Active vibration suppression with using centralized actuators and fibres

Ing. Tereza Kašpárková

Vedoucí práce: Doc. Ing. Zbyněk Šika PhD

Abstrakt

Obvykle se k tlumení vibrací kompozitových struktur používá velké množství aktuátorů. Tento článek se na rozdíl od tohoto obvyklého způsobu snaží tlumit vibrace kompozitové struktury s použitím malého množství centralizovaných aktuátorů.V článku je popsán návrh typu senzorů a aktuátorů a jejich umístění na základní struktuře. K zpětnovazebnému řízení je zde použita metoda LQR.

Klíčová slova

Aktivní tlumení vibrací, kompozitové struktury, LQR

1. Introduction

The composite materials are very popular because they have low weight high strength and possibilities of low temperature of expansibility. The planes, the bullet proof vests, some construction in space and the parts of cutting or shaping machine are manufacture of composite material. This material have some problems too. The biggest problem of this material is lower damping. The good solution of this problem is connection of active elements to the composite structure. The elements are piezoelectric patches and fibres. Mostly it is used a lot of elements. This concept have some problems with high electric voltage, high price and a lot of wire. This problems leads to idea of new concept of solution of the problem. Namely the use of only few linear drives and fibres correcting the drives and structure.

2. The concept of damping

The composite structures is four – corned, thin - walled beam [1]. This beam is made from carbon fibres. His length is 180 cm and his section is 10x10 cm. Each solution are calculated for the freely hung beam. Two linear drives are connected to each wall of the beam. The linear drives are placed in the middle of the wall of the beam. Two fibres connected each drives and suitable places of the beam (figure 1). The active elements (linear drivers and fibres) are placed inside the beam. The concept contain 16 sensors too. The sensors are accelerometers. Five accelerometers measure the axis x, five accelerometers measure the axis y and six accelerometers measure the axis z. They are placed on the outside walls of the beam.

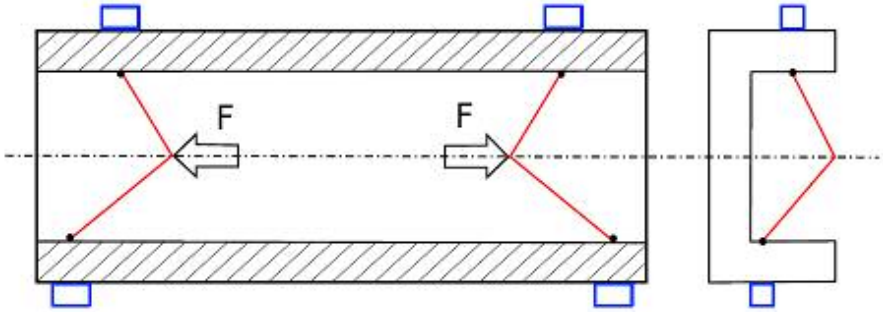

Fig.1*. New concept – red are acting elements and blue are sensing elements*

3. The placement of sensors and actuators

The H_2 Norm [2] is used for optimization of place for connecting sensors and fibres to the beam. It is calculated from the state space form (equation1) [1]

$$\begin{aligned} \dot{x} &= A_m \cdot x + B_r \cdot d + B_m \cdot u \\ y &= C_m \cdot x \end{aligned}, \tag{1}$$

where the matrix A_m is systems matrix, the matrix B_m is control input matrix, the matrix B_r is disturbing matrix and C_m is output matrix.

3.1 The sensors

The concept contain about two thousands and five hundred places for connection sensors. The H_2 Norm of the ith mode and kth sensor is in equation 2

$$\left\| G_{ik} \right\|_2 = \frac{\left\| B_{mi} \right\|_2 \left\| C_{mki} \right\|_2}{2\sqrt{\zeta_i \omega_i}}, \qquad i = 1 \ldots N, \qquad k = 1 \ldots R \tag{2}$$

where N is the number of all mode and R is the number of all places for connection sensors. The H_2 placement index if the ith mode and kth sensor is in equation 3

$$\sigma_{2ik} = w_{ik} \frac{\left\| G_{ik} \right\|_2}{\left\| G \right\|_2}, \qquad i = 1 \ldots N, \qquad k = 1 \ldots R \tag{3}$$

where $w_{ki} \geq 0$ is weight assigned and G is transfer function. The sensor placement matrix is made up from sensor placement indexes in this form

$$\sum\nolimits_2 = \begin{bmatrix} \sigma_{211} & \sigma_{212} & \cdots & \sigma_{21k} & \cdots & \sigma_{21R} \\ \sigma_{221} & \sigma_{222} & \cdots & \sigma_{22k} & \cdots & \sigma_{22R} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \sigma_{2i1} & \sigma_{2i2} & \cdots & \sigma_{2ik} & \cdots & \sigma_{2iR} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \sigma_{2n1} & \sigma_{2n2} & \cdots & \sigma_{2nk} & \cdots & \sigma_{2nR} \end{bmatrix} \tag{4}$$

where the ith row is set indexes of sensors for ith mode and the kth column consist of indexes of the kth sensors for every modes. The vector of sensor placement indexes is defined as $\sigma_S = \begin{bmatrix} \sigma_{s1} & \sigma_{s2} & \cdots & \sigma_{sR} \end{bmatrix}^T$, where σ_{sk} is rms sum kth sensor indexes over all modes

$$\sigma_{sk} = \sqrt{\sum_{i=1}^{n} \sigma_{ik}^2} \tag{5}$$

The index σ_{sk} characterizes the importance of the kth sensor. The places with the highest indexes are good for placing the sensors.

The concept contain five x-direction sensors, five y-direction sensors and five z-direction sensors. The index σ_{sk} is calculated for each three direction (figure 2, 3) and appropriate places are chosen from places with biggest σ_{sk} .

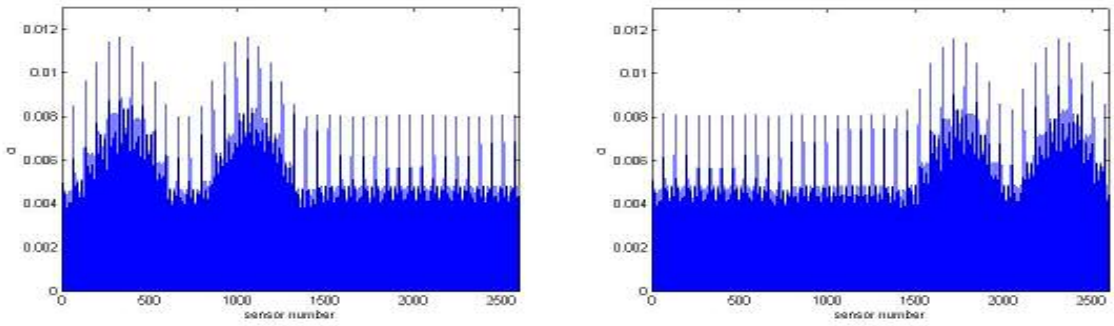

Fig. 2. Index σ_{sk} for direction x and y

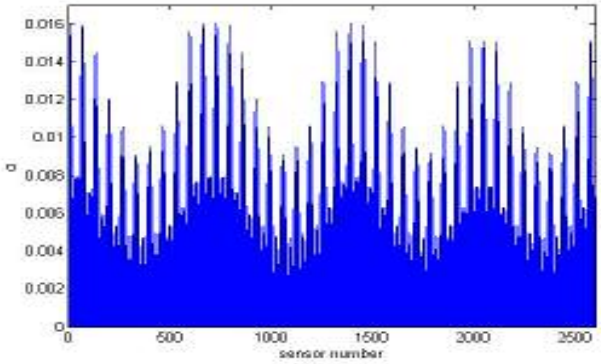

Fig. 3. Index σ_{sk} for direction z

3.2 The actuators

The beam is divided to the two parts for finding appropriate places to connecting the fibres to the beam. The linear drives make two sets. Each sets are connected to sets of places between linear drives and further end of the beam (figure 1). Both set of places for connecting fibres contain about one thousands place. The angles between fibre and the coordinate axes are very important for actuator efficiency and for every drives are different. This angel determine the force rate in axes x, y, z. The matrix Bm is made with reflect angles between fibre and beam. The H_2 Norms of the ith mode with the jth actuators is

$$\left\|G_{ij}\right\|_2 = \frac{\left\|B_{mij}\right\|_2 \left\|C_{mi}\right\|_2}{2\sqrt{\zeta_i \omega_i}}, \qquad i = 1 \ldots N, \qquad j = 1 \ldots S \tag{6}$$

where N is the number of all mode and S is the number of all places for connection fibres. The H_2 placement index if the ith mode and jth place for connecting fibre (actuators) is in equation 7.

$$\sigma_{2ij} = w_{ij} \frac{\left\|G_{ij}\right\|_2}{\left\|G\right\|_2}, \qquad i = 1 \ldots N, \qquad j = 1 \ldots S \tag{7}$$

where $w_{ji} \geq 0$ is weight assigned and G is transfer function. The actuator placement matrix is made up from actuator placement indexes in this form

$$\sum_2 = \begin{bmatrix} \sigma_{211} & \sigma_{212} & \cdots & \sigma_{21j} & \cdots & \sigma_{21S} \\ \sigma_{221} & \sigma_{222} & \cdots & \sigma_{22j} & \cdots & \sigma_{22S} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \sigma_{2i1} & \sigma_{2i2} & \cdots & \sigma_{2ij} & \cdots & \sigma_{2iS} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \sigma_{2n1} & \sigma_{2n2} & \cdots & \sigma_{2nj} & \cdots & \sigma_{2nS} \end{bmatrix} \tag{8}$$

where ith row is set indexes of actuators for ith mode and the jth column consist of indexes of the jth actuators for every modes. The vector of actuator placement indexes is defined as $\sigma_a = [\sigma_{a1} \quad \sigma_{a2} \quad \cdots \quad \sigma_{aS}]^T$, where σ_{aj} is rms sum jth actuator indexes over all modes

$$\sigma_{aj} = \sqrt{\sum_{i=1}^{n} \sigma_{ij}^2} \tag{9}$$

The index σ_{aj} characterizes the importance of the jth place for connecting fibre to beam. The places with the highest indexes are good for connecting fibres.

The concept contain 16 fibres and eight linear drives. The index σ_{aj} is calculated for each eight drives and appropriate places for connecting two fibres to beam are chosen from places with biggest σ_{aj} .Some figure with σ_{aj} for the firs set of places are in the figure 4,5.

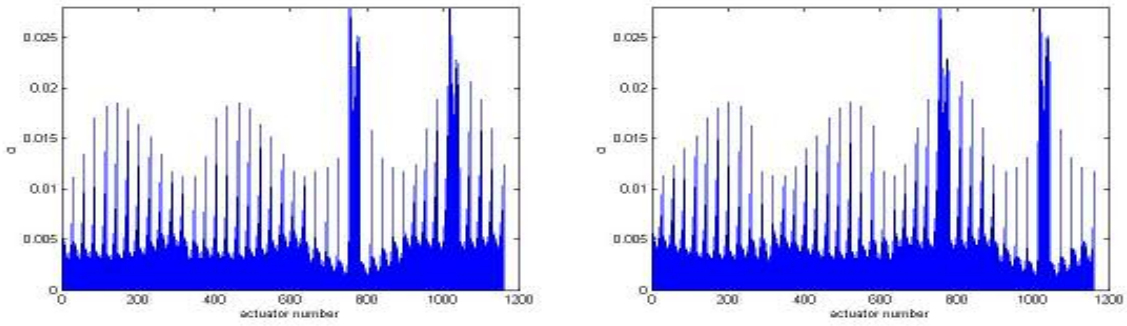

Fig. 4. Index σ_{aj} for drives 1 and 2

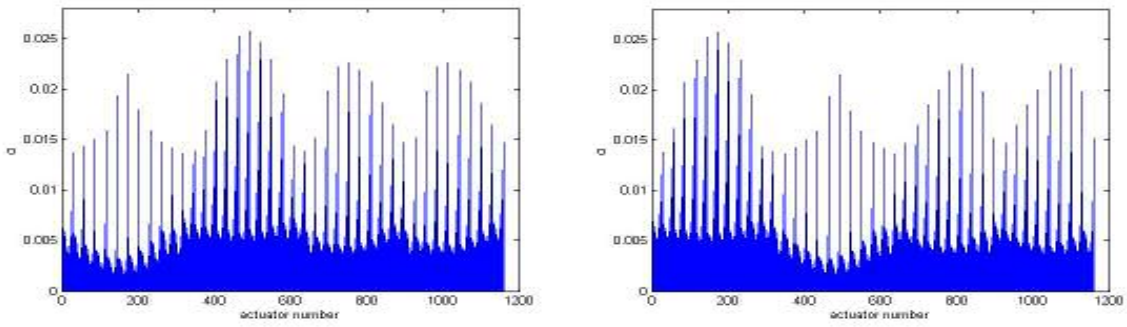

Fig. 5. Index σ_{aj} for drives 3 and 4

4. The control

The actuators are under control of the state space feedback control [3]. The state space definition of system (equation 1) is necessary for the control synthesis. The control input is define by the state space feedback as

$$u = -K \cdot x \tag{10}$$

where K is the gain of feedback control. From equations 1 and 10 follow

$$\dot{x} = (A_m - B_m \cdot K)u + B_r \cdot d \tag{11}$$

The LQR [4] is method of state space feedback control. This method is used to finding the gain K. The gain K is determined as minimization of quadratic cost function J in equation 12

$$J = \frac{1}{2}\int_0^\infty \left(x^T \cdot Q \cdot x + u^T \cdot T \cdot u\right)dt \tag{12}$$

where Q and T represent weights on the different states and control channels. The matrix Q have to be symmetric semi-positive definite and T have to be symmetric positive definite. The matrix Q [5] is

$$Q = H^T H \tag{13}$$

where H is the vector of weight of the state. The gain matrix K is

$$K = T^{-1} B_m{}^T P \tag{14}$$

where P is calculated from Riccati equation

$$PA + A^T P + Q - PB_m T^{-1} B_m{}^T P = 0 \tag{15}$$

The control is used to damping 16 modes so the vector H have size 1x16. The weights of the control channels T is 0.001.

5. The results

The final results are composed from the results of placement and the results of control. The results of placement are the optimum places for sixteen sensors and sixteen fibres. The results of the control is the gain matrix K. The gain matrix is modified with respect to the coordinate systems on the beam wall. The final results are in the figure 6. They are transfer function of controlled and uncontrolled system

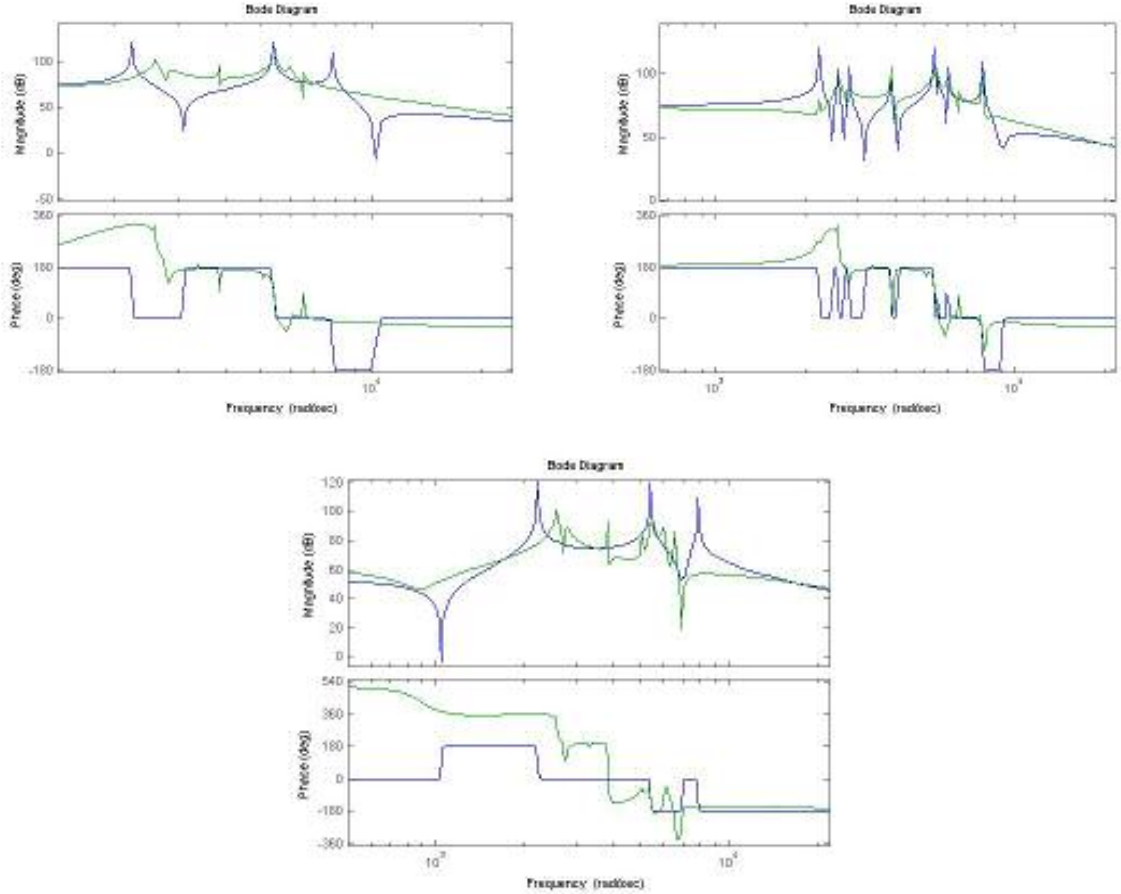

***Fig. 6**. Transfer function to the tree another sensors*

6. The conclusion

The paper presents the new concept of active vibration suppression of composite beam. Eight linear drives with sixteen fibres connecting the drives and the structure are used as actuation system. 16 one-axis accelerometers are connected to the beam as a sensors. The method uses only a few actuators. This is the main advantage with respect to the concept with high number of distributed piezoelectric actuators. The appropriate placing of sensors and fibre connection is very important for the efficiency. The control law synthesis is realized by the state space feedback control using LQR.

7. The Acknowledgements

The authors appreciate the kind support of the grant "Research of new principle of mechanical and biomechanical systems with a intelligent behaviour" (GD101/08/H068)

The list of the symbols

x	system state (modal state)	[1]
u	control input	[kg.m.s^{-2}]
d	disturbing input	[kg.m.s^{-2}, kg.m^2 s^{-2}]
y	measure output	[m.s^{-2}]
A_m	system matrix	[-]

B_m	input control matrix	[-]
B_r	disturbing matrix	[-]
C_m	output matrix	[-]
$\|G\|_2$	system H_2 Norm	[-]
$\|B_m\|$	norm of input control matrix	[-]
$\|C_m\|$	norm of output matrix	[-]
w	weight matrix	[-]
ζ	modal damping	[-]
ω	natural frequence	[1/s]
σ_2	H_2 norm placement index	[-]
σ	vector of placement indexes	[-]
Σ	placement matrix	[-]
K	gain matrix	[-]
J	quadratic cost function	[-]
Q	weight matrix	[-]
T	weight matrix	[-]
H	vector of weight of the state	[-]
i	coefficient	[-]
k	coefficient	[-]
j	coefficient	[-]
N	number of mode	[-]
S	number of places for actuator	[-]
R	number of places for sensor	[-]

References

[1] Kašpárková, T., Šika Z, Active vibration soppression of composite beam, *11th Workshop on Applied Mechanics, Proceeding,* 2010,.

[2] Gawronski, W. K., Advanced Structural Dynamics and Active Control of Structures, *Mechanical Engineering Series*, Springer, 2004

[3] Kejval, J., *Tlumení vibrací strojů poloaktivním dynamickým hltičem.* Disertační práce, ČVUT, Praha, 2001

[4] G. E. Stavroulakis, G. Foutsitzi, E. Hadjigeorgiou, D. Marinova, C. C. Baniotopoulos, Design and robust optimal control of smart beams with application on vibrations suppression, *Advances in Engineering Software,* **36**, 2005, 806-813.

[5] Šika, Z., *Aktivní a poloaktivní snižování mechanického kmitání strojů*. Habilitační práce, ČVUT, Praha, 2004