Twyman - Green Interferometer with Automatic Sensing and Interference Field Evaluation

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Abstrakt

Bezkontaktní metody vyhodnocování tvaru optických ploch patří mezi velice žádané nástroje právě z důvodu eliminace fyzického kontaktu měřící a měřené součásti, který zvyšuje pravděpodobnost poškození měřeného vzorku. V práci je popsána realizace konstrukce Twyman – Greenova interferometru kompaktních rozměrů (průměr měřícího svazku 10 mm) s automatickým vyhodnocováním interferenčního pole, včetně uvedení některých z použitých vyhodnocovacích algoritmů.

Keywords

surface topography, interference of light, evaluation algorithm, Twyman – Green interferometer

1. Introduction

The first time the interference phenomenon was demonstrated was around the year 1807. Since the time the interference methods have become very strong and an extensive tool for many measuring tasks, e.g. determination of wave length, surface quality, optical layer thickness etc.

Devices using interference for exact measuring are called interferometers [15, 16] and in accordance with their usage we can speak about a few kinds of them. For the length measurement we use an interference comparator, for a speed of light relative shift measurement (by which means we can determine refractive indexes of gasses and liquids) we use interference refractometers. Another type are interference spectroscopes for spectral lines fine structure determination. In praxis (the optical industry especially) there are the most important interferometers for a surface topology measurement. In this branch there are used two types of interferometers: Fizeau and Twyman-Green. More information about this topic it's possible to be found in [17].

In this article there is described an engineering design of the Twyman-Green interferometer (a modification of the Michelson interferometer) of compact dimensions which can be used for measuring reflexive, refractive, planar and spherical surfaces. The main advantages of this design are its small dimensions and availability of full automatic measurement process (automatic sensing with following evaluation of snapped interferograms).

2. Two - beam interference of light

In the figure 1 there is an experimental setup scheme for two-beam interference depicted. S_0 and S_R are sources of the coherent wave fields. These fields interfere with each other and this interference field characterized by the interference fringes we can observe on the screen.

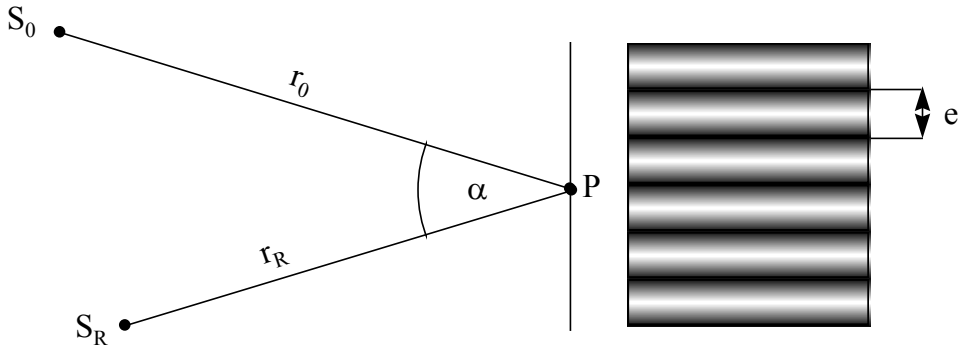

Fig. 1. *Two beam interference experimental scheme setup.*

For the phase difference φ of these fields in the interference field point *P(x,y)* we can write

$$\varphi = \varphi_R - \varphi_0 = \frac{2\pi}{\lambda} r_R + \delta_R - \frac{2\pi}{\lambda} r_0 - \delta_0 = \frac{2\pi}{\lambda}(r_R - r_0) + \delta_R - \delta_0 = \frac{2\pi}{\lambda} W + \delta_R - \delta_0 \tag{1}$$

Where λ is the interfering light wavelength, W the path difference and δ is the initial phase. The adjacent fringes distance e for small angles α is then given by the formula [1, 2, 3]

$$e = \frac{\lambda}{\alpha} \tag{2}$$

Now in the general case we have two coherent, linearly polarized wave fields with complex amplitudes $U_0 = A_0 \exp(i\varphi_0)$ and $U_R = A_R \exp(i\varphi_R)$ where U_0 is the unknown wave field and U_R is the known reference field; A_0, A_R are amplitudes of the unknown and reference field and φ_0, φ_R are phases of unknown and reference fields. These two fields then interfere with each other and in case the polarization directions are the same, the intensity *I(x,y)* in the interference field point *P(x, y)* is given by the formula $(I \sim A^2)$ [1, 2, 3]

$$I = I_0 + I_R + 2\sqrt{I_0 I_R}\cos(\varphi_R - \varphi_0) = A + B\cos(\varphi) \tag{3}$$

The phase difference φ we can indentify by the interference fringes evaluation. Intensity maxima (light interference fringe) will occur in places for which we can write

$$\varphi = \pm 2N\pi\,,\ N = 0,1,2,3,... \tag{4}$$

For places with the intensity minima (black interference fringes) we can write

$$\varphi = \pm 2(N+1)\pi\,,\ N = 0,1,2,3,... \tag{5}$$

For intensities I_{max}, I_{min} and contrast K then applies

$$I_{\max} = I_0 + I_R + 2\sqrt{I_0 I_R}\,,\ I_{\min} = I_0 + I_R - 2\sqrt{I_0 I_R} \tag{6}$$

$$K = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{2\sqrt{I_0 I_R}}{I_0 + I_R} = \frac{B}{A} \tag{7}$$

From formula (7) we can see the contrast K will be maximal ($K = 1$) for the same intensity values $I_0 = I_R$.
Let's assume for the phase difference φ in the interference field evaluation plane we can write

$$\varphi = \varphi(x, y) \tag{8}$$

After differentiation we get

$$\mathrm{d}\varphi = \frac{\partial \varphi}{\partial x}\mathrm{d}x + \frac{\partial \varphi}{\partial y}\mathrm{d}y \tag{9}$$

Now we choose an interference fringe and we move along them. Because the phase difference φ is constant along this interference fringe i.e. $\varphi = \varphi(x, y) = C$ (where C is a constant), from (9) we get

$$\mathrm{d}\varphi = \frac{\partial \varphi}{\partial x}\mathrm{d}x + \frac{\partial \varphi}{\partial y}\mathrm{d}y = 0 \tag{10}$$

and then

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{\frac{\partial \varphi}{\partial x}}{\frac{\partial \varphi}{\partial y}} \tag{11}$$

This equation gives us a tangent direction in any place of an interference fringe. If we know the direction of an interference fringe in any point, we know also the interference fringe shape. For determination of the distance between two adjacent interference fringes we need to evaluate the phase difference change in the direction of normal to the interference fringe. Let's write

$$\mathrm{d}\varphi = \frac{\partial \varphi}{\partial n}\mathrm{d}s = \sqrt{\left(\frac{\partial \varphi}{\partial x}\right)^2 + \left(\frac{\partial \varphi}{\partial y}\right)^2}\,\mathrm{d}s \tag{12}$$

where ds is the distance change in the direction of the normal to the interference fringe leading to the phase difference dφ. Because the phase difference dφ of two adjacent interference fringes equals $\mathrm{d}\varphi = 2\pi$, the distance ds is equal to the interference fringes distance e. From previous equation we get

$$\mathrm{d}\varphi = e\sqrt{\left(\frac{\partial \varphi}{\partial x}\right)^2 + \left(\frac{\partial \varphi}{\partial y}\right)^2} = 2\pi \tag{13}$$

and

$$\mathrm{e} = \frac{2\pi}{\sqrt{\left(\frac{\partial\varphi}{\partial x}\right)^2 + \left(\frac{\partial\varphi}{\partial y}\right)^2}} \tag{14}$$

Let's have *W(x, y)* the path difference of two interfering fields in the point of interference field *(x, y)*. We can write

$$\varphi(x, y) = \frac{2\pi}{\lambda} W(x, y) + \delta \tag{15}$$

and after substitution into previous equation we get the relation for the interference fringes distance in dependence on the path difference

$$e = \frac{\lambda}{\sqrt{\left(\frac{\partial W}{\partial x}\right)^2 + \left(\frac{\partial W}{\partial y}\right)^2}} \tag{16}$$

The path difference it's possible to be written in the Seidel polynomials form. For polar coordinate system we have

$$\begin{aligned} W(r,u) = {} & W_{1x} r \sin u + W_{1y} r \cos u + W_{20} r^2 + W_{40} r^4 + W_{60} r^6 + W_{31} r^3 \cos u + \\ & + W_{51} r^5 \cos u + W_{22} r^2 \cos 2u \end{aligned} \tag{17}$$

and for Cartesian coordinate system then

$$\begin{aligned} W(x, y) = {} & W_{1x} x + W_{1y} y + W_{20}\left(x^2 + y^2\right) + W_{40}\left(x^2 + y^2\right)^2 + W_{60}\left(x^2 + y^2\right)^3 + \\ & + W_{31}\left(x^2 + y^2\right) y + W_{51}\left(x^2 + y^2\right)^2 y + W_{22}\left(y^2 - x^2\right) \end{aligned} \tag{18}$$

Where coefficients are:
W_{1x}…x - axis tilt coefficient
W_{1y}…y – axis tilt coeff.
W_{20}…longitudinal defocusing coeff.
W_{40}…3rd order spherical aberration coeff.
W_{60}…5th order sph. aberr. coeff.
W_{31}…3rd order coma aberr. coeff.
W_{51}…5th order coma aberr. coeff.
W_{22}…3rd order astigmatism aberr. coeff.

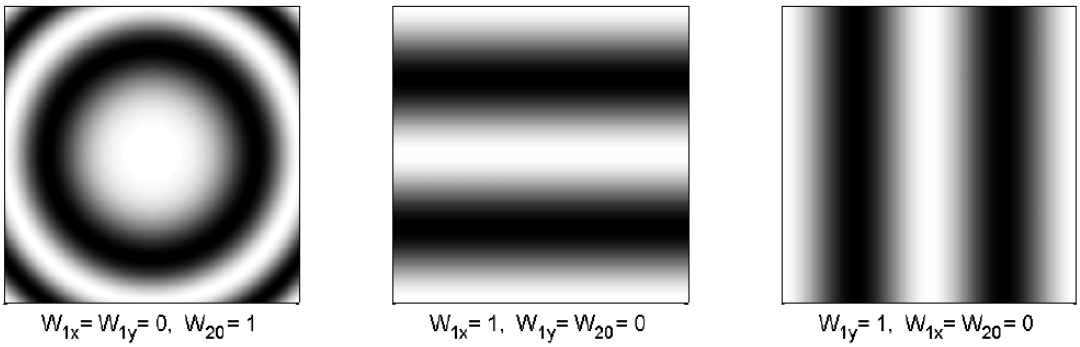

Fig. 2. *Interference field for various coefficients W_{1x}, W_{1y} and W_{20} values combinations.*

3. Interference field evaluation methods

In the equation for intensity evaluation

$$I = I_0 + I_R + 2\sqrt{I_0 I_R}\cos(\varphi_R - \varphi_0) = A + B\cos(\varphi) \tag{19}$$

there are three unknown values A, B and φ. In order to evaluate these values we need to perform at least three intensity I measurements (three steps). To perform this we will change the reference field phase shift φ_R of value g. Let's have

$$I_i = A + B\cos(\varphi + g_i),\ I = 1, 2, 3, \ldots, N \tag{20}$$

In this equation N is the count of steps (number of intensity changes and measurements) we need to perform in order to evaluate the phase shift φ in this place. For $N \geq 3$ and phase shift values g_i we get the system of N equations from which we can evaluate the values A, B and φ. The phase shift φ at a given point is expressed through the measured intensities at this point of the interference field and therefore from many algorithms for the phase shift evaluation we need to choose the proper one - the one with the minimal final $\delta\varphi$ deviation attached to the deviations arisen during intensity measurements performing. The same fact is necessary to be taken in account in case of phase shifts g which we need to have precise measured as well. Optimal algorithm for us is the one with the minimal sensitivity to the measurements errors of intensity δI and phase shift $\delta\varphi$. To minimize this sensitivity we can use algorithms with more steps (bigger N value) and furthermore we need to choose the suitable values of g. Using more steps has the advantage of sensitivity decrease but on the other side we need to perform more measurements what is more time consuming and in some practical cases not very suitable. In the following there will be described some algorithms possible to be used in praxis.

3.1 Three - step algorithms N = 3

As stated in previous section the minimal count of steps is $N = 3$ (three - step algorithms). The equation system is then

$$\begin{aligned} I_1 &= A + B\cos g_1 \cos\varphi - B\sin g_1 \sin\varphi \\ I_2 &= A + B\cos g_2 \cos\varphi - B\sin g_2 \sin\varphi \\ I_3 &= A + B\cos g_3 \cos\varphi - B\sin g_3 \sin\varphi \end{aligned} \tag{21}$$

We get

$$I_1 - I_3 = B(\cos g_1 - \cos g_3)\cos\varphi - B(\sin g_1 - \sin g_3)\sin\varphi \tag{22}$$

$$2I_2 - I_3 - I_1 = B(2\cos g_2 - \cos g_3 - \cos g_1)\cos\varphi - B(2\sin g_2 - \sin g_3 - \sin g_1)\sin\varphi \tag{23}$$

By dividing these equations we have

$$C = \frac{I_1 - I_3}{2I_2 - I_3 - I_1} = \frac{C_1 - S_1 tg\varphi}{C_2 - S_2 tg\varphi} \tag{24}$$

where

$$\begin{aligned} C_1 &= \cos g_1 - \cos g_3 \\ S_1 &= \sin g_1 - \sin g_3 \\ C_2 &= 2\cos g_2 - \cos g_3 - \cos g_1 \\ S_2 &= 2\sin g_2 - \sin g_3 - \sin g \end{aligned} \tag{25}$$

From this equation we can evaluate the phase shift φ

$$\varphi = \arctan\left[\frac{C_1 - C\cdot C_2}{S_1 - C\cdot S_2}\right] \tag{26}$$

Following examples demonstrate some forms of eq. (15) for give values g_1, g_2 and g_3:

$$g_1 = -\alpha,\ g_2 = 0,\ g_3 = \alpha,\ \varphi = \arctan\left[\left(\frac{1-\cos\alpha}{\sin\alpha}\right)\frac{I_3 - I_1}{I_3 - 2I_2 + I_1}\right] \tag{27}$$

$$g_1 = -\frac{\pi}{2},\ g_2 = 0,\ g_3 = \frac{\pi}{2},\ \varphi = \arctan\left[\frac{I_3 - I_1}{I_3 - 2I_2 + I_1}\right] \tag{28}$$

$$g_1 = -\frac{2\pi}{3},\ g_2 = 0,\ g_3 = \frac{2\pi}{3},\ \varphi = \arctan\left[\sqrt{3}\frac{I_3 - I_1}{I_3 - 2I_2 + I_1}\right] \tag{29}$$

$$g_1 = \frac{\pi}{4},\ g_2 = \frac{3\pi}{4},\ g_3 = \frac{5\pi}{4},\ \varphi = \arctan\left[\frac{I_3 - I_2}{I_1 - I_2}\right] \tag{30}$$

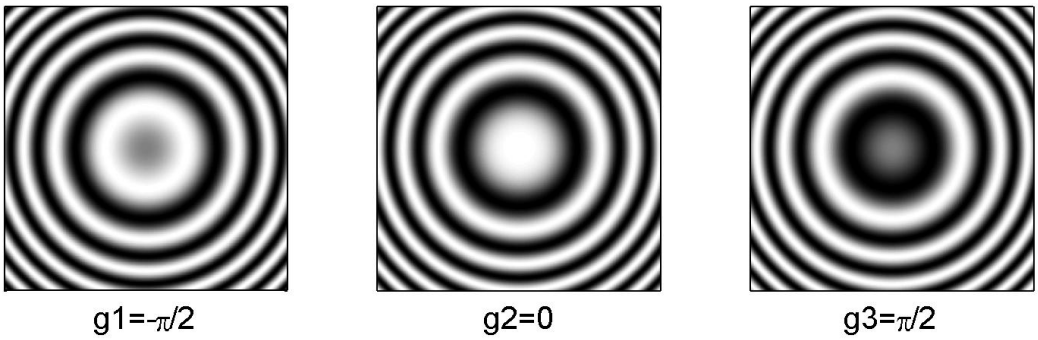

Fig. 3. *Interference fields in dependency on a phase shift g with coefficient $W_{20} = 3$*

3.2 Four - step algorithms N = 4

Like for three - step algorithms we can get many four steps-algorithms, e.g.

$$g_1 = 0,\ g_2 = \frac{\pi}{2},\ g_3 = \pi,\ g_4 = \frac{3\pi}{2},\ \varphi = \arctan\left[\frac{I_4 - I_2}{I_1 - I_3}\right] \tag{31}$$

$$g_1 = \frac{\pi}{4},\ g_2 = \frac{3\pi}{4},\ g_3 = \frac{5\pi}{4},\ g_4 = \frac{7\pi}{4},\ \varphi = \arctan\left[\frac{I_1 + I_2 - I_3 - I_4}{I_1 - I_2 - I_3 + I_4}\right] \tag{32}$$

3.3 Five-steps algorithms N = 5

$$g_1 = -\pi,\ g_2 = -\frac{\pi}{2},\ g_3 = 0,\ g_4 = \frac{\pi}{2},\ g_5 = \pi,\ \varphi = \arctan\left[\frac{I_2 - I_4}{2I_3 - I_5 - I_1}\right] \tag{33}$$

$$g_1 = -\frac{\pi}{2},\ g_2 = 0,\ g_3 = \frac{\pi}{2},\ g_4 = \pi,\ \varphi = \arctan\left[\frac{I_2 - I_4}{2I_3 - I_5 - I_1}\right] \tag{34}$$

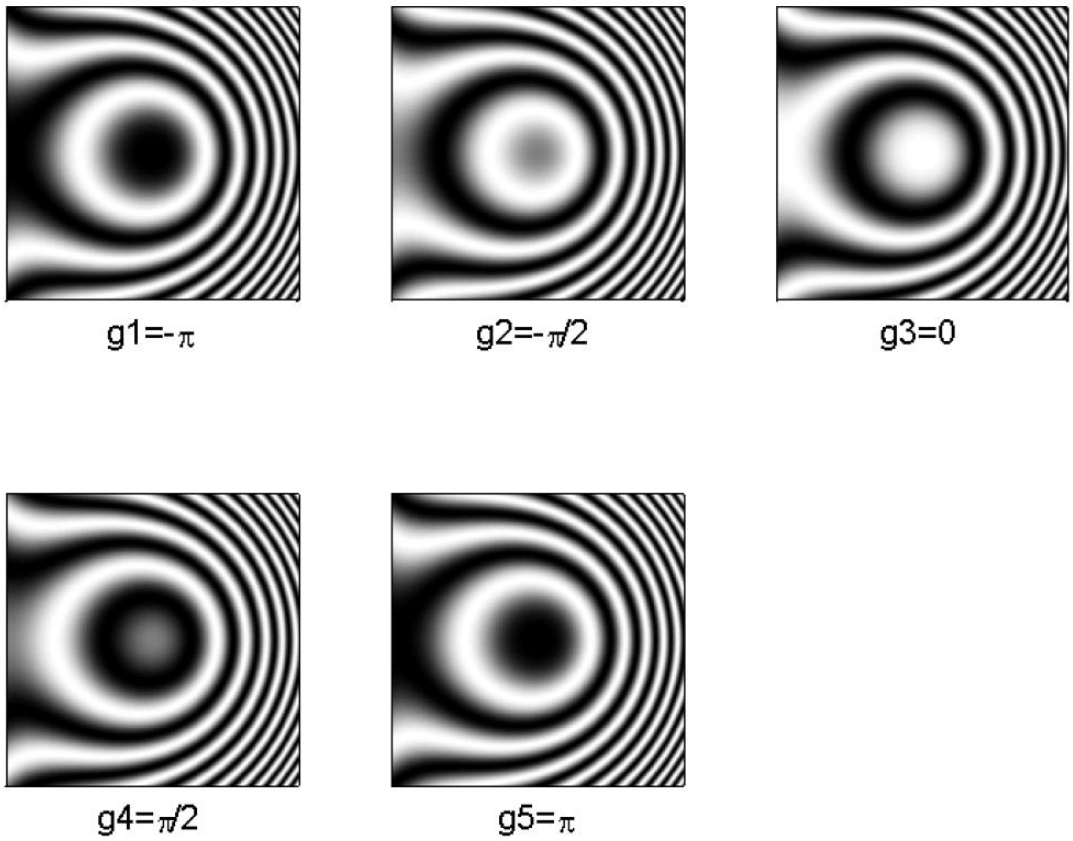

Fig. 4. *Interference fields in dependency on a phase shift with coefficients W_{20} = 3, W_{31} = 2*

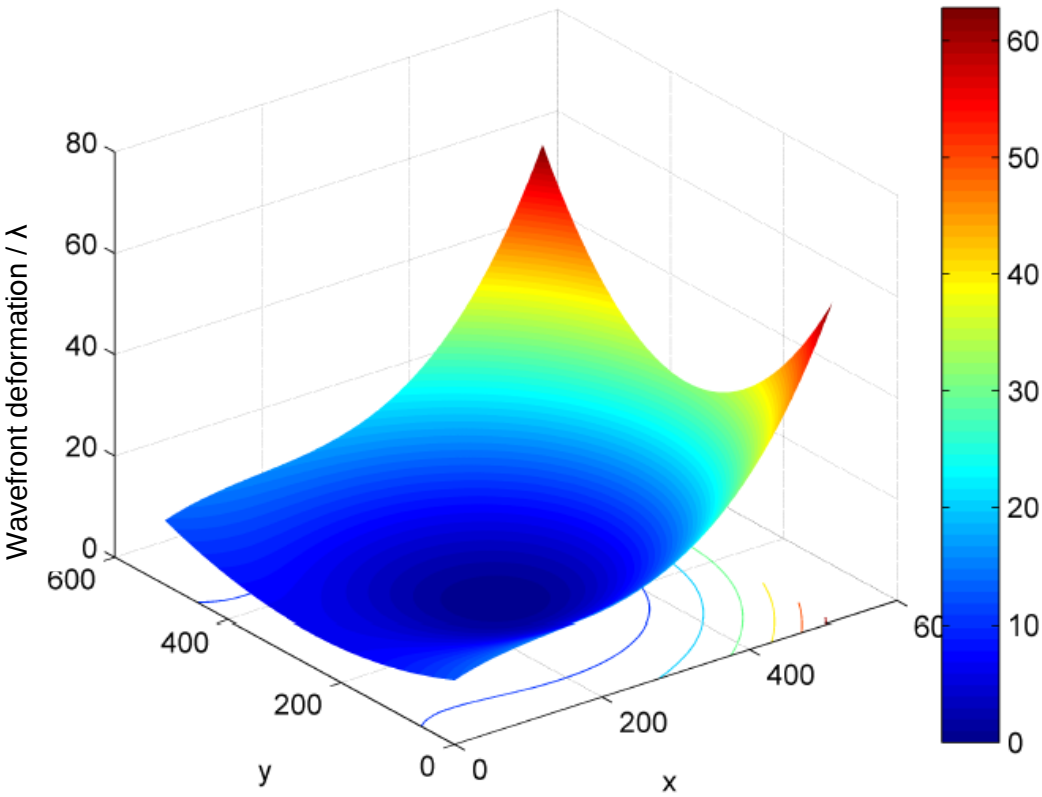

Fig. 5. *Interference fields in dependency on a phase shift g with coefficient W_{20} = 3*

3.4 Seven - step algorithms N = 7

$$g_1 = \frac{-3\pi}{2}, \qquad g_2 = -\pi, \qquad g_3 = -\frac{\pi}{2}, \qquad g_4 = 0, \qquad g_5 = \frac{\pi}{2}, \qquad g_6 = \pi, \qquad g_7 = \frac{3\pi}{2},$$

$$\varphi = \arctan\left[\frac{7(I_3 - I_5) - (I_1 - I_7)}{8I_4 - 4(I_2 - I_6)}\right] \tag{35}$$

$$g_1 = 0, \qquad g_2 = \frac{\pi}{3}, \qquad g_3 = \frac{2\pi}{3}, \qquad g_4 = \pi, \qquad g_5 = \frac{4\pi}{3}, \qquad g_6 = \frac{5\pi}{3}, \qquad g_7 = 2\pi,$$

$$\varphi = \arctan\left[-\sqrt{3}\,\frac{I_2 + I_3 - I_5 - I_6}{I_1 + I_2 - I_3 - 2I_4 - I_5 + I_6 + I_7}\right] \tag{36}$$

It would be possible to continue with another more steps algorithms but in praxis the sense is not very beneficial. The most important are algorithms with three and five steps which are enough precise in most of practical needs.

4. Experimental spherical surface shape measuring methods through the use of the Twyman – Green interferometer

The fundamental scheme of the Twyman – Green interferometer for spherical surfaces measurement is depicted in Fig. 6 [6]. Light from the source *S* placed in the objective O_1 object focal plane is by this objective transformed to a parallel pencil of rays (plane wave) that impacts on the beam splitter *Z*. The first part of light (reference wave) glances off the mirror and impacts on the reference plane mirror Z_R. Reference wave reflects from the reference mirror Z_R and passes through the beam splitter *Z* to the objective O_3. The second part of light passes through the beam splitter *Z* and impacts on the objective O_2 that transforms it to spherical wave. This spherical wave impacts on a measured surface. A centre of curvature of this measured surface is practically coincident with a centre of curvature of spherical wave coming out from the objective O_2. After reflection from measured surface spherical wave passes through the objective O_2, glances off the mirror *Z* and impact on the objective O_3. In this space these two waves interfere. Objective O_3 displays image of source *S* into the point *S″* coincident with an image focus of this objective. Behind the objective O_3 there is situated the objective O_4 whose entrance pupil centre is coincident with the point *S″*. Objectives O_3 and O_4 display point *A* (the point of an interference field situated on the reference surface of the mirror Z_R) into the plane of detector *D* as a point *A′*. The detector can record an interference field distribution in the form of interference fringes. Now for an interference field evaluation we can use one of the methods mentioned in the chapter 3. Reference field phase shift we acquire through the mirror Z_R movement (by the help of the piezo actuator).

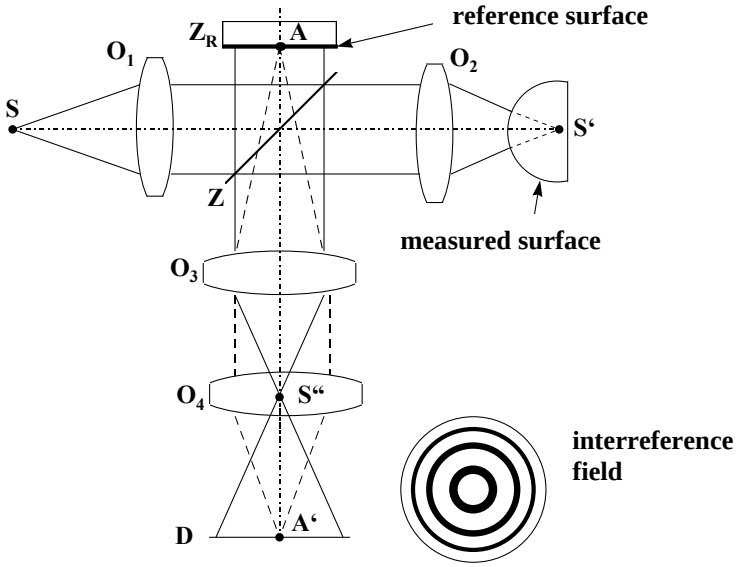

Fig. 6. *Twyman – Green interferometer principal scheme.*

4.1 The engineering design description

For an interference field evaluation we use the method of a phase shift in the reference branch. Manners for the phase evaluation are described in the section 3 and in [1, 2, 3]. For the reference phase shift we use a piezo actuator LISA P-753.1CD [4]. This piezo actuator is pc-controlled and customized operating software synchronized with camera image sensing software for an automatic interference fields snapping and evaluation was written.

The light source is the stabilized HeNe laser SL02/1 [5] from the SIOS Messtechnik GmbH Company, the used wavelength is $\lambda = 632{,}8nm$. Laser beam is via fibre optic link brought to the front focus of a collimating lens (*f’ = 75mm)*. The measuring beam diameter inside is 10 *mm* and an interference field is sensed by a CCD camera. One of the biggest advantage of this design are its small dimensions ($110x70x57mm$ without an objective).

Mirror holder in reference branch is 3-axis adjustable and mounted on the piezo actuator. Closed-loop travel of this actuator is $12\mu m$ and resolution $0{,}05nm$. The surface flatness of the reference mirror and the cube beamsplitter is $\lambda / 20$.

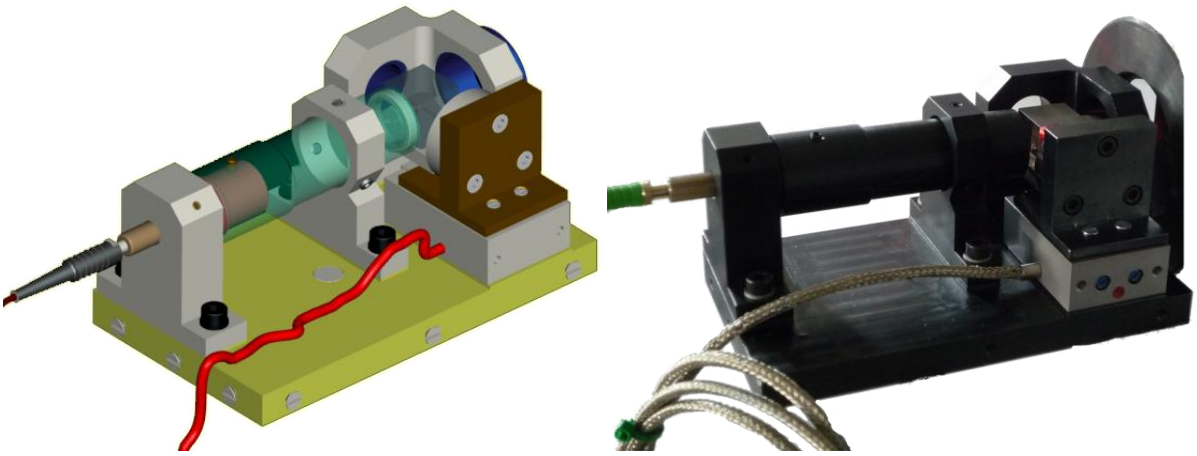

Fig. 7. *Twyman – Green interferometer with automatic interference field sensing and evaluation*

The piezo – actuator operating and CCD camera synchronizing software Nanoflea v1.0 is written in C++ programming language and for evaluation of interferograms and graphical representation is used the Matlab software.

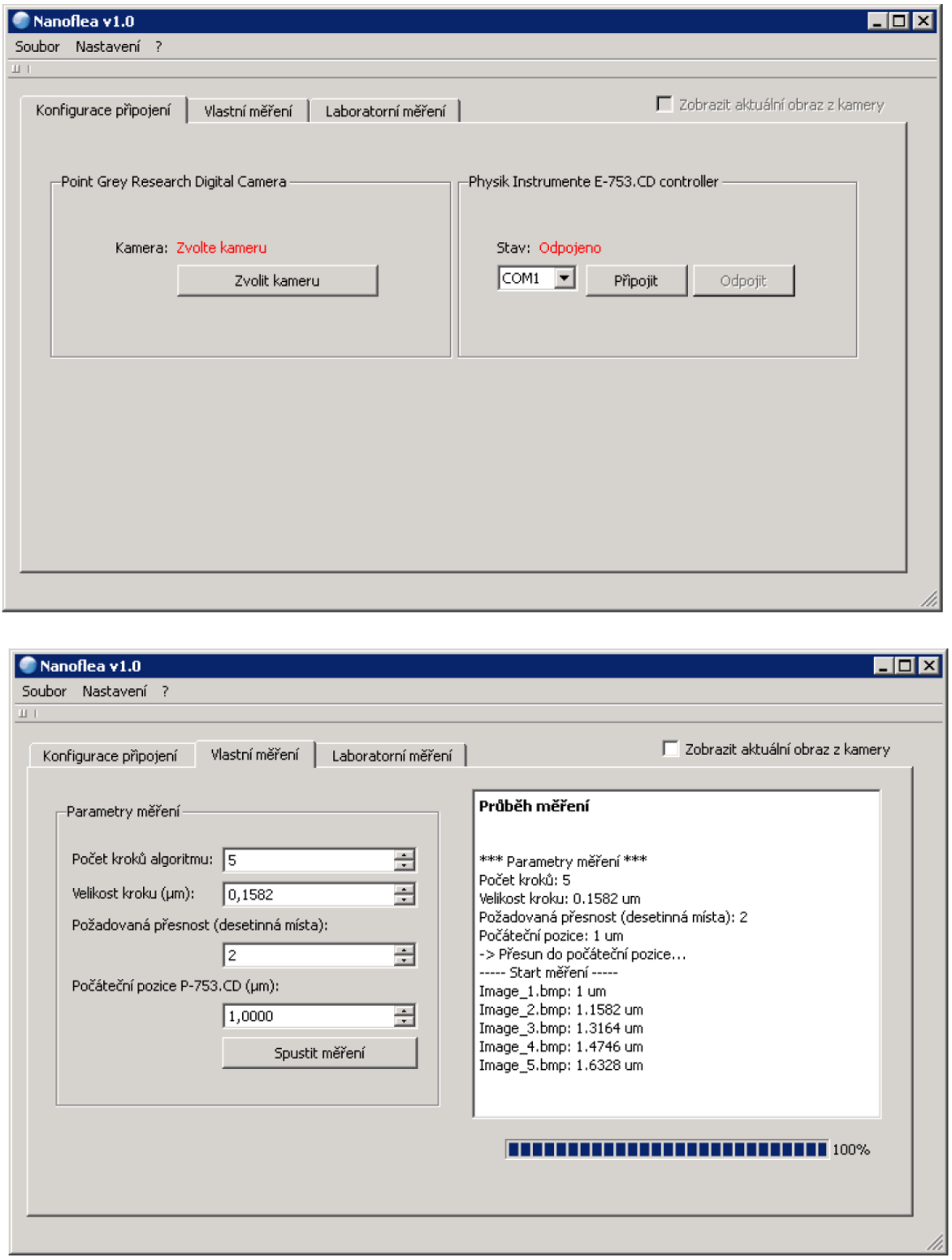

Fig. 8. *Operating software environment demonstration*

5. Conclusion

In this article there was described an engineering design of the Twyman-Green interferometer with full automatic measurement process. This enables automatic interferograms sensing (count of interferograms depends on currently used algorithm) with following evaluation of snapped interferograms in Matlab (including the wavefront shape graphical representation as well).

Acknowledgements

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Used symbols

φ	phase difference	[rad]
λ	wavelength	[μm]
W	path difference	[μm]
δ	initial phase	[rad]
e	adjacent interference fringes distance	[μm]
α	angle between two light beams	[rad]
I	light intensity	[arb.]
K	contrast	[1]
W_{1x}	x - axis tilt coefficient	[1]
W_{1y}	y - axis tilt coefficient	[1]
W_{20}	longitudinal defocusing coeff.	[1]
W_{40}	3rd order spherical aberration coeff.	[1]
W_{60}	5th order sph. aberr. coeff.	[1]
W_{31}	3rd order coma aberr. coeff.	[1]
W_{51}	5th order coma aberr. coeff.	[1]
W_{22}	3rd order astigmatism aberr. coeff.	[1]

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