surface theory“ or DLM. On this matrix is applied spline function and is also reduced to obtaining the matrix in generalized form. The equations (4) has to be rewritten to matrix form for solutions in Nastran (5).

\[ [A - pI] \{ u_h \} = 0 \]

where \([A]\) is the real matrix

\[
[A] = \begin{bmatrix}
0 & 0 & 0 & 0 & -I \\
-\frac{M_{hh}}{k} & \frac{1}{2} \rho V^2 & 0 & 0 & -\frac{M_{hh}}{k} \\
0 & -\frac{M_{hh}}{k} & 0 & 0 & -\frac{M_{hh}}{k} \\
0 & 0 & -\frac{M_{hh}}{k} & 0 & -\frac{M_{hh}}{k}
\end{bmatrix}
\]

(5)

And for real roots of (5) is the damping expressed as (6). Obtaining the roots from equations (5) is iteration process.

\[ g = 2\gamma = \frac{2pc}{(\ln 2)\nu} \]

Via damping we can determine when the flutter occurs. The computed damping is aerodynamic damping, in this case we do not know structural one so the FAA regulations „FAR 23.629 Flutter“ estimate as critical damping value 0.03. But if the curve slop of damping is too high, critical velocity is on line of zero damping. The flutter calculations was done for the same model as in Normal modes solutions. Results are summarize in Tab.4 & Tab.5 and critical modes are plotted in Pic.15 & Pic.16.

### Tab 4. – Results of flutter analysis

<table>
<thead>
<tr>
<th>Mode</th>
<th>1st mode</th>
<th>2nd mode</th>
<th>3rd mode</th>
<th>4th mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>H=0m</td>
<td>OK</td>
<td>g= - 0,02</td>
<td>g= - 0,02</td>
<td>OK</td>
</tr>
<tr>
<td>H=1500m</td>
<td>OK</td>
<td>V_{FL}=402 km/h</td>
<td>g= - 0,00015</td>
<td>OK</td>
</tr>
<tr>
<td>H=3000m</td>
<td>OK</td>
<td>V_{FL}=397 km/h</td>
<td>g= - 0,0001</td>
<td>OK</td>
</tr>
</tbody>
</table>

### Tab 5. – Results of flutter analysis-continue

<table>
<thead>
<tr>
<th>Mode</th>
<th>5th mode</th>
<th>6th mode</th>
<th>7th mode</th>
<th>8th mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>H=0m</td>
<td>OK</td>
<td>OK</td>
<td>OK</td>
<td>OK</td>
</tr>
<tr>
<td>H=1500m</td>
<td>OK</td>
<td>OK</td>
<td>OK</td>
<td>OK</td>
</tr>
<tr>
<td>H=3000m</td>
<td>OK</td>
<td>OK</td>
<td>OK</td>
<td>OK</td>
</tr>
</tbody>
</table>

OK     Flutter free to 1.2V_{D}
V_{FL}  Velocity of flutter
\(g\)     maximal damping between computed velocity V=0 km/h and 1,2V_{D}=408 km/h, only for this modes that are too close of line zero damping.
**Pic. 15.** V-g plot for 2\textsuperscript{nd} mode

**Pic. 16.** V-g plot for 3\textsuperscript{rd} mode
From results we can see that critical modes are two 2nd & 3rd. The 2nd mode crosses the stability axis and the slope is steep. In actual flight may be a 20 kilometers an hour between completely stable and extremely unstable plane. Flutter occurs at velocity close under $1.2V_D$. The 3rd mode have trend going to unstable area but the slope of curve is not steep. In investigate s velocities flutter will not occur, but it is sure that at the speed little bit higher than $1.2V_D$ flutter will occur. Rest of the investigate modes have no trends of instability.

4. Control reversal

Control reversal is static aeroelasticity problem, thus it’s without time depending and do not have oscillation character of deformation. We consider aerodynamic and elastic forces only, in solution of static aeroelasticity. Control reversal lead to loss of controllability of the plane, but do not lead to destruction of the structure. A limiting reversal speed is reached when the change in lift due to control surface rotation is nullified by the change in lift due to twist of the lifting surface.

![Pic. 17. Principle of control reversal](image)

For control reversal problem was used solution SOL 144 „Static Aeroelastic Analysis“ where is possible to define certain flight parameters of the model (such as angle of attack, deflection of control surface, flight speed and so on.) and watch the final movement of the model (via non-dimensional stability and control derivative coefficients, trim parameters and so on...). For solutions of control reversal problem was used this setting of model:

Definition of constant deflection of aileron $\delta=0.3$ [rad] and released model for rotation about axis of symmetry (x-axis). The setting was done in source code of input Nastran file *.bdf as:

- **Boundary condition:** Front and rear hinge: SCP 123 56
- **Support Rigid body DOF:** NODE 7895 DOF:4
- **Rigid Body Motion Trim Variables:** ROLL; URDD4
- **Trim Parameters for Subcase:** URDD4=0.0; AILE= 0.3; M=0.0
- **Aeroelastic Model Parameters:** PARAM AUNITS 1.0
- PARAM BAILOUT -1
- **Symmetry of aerodynamic motion:** SYMXZ -1; SYMXY 0 (Default)

Note: „SCP 123 56“ mean that was restricted all motion in DOF 12356 except 4DOF thus rotation about x-axis.

Monitoring of model response is via parameter ROOL which is one of the printed output and is defined by equations (7):

$$ROLL = \frac{p^{(1/2)}}{2v} \quad [-]$$

(7)
Where:

- $l$ .......... wing span [m]
- $p$ .......... "Rool rate" angular velocity of rotation about x-axis [rad/sec]
- $v$ .......... velocity [m/s]

If is equations (7) divide by deflection of aileron in [rad] we can obtain ailerons effectiveness (8)

$$
\eta_x = \frac{\pi (l/2)}{2\pi \delta} \quad [\text{–}]
$$

Where:

- $\eta_x$ .......... ailerons effectiveness [-]
- $\delta$ .......... deflection of aileron [rad]

The calculations was done for dynamic pressures corresponding to speeds from 0 km/h to 450 km/h and altitude 0m and 3000m. The results are summarized in Tab. 6 and Pic. 18.

**Tab 6. – Results of control reversal analysis**

<table>
<thead>
<tr>
<th>Dynamic pressure</th>
<th>Effectiveness $\eta_x$ [-]</th>
<th>$V_{TAS}$ [km/h]</th>
<th>$V_{TAS}$ [km/h]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q$ [MPa]</td>
<td></td>
<td>$V_{TAS}$ [km/h]</td>
<td>$V_{TAS}$ [km/h]</td>
</tr>
<tr>
<td>4,73E-08</td>
<td>0,44</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2,66E-04</td>
<td>0,43</td>
<td>75</td>
<td>87</td>
</tr>
<tr>
<td>1,06E-03</td>
<td>0,38</td>
<td>150</td>
<td>174</td>
</tr>
<tr>
<td>1,89E-03</td>
<td>0,35</td>
<td>200</td>
<td>232</td>
</tr>
<tr>
<td>2,95E-03</td>
<td>0,29</td>
<td>250</td>
<td>290</td>
</tr>
<tr>
<td>4,25E-03</td>
<td>0,20</td>
<td>300</td>
<td>348</td>
</tr>
<tr>
<td>4,99E-03</td>
<td>0,14</td>
<td>325</td>
<td>377</td>
</tr>
<tr>
<td>5,79E-03</td>
<td>0,06</td>
<td>350</td>
<td>406</td>
</tr>
<tr>
<td>6,65E-03</td>
<td>-0,02</td>
<td>375</td>
<td>435</td>
</tr>
<tr>
<td>7,56E-03</td>
<td>-0,14</td>
<td>400</td>
<td>464</td>
</tr>
<tr>
<td>8,54E-03</td>
<td>-0,28</td>
<td>425</td>
<td>493</td>
</tr>
<tr>
<td>9,57E-03</td>
<td>-0,44</td>
<td>450</td>
<td>522</td>
</tr>
</tbody>
</table>

**Pic. 18. Aileron effectiveness plot**
Velocity of control reversal $V_{REV}$ is when aileron effectiveness decrease on zero $\eta_x = 0$. The critical speed was determine by linear interpolation and results are in Tab.7

**Tab 7. Control reversal speeds**

<table>
<thead>
<tr>
<th>Altitude</th>
<th>$V_{REV}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 m</td>
<td>386 km/h</td>
</tr>
<tr>
<td>3000 m</td>
<td>427 km/h</td>
</tr>
</tbody>
</table>

4. Wing torsion divergence

Torsion divergence is also problem of static aeroelasticity as control reversal. But it is problem which leads do destruction of the structure. Divergence may occur without warning.

**Pic. 19. Principle of torsion divergence**

For understanding the problem, assume a wing in horizontal flight with small angle of attack $\alpha$. The aerodynamic lift force $Y$ acting in aerodynamic center (A.O.) creates a torque $M_{z0}$ to elastic axis (E.O.). This torque causes a torsion deformation of a wing, and increasing angle of attack $\Theta$. This is flowed by increasing aerodynamic lift forces. With increasing speed the torsion deformation is also increasing. In the moment when a structure is not capable to damp difference of torque, the torsion divergence of wing occurs. This critical speed is called $V_{DIV}$.

For the calculations was used SOL 144 as in chapter 3. The process for obtain $V_{DIV}$ was following. The model of wing was released in rotation about y-axis, thus in mean of change angle of attack. Also was defined condition of the flight at constant flight level. Investigate will the motion of model with increasing dynamic pressure. The critical speed $V_{DIV}$ can be obtain from aerodynamic derivation $C_{Z\alpha}$ also know as $C_y \alpha$. This derivation show change of aerodynamic lift force witch changing angle of attack. In area of $V_{DIV}$ the $C_{Z\alpha}$ will grow to extreme high values. It is given by torsion deformation of wing and great difference of lift in small difference of angle of attack. The setting was done in source code of input Nastran file *.bdf as:

- **Boundary condition:**
  - Rear hinge: NODE 7853 SCP 26
  - Rear hinge: NODE 7905 SCP 126
  - Front hinge: NODE 7650 SCP 26

- **Support Rigid body DOF:**
  - NODE 7905 35
  - NODE 849 5 (ailerons)

- **Rigid Body Motion Trim Variables:**
  - ANGLEA; PITCH; URDD3; URDD5
  - ROLL; URDD4

- **Trim Parameters for Subcase:**
  - ANGLEA=FREE; PITCH=0.0; URDD3=-1.0;
  - URDD5=FREE; ROLL=0.0; URDD4=0.0
  - AILE= FREE

- **Aeroelastic Model Parameters:**
  - PARAM AUNITS 1.0193E-04
Symmetry of aerodynamic motion: SYMXZ 1; SYMXY 0 (Default)

**Table 8.** – Results of divergence analysis

<table>
<thead>
<tr>
<th>Velocity</th>
<th>Derivation</th>
</tr>
</thead>
<tbody>
<tr>
<td>VEAS [km/h]</td>
<td>CZa [-]</td>
</tr>
<tr>
<td>100</td>
<td>-15,8488</td>
</tr>
<tr>
<td>200</td>
<td>-0,2956</td>
</tr>
<tr>
<td>300</td>
<td>-0.0550</td>
</tr>
<tr>
<td>400</td>
<td>-0.0358</td>
</tr>
<tr>
<td>450</td>
<td>0.0177</td>
</tr>
<tr>
<td>500</td>
<td>0.1295</td>
</tr>
<tr>
<td>550</td>
<td>0.4936</td>
</tr>
<tr>
<td>562</td>
<td>0.7548</td>
</tr>
<tr>
<td>575</td>
<td>1.3260</td>
</tr>
<tr>
<td>580</td>
<td>1.8061</td>
</tr>
<tr>
<td>585</td>
<td>2.7324</td>
</tr>
<tr>
<td>590</td>
<td>5.2853</td>
</tr>
<tr>
<td>595</td>
<td>46,1169</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Velocity</th>
<th>Derivation</th>
</tr>
</thead>
<tbody>
<tr>
<td>VEAS [km/h]</td>
<td>CZa [-]</td>
</tr>
<tr>
<td>600</td>
<td>-7,2340</td>
</tr>
<tr>
<td>605</td>
<td>-3,4215</td>
</tr>
<tr>
<td>610</td>
<td>-2,2630</td>
</tr>
<tr>
<td>620</td>
<td>-1,3616</td>
</tr>
<tr>
<td>630</td>
<td>-0,9716</td>
</tr>
<tr>
<td>640</td>
<td>-0,7452</td>
</tr>
<tr>
<td>650</td>
<td>-0,5910</td>
</tr>
<tr>
<td>660</td>
<td>-0,4743</td>
</tr>
<tr>
<td>675</td>
<td>-0,3359</td>
</tr>
<tr>
<td>700</td>
<td>-0,1288</td>
</tr>
<tr>
<td>900</td>
<td>-0,0530</td>
</tr>
<tr>
<td>800</td>
<td>-0,1402</td>
</tr>
<tr>
<td>1000</td>
<td>-0,1341</td>
</tr>
</tbody>
</table>

**Pic. 20.** Dependent of CZa at velocity plot for H=0m

Velocity of wing torsion divergence was determined by analysis as V_{DIV}=601 km/h for H=0m and V_{DIV}=695 km/h for H=3000m

**5. Conclusions**

This paper deals with aeroelastic analysis in FEM software MSC.Nastran. The analysis determines that flutter may occur at speed V_{FL}=397 km/h, control reversal V_{REV}=368 km/h and torsion divergence V_{DIV}=601 km/h.
This analysis will be useful for investigation aeroelastic phenomenon. And for determination of which structure parameters have significant influence on those aeroelastic phenomenon. Also this method will compare with experimental investigation of flutter phenomena on a real aircraft structure.

**List of symbols**

- $[B_{hh}]$ damping matrix
- $C \ [m]$ reference length
- $C_{Za} [-]$ Aerodynamic derivation $C_y \alpha$
- $f \ [Hz]$ Natural frequency
- $g [-]$ damping
- $h \ [m]$ altitude
- $k$ Reduced frequency
- $[K]$ Stiffness matrix
- $k_{hh}$ Modal stiffness matrix
- $l \ [m]$ Wing span
- $[M], [M_{hh}]$ Mass matrix
- $M_{zo} [N.m]$ torque
- $p [-]$ eigenvalue
- $p_r [rad/sec]$ Rool rate
- $Q \ [Mpa]$ Dynamic pressure
- $Q_{hh}^l$ modal aerodynamic damping matrix, imaginary part
- $Q_{hh}^r$ modal aerodynamic damping matrix, real part
- $S \ [m^2]$ Wing surface
- $\{u\}$ Harmonic solution
- $\{u_h\}$ Modal amplitude vector
- $V [km/h]$ Velocity
- $V_D [km/h]$ Design speed
- $V_{DIV} [km/h]$ Flutter critical velocity
- $V_{ESA} [km/h]$ Equivalent air speed
- $V_{FL} [km/h]$ Flutter critical velocity
- $V_{MIN} [km/h]$ Stall speed
- $V_{REV} [km/h]$ Flutter critical velocity
- $Y [N]$ Lift force

- $\alpha \ [rad]$ Angle of attack
- $\delta \ [rad]$ deflection of aileron
- $\eta_x [-]$ Ailerons effectiveness
- $\Theta \ [rad]$ Increment angle of attack
- $\lambda \ [-]$ Aspect ration
- $P \ [kg/m^3]$ Fluid density
- $\{\phi\}$ The eigenvector or mode shape
- $\omega$ The circular natural frequency

**List of abbreviations**

- A.O. Aerodynamic axis
- DOF Degree of freedom
- DLM Double lattice method
E.O. Elastic axis
FAA Federal Aviation Administration
FAR Federal Aviation Regulations
FEM Finite elements method
MPC Multi constraint points
SOL103 Normal modes analysis
SOL144 Static Aeroelastic Analysis
SOL145 Dynamic Flutter Analysis

References

Internet sources
[2] FAR 23.629 Flutter

MSC.Software manuals for MSC.Nastran/Patran
[M1] MSC.Patran User’s Guide
[M3] MSC Advanced Dynamic Analysis
[M8] MSC.Nastran Basic Dynamic Analysis Users Guide

Used software:
FindText
FlutterPlotter
MSC.Patran 2005
MSC.Nastran 2005