Determining Resonance Frequencies of an Airplane Wing

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Abstract in Czech

Cílem projektu bylo experimentální stanoveni resonančních frekvencí křídla lehkého sportovního letounu. Resonanční frekvence je tá, při které i poměrně malá buzení způsobuje kmitání s velkými odchylkami (s velkou amplitudou), proto je děj resonance v mnoha případech nebezpečný. Abychom mohli nežadoucí resonanci zabránit, je potřeba nejprve zjistit při jakých okolnostech, zejména při jakých budicích frekvencích k ní dochází.
Výsledkem tohoto projektu jsou stanovená resonanční frekvence křídla i analýza provedených experimentů. Důraz je kladen na použití různých metod buzení konstrukce a porovnání výsledků, dosažených jednotlivými metodami.

Abstract in English

The goal of the proposed project was to determine experimentally resonance frequencies of a light sport airplane wing. Structure oscillations at resonance frequencies are often dangerous because relatively small excitation forces result then in large-amplitude vibrations, which can lead to failure of the structure. In order to prevent such resonance phenomenon, it is necessary to investigate under what conditions the resonance occurs.
The result of this project is determined resonance frequencies of the wing and analysis of conducted experiments. The emphasis is put on applying different excitation signals and comparing corresponding results.

Key concepts

Accelerometer, Aeroelasticity, Data Acquisition, Fourier Transformation, Frequency Response, Modal Testing, Natural Frequency, Noise Signal, Oscillation, Resonance, Vibrations.

1. Introduction

Resonance phenomenon is an important concept in engineering. In most cases this phenomenon is undesirable because it can lead to a fail of a structure. When engineers try to prevent it, they investigate under what circumstances the resonance occurs. More fundamentally it means finding at which frequencies the structure oscillates when in resonance. The next step to solve this problem is exploring dependencies between the structure's oscillation frequencies and its operational conditions (e.g. flight speed or Mach number, engine rpm or other cyclic loading, which makes the structure to oscillate). The simplest way to avoid the resonance is avoiding corresponding operational conditions (e.g. certain engine rpm range). However, it is often not possible for a number of reasons. In such case, the structure design changes are necessary. This is why vibration phenomena should be taken into consideration while developing a certain mechanical structure that will be subjected to cyclic loading.

1.1 Project objectives

The primary goal of the proposed project was to determine resonance frequencies of the wing. The secondary goal was to try various excitation signal types and determine how big the influence of a signal type on the result is.

2. Approach and technical background

With regard to the complexity of the system, it was expected that the wing had more than one resonance frequencies, so the careful analysis had to be performed. The approach was experimental and the technique that we used is called Modal Testing. It implies introducing the structure to an excitation from a vibration source (called exciter or shaker) and simultaneous measurement of the structure's response to this excitation. The shaker's frequency range was set to vary from 1 Hz to 100 Hz, which corresponds to a common oscillation range for a light airplane. A typical setup of such experiment is displayed in figure 1 below.

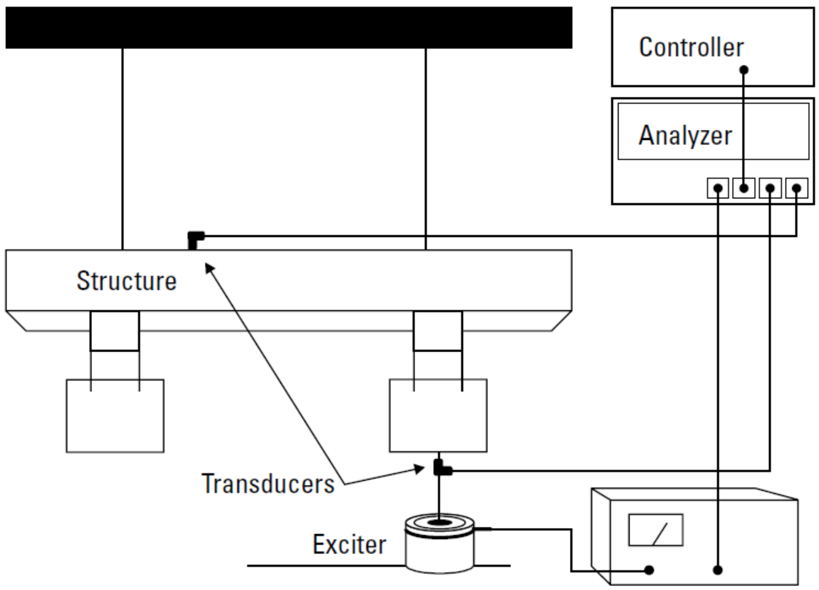

Fig. 1. Typical experiment setup for the Modal Testing.

Fig. 1 shows that the structure (in the centre) is being excited by an exciter (centre bottom), which is controlled by a controlling device (up right corner). Often the controlling device and analyzer are integrated into one piece of hardware, as it was in our case too. The response measurement, performed by acceleration transducers is described closer in the following section. Often, the most challenging and demanding part of such experiments is processing and evaluation of the measured data.

2.1 Accelerometers

Accelerometers, marked in the fig. 1 as transducers, are attached to the surface of the structure (fig. 2 and 3). Also, one sensor is attached to each exciter (fig. 4). The accelerometers measure acceleration, which is the way to quantify the structure's response. Various accelerometers might employ different principles. The most common one is based on the piezoelectric effect. Usually, many sensors are placed over the structure, which allows measuring with higher precision and determining so called mode shapes of oscillation, which are closer discussed in section 7.3 located in the attachment to this paper.

2.2 Shakers

In our experiments, the shakers were sitting on adjustable tripods (fig. 2 and 4) and were screwed to the wing for the best attachment via a thin, strong steel road (fig 4). Each shaker had an additional accelerometer (called a load cell in this case) attached to it in order to quantify the loading applied to the structure by an individual shaker. Unlike the other sensors that measure response in m/s^2, the load cells are calibrated to measure in Newtons since their purpose is to evaluate the load input to the structure. Otherwise they function as typical accelerometers.

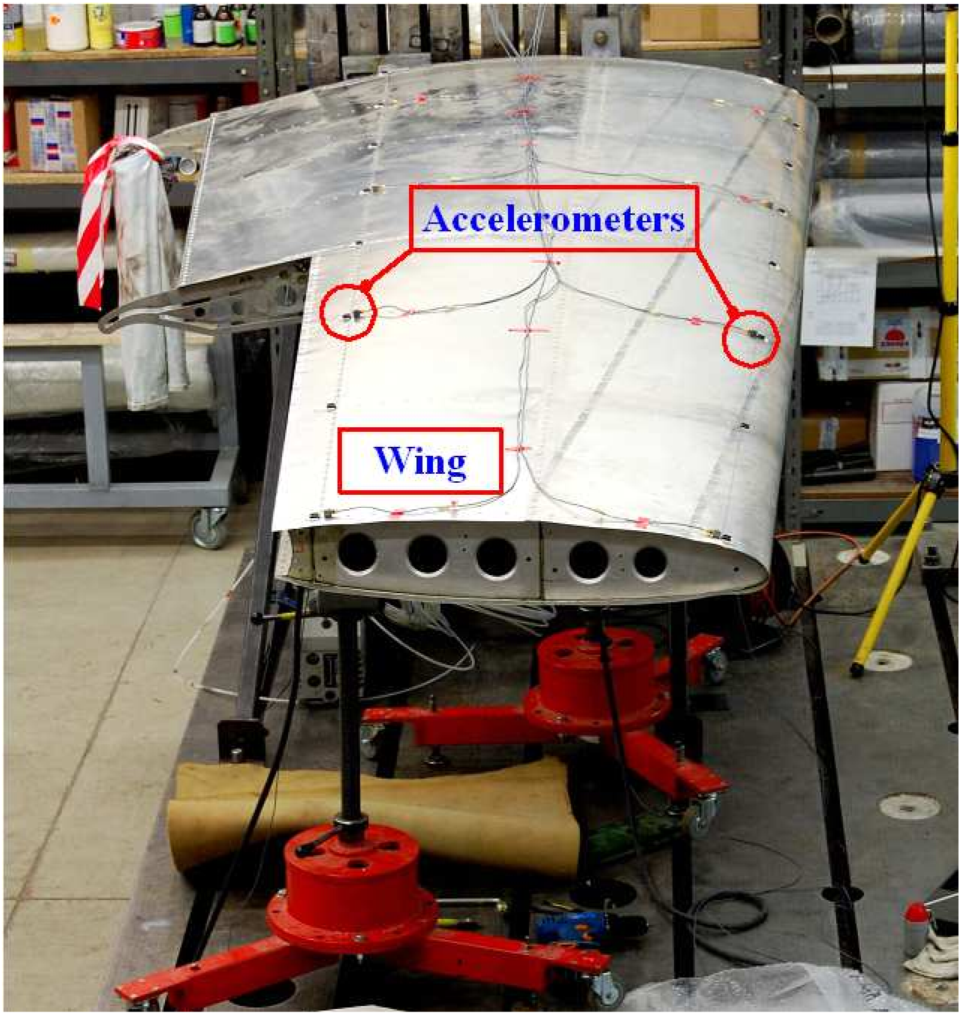

Fig. 2. Wing with attached accelerometers and shakers.

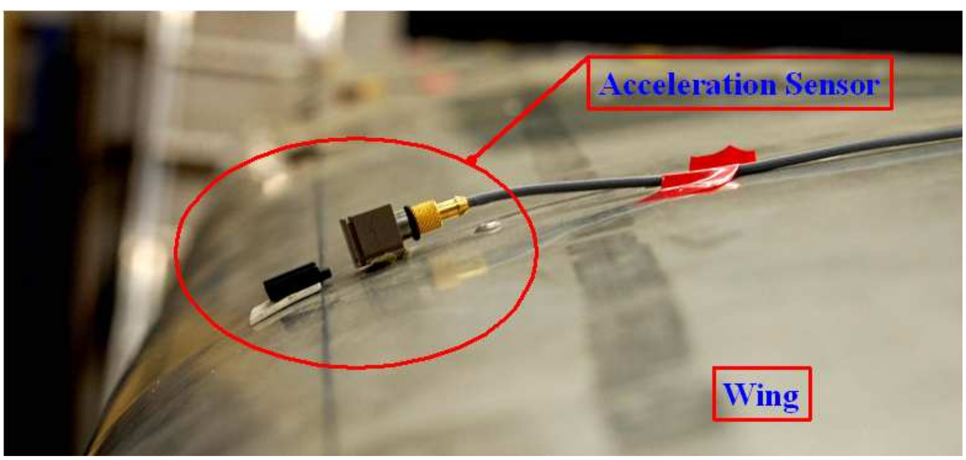

Fig. 3. An accelerometer attached to the wing.

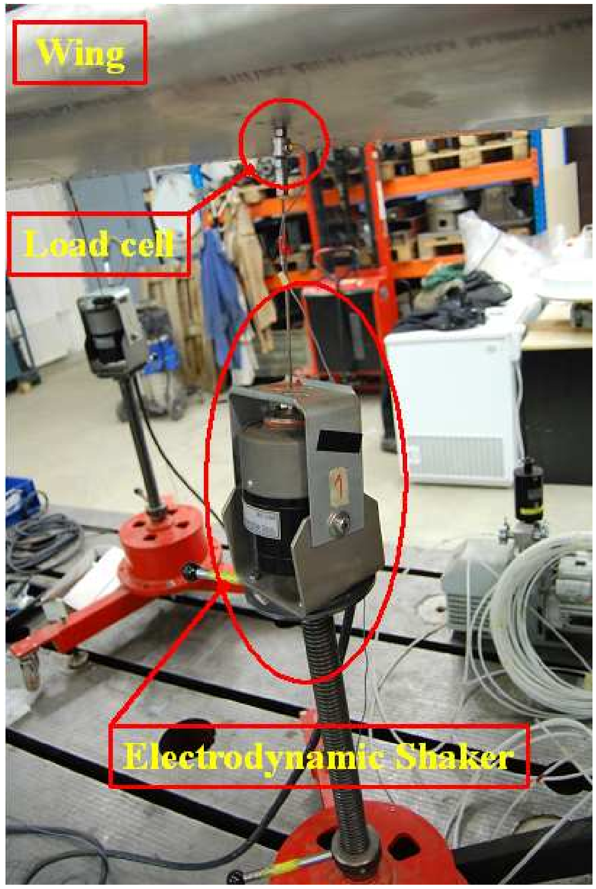

Fig. 4. Shaker attached to the wing via a thin strong rod with a sensor.

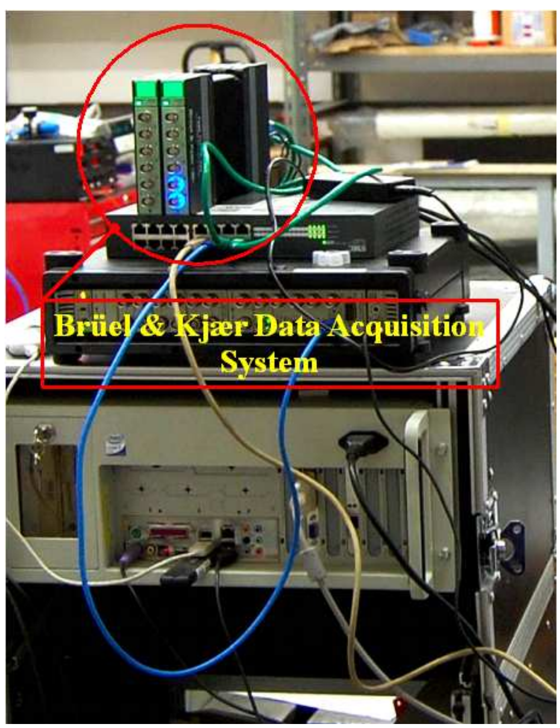

Fig. 5. Brüel & Kjær LAN-XI Data Acquisition.

2.3 Controller and Analyzer

The controller and analyzer functions were accomplished by a special purpose high-frequency data acquisition unit from Brüel & Kjær Company (fig. 5), which was used to control the shaker and gather experiment data. This state-of-the-art device, designed specifically for Modal Testing was kindly lent by the company to the Aerospace Department Lab for several days to perform the presented experimental work. Further data processing was conducted on a computer in ME'ScopeVES program, particularly developed for analyzing and processing frequency tests data. The program and method of data processing is closer discussed in section 5.

3. Excitation signals

The secondary project goal was to determine the influence of an excitation signal type on the structure's response. Various excitation signals were applied. Specific signal types are listed in table 1 below. All the signals with exception of those called "external" were generated by the data acquisition system described in section 2.3. The external white and pink noises were generated by a separate device "Minirator MR1" from NTI AG Company. This device was functioning as a signal generator of either white or pink noise and was used as an alternative signal source to the Brüel & Kjær control unit.

Table 1. – Excitation signal types and their further modifications.

Signal type	Swept sine	Random	Pseudo random	Burst
Further signal modifications	6.133 Hz/s	White noise	White noise	4 sec + 4 sec
	3.156 Hz/s	Pink noise	Pink noise	2 sec + 2 sec
	12.625 Hz/s	Periodic random		1 sec + 1 sec
		External white noise		0.5 sec + 0.5 sec
		External pink noise		

Excitation signals listed were controlling the shaker directly. For example, if the signal type was Random white noise, it means that the shaker was exciting the structure with random frequencies in the range of (0 ÷ 100) Hz. There were also other modifications of Random signal applied (pink, periodic and external noises). The following part of section 3 briefly reviews these various signal types.

3.1 Swept sine

Swept sine is a type of a sinusoidal signal whose frequency gradually changes with time, while it is possible to set the speed of this change in advance. In the swept sine that we used, the frequency was increasing with time. This change in frequency was characterized by the speed of change in Hz per second. Table 1 indicates that we used three different modifications of the swept sine that had different speeds of the frequency increase (6.133, 3.156 and 12.625 Hz/second).

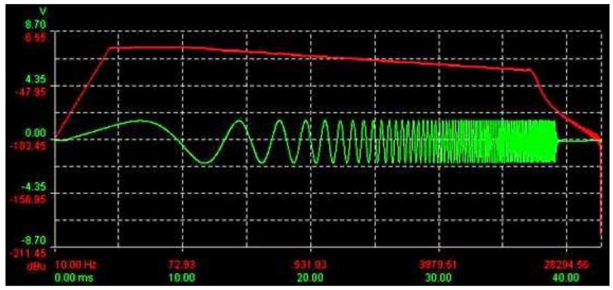

Fig. 6. Swept sine signal.

3.2 White noise

White noise is a type of a random signal (fig. 7), which means that its frequencies are not orderly arranged, e.g. it may start with a 6 Hz-frequency and the next moment jump to 51 Hz. The peculiarity of white noise is that it has an equal intensity of all frequencies within an interval of (0 ÷ 20) kHz, or in other words, all frequencies are equally represented in this type of signal (fig. 8). In our experiment however, only the interval of (0 ÷ 100) Hz was used since it was the range of frequencies we were interested in.

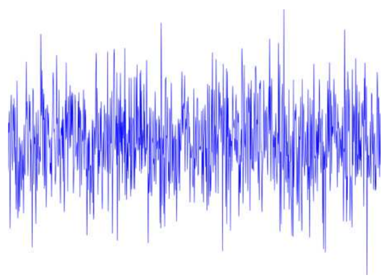

Fig. 7. Random noise signal.

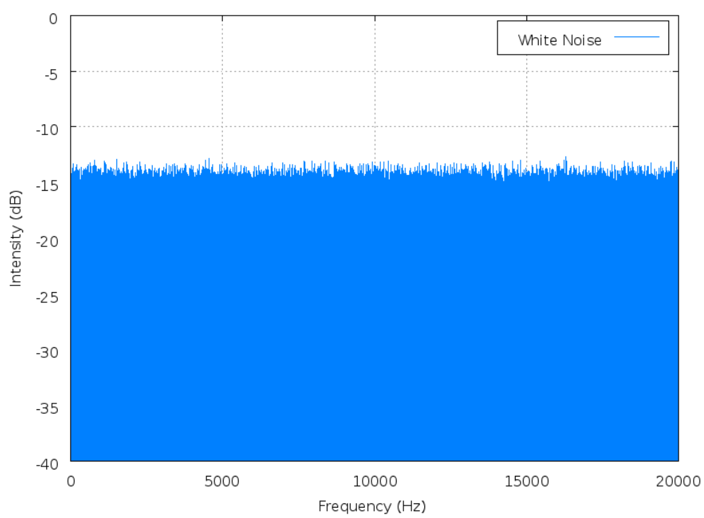

Fig. 8. White noise Frequency vs. Intensity distribution (constant with time).

3.3 Pink noise

Pink noise differs from white by its variable intensity of frequencies. In the pink noise, lower frequencies are represented stronger (this pattern is depicted in fig. 9). This is why pink noise feels heavier that the white; it is analogous to acoustic sound, when lower frequencies often cause heavier disturbance than higher frequencies.

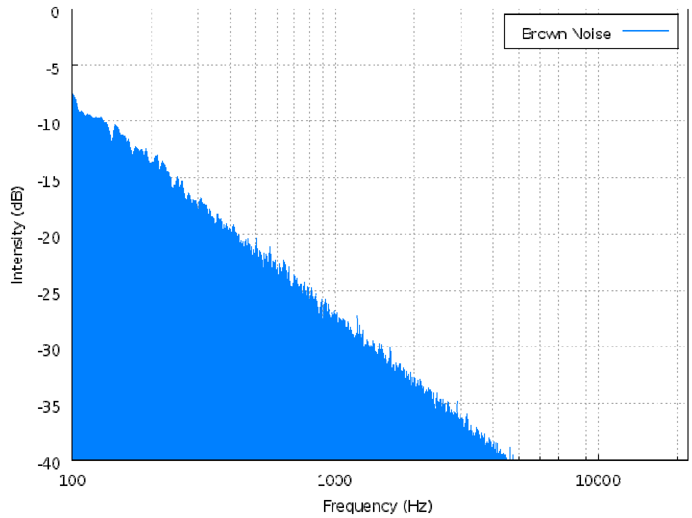

Fig. 9. Pink noise Frequency vs. Intensity distribution (non-constant with time).

3.4 Periodic random noise

Periodic random noise is a summation of sinusoidal signals with the same amplitudes but with random phases. So, the signal itself has some sort of a systematic headway. As in case of white noise, all frequencies are equally represented.

3.5 External white and pink noises

These were normal white and pink noises generated however, by a separate device “Minirator” – instead of the Brüel & Kjær control unit, as described in section 2.3 and the introduction to section 3.

3.6 Pseudo random noise signal

Pseudo random signal consists of a fragment of either white or pink noise, which repeats itself. This type of signal is very similar to normal noise; the only difference is that pseudo random signal has the described above pseudo periodicity.

3.7 Burst signal

Burst signal is formed by a fragment of white noise that repeats itself after a certain dwell time. During the dwell, no signal is being generated. For example, the modification “4 sec + 4 sec” means that the random signal lasts for 4 seconds, and then the dwell lasts for 4 seconds as well. Then this block of signal and the dwell is repeated.

4. Measurements

All measurements were organized into two groups called Experiment 1 and Experiment 2 (tables 2 and 3). The former consisted of six measurements and the latter of eight, where each measurement was different in terms of excitation signal type and its characteristics. The measurements were conducted in a raw, one after another, on exactly the same equipment and same testing parameters. The result of the measurement was recorded for further processing. These results included excitation force and corresponding response of the structure to this excitation as a function of time. During further signal processing, the time domain was converted into frequency domain (more about data processing in section 5.1).

Table 2. – Experiment 1 measurements.

Experiment 1		
Measurement number	**Signal type**	**Signal characteristics**
1	Swept sine	6.313 Hz/s
2	Swept sine	3.156 Hz/s
3	Swept sine	12.625 Hz/s
4	Random	White noise
5	Random	Pink noise
6	Pseudo random	White noise

Table 3. – Experiment 2 measurements.

Experiment 2		
Measurement number	**Signal type**	**Signal characteristics**
1	Pseudo random	Pink noise
2	Periodic random	White noise
3	Burst	4 sec + 4 sec
4	Burst	2 sec + 2 sec
5	Burst	1 sec + 1 sec
6	Burst	0.5 sec + 0.5 sec
7	External	White noise
8	External	Pink noise

5. Data processing

Data processing was performed according to a modern approach widely used in industry, which is by means of Modal Testing software. The data were processed on a computer equipped with ME'ScopeVES program (version 4.0.0.84 from year 2006), developed by Vibrant Technology Company. This program was designed for a complete processing of the frequency tests data, starting with pre-processing data arrangements and ending with specific results in terms of calculated resonance frequencies and damping. The algorithms and mathematical background of these calculations are rather complicated and will not be discussed in the proposed paper (more information can be found in reference literature, mainly [1], [2] and [5]). Basic calculation steps and their order are discussed in several following sections.

5.1 Theoretical background

The first step in the data processing was performing pre-calculation. Since the response was recorded as a function of time, this calculation included transferring the measured response from time to frequency domain with a use of Fourier transformation. A simple example of such time-to-frequency transformations is illustrated in fig. 10.

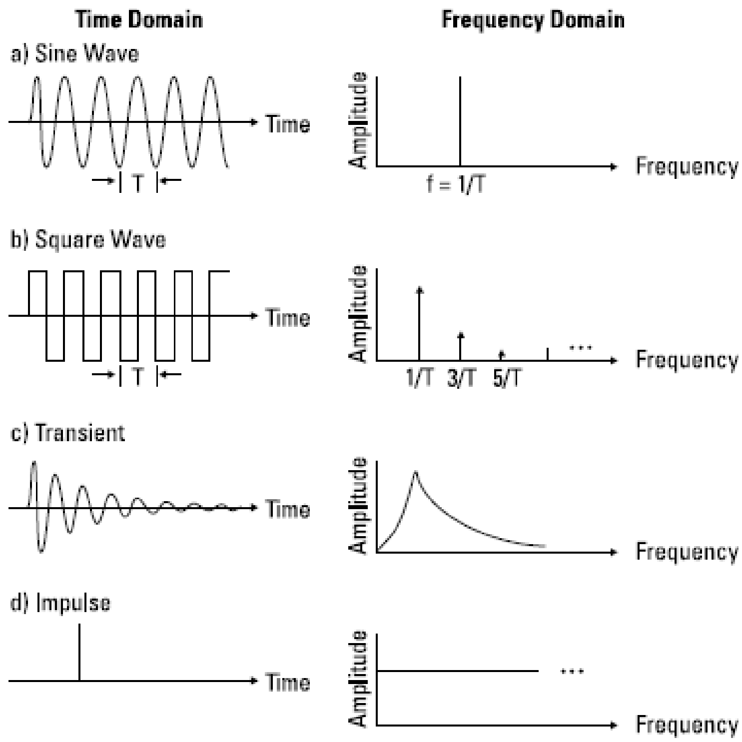

Fig. 10. Fourier transformation (time domain to frequency domain).

In this paper, measured response means the ratio of acceleration, measured by an accelerometer, and force, generated by an exciter at a certain time instant. Hence, if applied force is small but it still causes large accelerations of the structure (and therefore large deformations), the acceleration to force ratio has a greater magnitude, which indicates higher response. Such instances are then reflected in form of peaks in the frequency-response diagram, as illustrated in fig. 10).

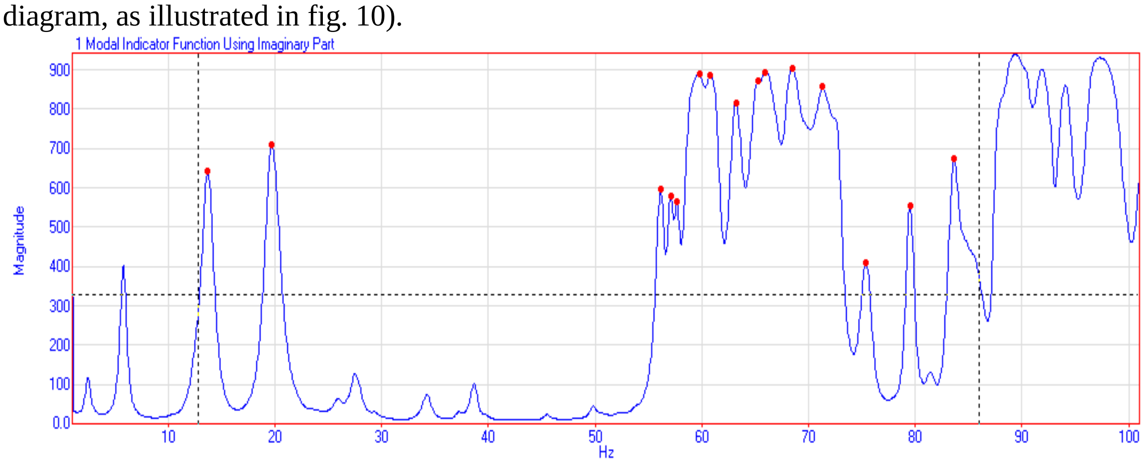

Fig. 11. Diagram of Frequency vs. Response vector magnitude (peaks indicate resonance condition).

Essentially, this is the principle of determining resonance frequencies. These are the frequencies at which small excitation forces cause large response (large amplitudes of oscillation). In the frequency-response diagram, these points are located at peaks. The steeper is the peak, the more distinct and the stronger is corresponding resonance frequency (fig 12).

Strong resonance phenomenon (fig. 12a) is also characterized by low damping. On the other hand, if the damping in the structure is large, the resonance is not severe (fig. 12b).

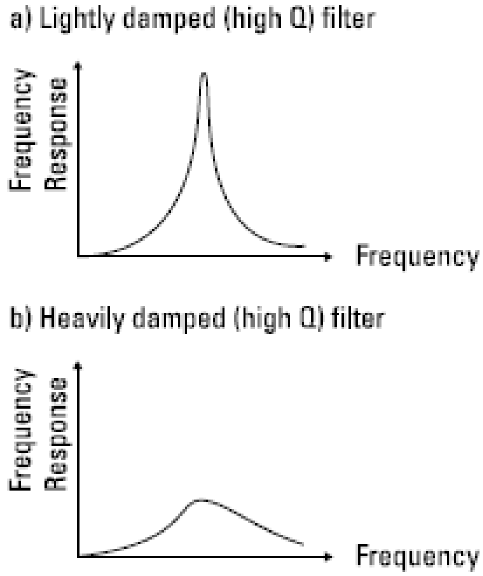

Fig. 12. Strong (a) vs. Light (b) resonance phenomena.

5.2 Processing software

A simple model of the wing (fig. 13) was created in the software used for data processing, ME'ScopeVES program (user interface is displayed in fig. 14). This program was also capable to split the response vector into real and imaginary components. The theory proves that when resonance occurs, the imaginary component has its maximum value, while the real component is zero. This property of the response vector was very useful in tracking resonance frequencies (upper graph in fig. 14). We could pre-determine potential resonance frequencies by manually analyzing all components of the response vector: real, imaginary and magnitude.

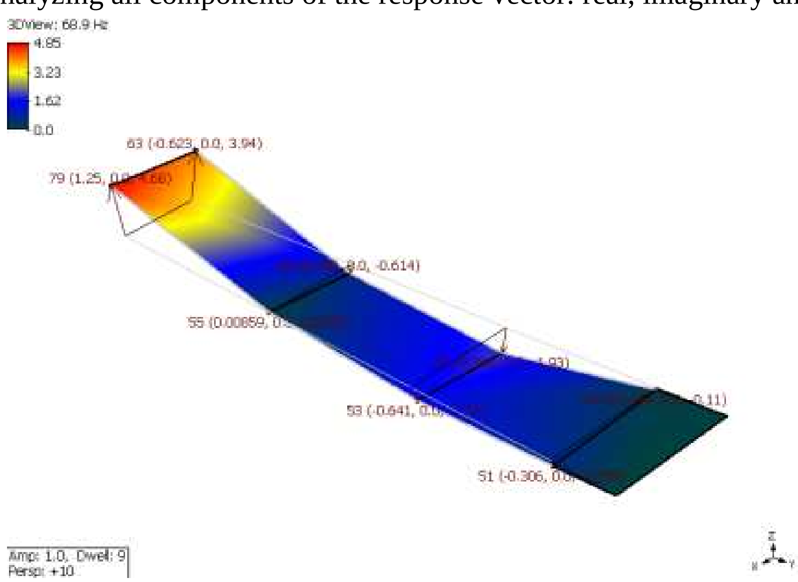

Fig. 13. ME'ScopeVES model of the wing and its response for 68.9 Hz.

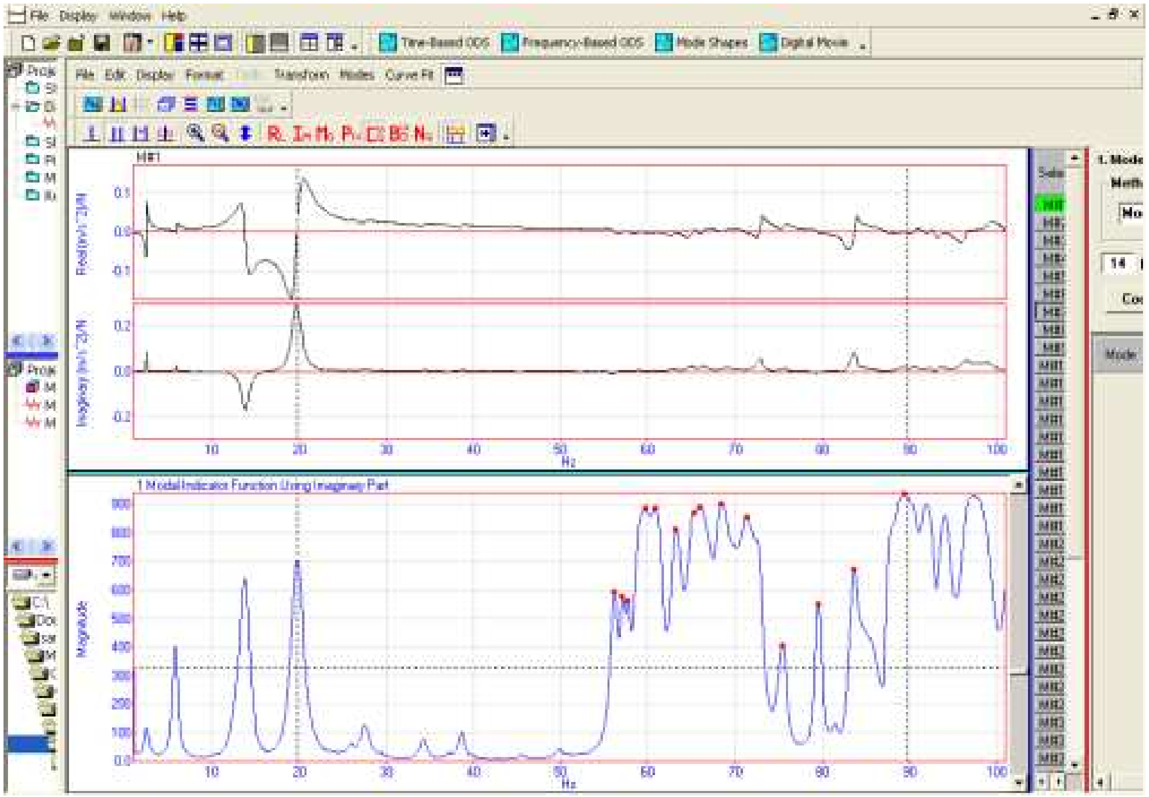

Fig. 14. ME'ScopeVES data processing (graph of Imaginary part vs. Frequency is on top, and Magnitude vs. Frequency is on the bottom).

The program could not only visualize the data, but also contained a code for evaluating resonance frequencies numerically. However, it was important to assist the program in doing this in order to correct possible uncertainties (the program was sometimes marking as resonance those frequencies that were not such in fact). The manual control over the computational process was needed.

6. Results

By the approach described in section 5, each measurement in both experiments was carefully analyzed and resonance frequencies were determined. These frequencies, however, were not always exactly the same for every measurement. The difference was usually in the order of tenths of Hertz, which is a small number for most mechanical applications. Table 4 lists the averaged resonance frequencies of the wing. The following, seventh section, offers detailed results analysis.

7. Analysis of the results

As it was mentioned in section 6, resonance frequencies obtained from different measurements slightly varied in value. Also, not every of the eleven frequencies in table 4 were represented in each measurement. As an example, the results from two measurements, Experiment 1 Measurement 3 (E1 M3) and Experiment 2 Measurement 4 (E2 M4) are listed in tables 5 and 6, respectively.

Table 4. - Final results: resonance frequencies.

Resonance Frequencies		
Mode	Frequency (Hz)	Damping (%)
1	2.47	1.53
2	5.95	1.55
3	13.66	3.01
4	19.58	3.23
5	62.33	0.38
6	66.37	0.5
7	68.96	0.66
8	71.53	0.37
9	73.06	0.44
10	83.74	0.53
11	96.35	0.62

Table 5. - Results of Measurement 3 from Experiment 1 (E1 M3).

Resonance Frequencies		
Mode	Frequency (Hz)	Damping (%)
1	2.47	1.53
2	5.95	1.6
3	13.6	3.16
4	19.6	3.28
5	62	0.636
6	65.3	0.154
7	66.3	0.55
8	68.8	0.553
9	73	0.5
10	83.8	0.587
11	96.9	2.67

The colors are used to link similar frequencies in these two measurements, which helps to recognize that not all frequencies were matched but only nine of them. For example, the 7th frequency in E2 M4, which is 69.7 Hz, does not occur in E1 M3 as resonance frequency. However, since it occurs in most of the other measurements, is present in the final results table (listed under number 7). Sometimes it happened that a resonance frequency resulting from a particular measurement was not present in most of the other measurements (e.g. 11th frequency from E2 M4, which is 94 Hz). In such case, this frequency was not included in the final results.

Table 6. – Results of Measurement 4 from Experiment 2 (E2 M4).

	Resonance Frequencies	
Mode	Frequency (Hz)	Damping (%)
1	2.46	1.18
2	5.94	1.12
3	13.8	3.84
4	19.6	3.57
5	62.7	0.122
6	65.3	0.202
7	69.7	1.31
8	73.2	0.478
9	79	0.674
10	83.5	0.585
11	94	0.526
12	96.3	0.558

7.1 Similarities in the results

As it is evident from comparison of tables 5 and 6, the frequencies in the range of (0 ÷ 20) Hz are almost identical for both measurements. The same can be noted about the rest of the measurements: similarities in the results for lower frequencies are higher. Repeated analysis of fig.11 leads to the conclusion that resonance frequencies in the range of (0 ÷ 20) Hz are more distinct and are situated far from each other, while in the range of (50 ÷ 75) Hz there are many peaks located densely over a short frequency span. Hence, in the range of (50 ÷ 75) Hz, the frequencies interfere with each other greater, and this is why it is harder to work precisely in this range. This also causes the results for this range to be different for different measurements.

Fig. 15 and 16 present the deviations of individual resonance frequencies of each measurement from the corresponding mean (nominal) frequency from the final results. This deviation is calculated according to equation 1 shown below. It is notable that deviations in computed resonance frequencies do not exceed 1.2%.

$$Deviation(\%) = \frac{|Measured\ Frequency - Mean\ Frequency|}{Mean\ Frequency} \times 100 \qquad [Equation\ 1]$$

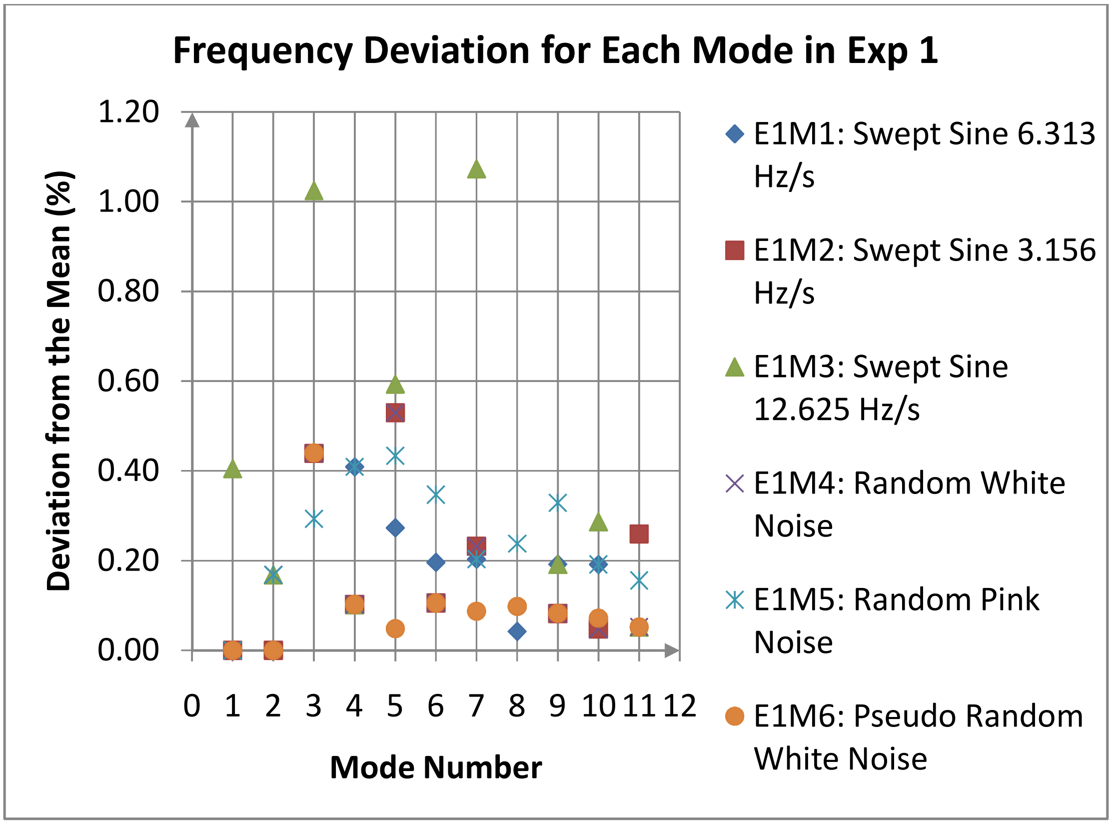

Fig. 15. Frequency deviation from the mean for Experiment 1.

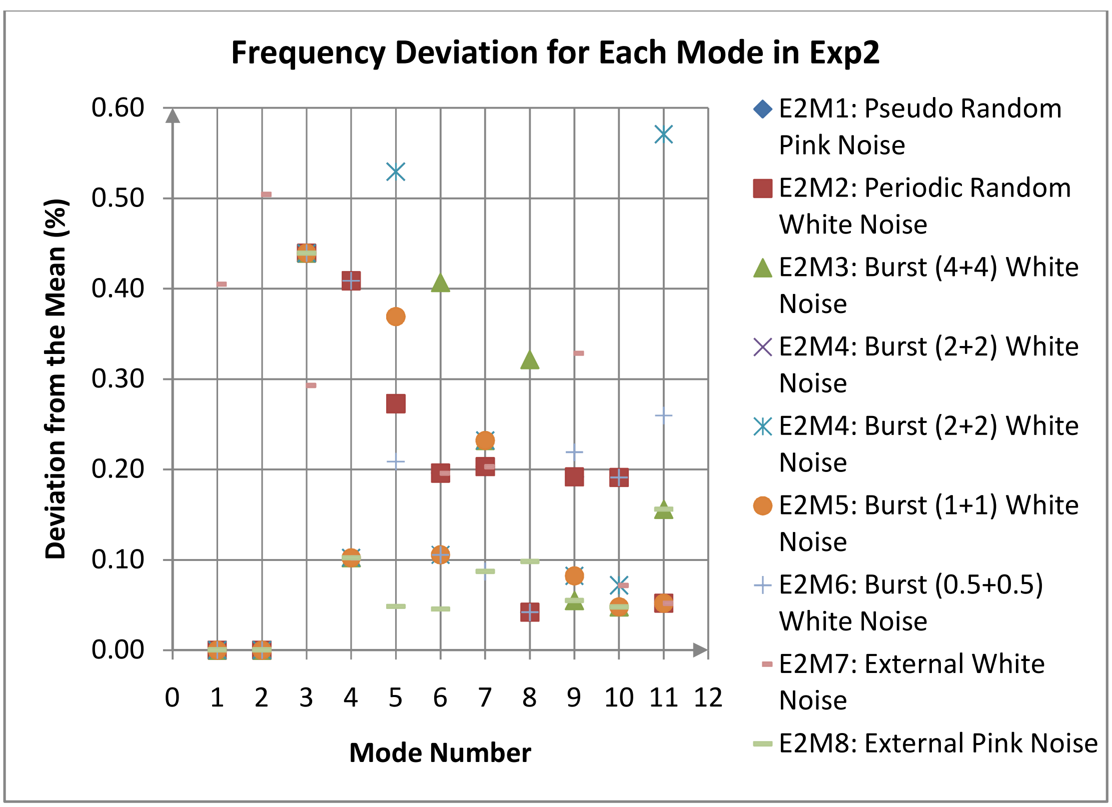

Fig. 16. Frequency deviation from the mean for Experiment 2.

7.2 Non-linearities in the system

An airplane wing is a complex structure consisting of many parts. Such structures tend to demonstrate non-linear behavior, which means that their response is influenced by interactions between individual structure components. The source of such interactions can often be allowances – either designed or caused by imperfect assembling of the structure. The wing we worked on had eight ribs and many rivets, so some undesired allowances were definitely present, causing non-linearity of the system.

Non-linear behavior usually does not occur while the structure is being excited by low frequencies but becomes stronger at medium and then might disappear again at very high frequencies. This behavior is also relevant in our case, which can be noted from fig. 11: non linear behavior, characterized by high density of resonance peaks, is present in the frequency range of (50 ÷ 75) Hz.

8. Conclusion

The primary goal of the project, which was determining resonance frequencies of the structure, was successfully achieved (the results are listed in table 4). The analysis of the experiment outcomes pointed out similarities in the results that were obtained by using different excitation signals: all of the applied signal types led to very similar results, which means that for such experiments, a signal type does not have an influence large enough to be taken into consideration. This conclusion satisfies the secondary project goal.

Another conclusion concerns the oscillation mode shapes. Those were different for all modal frequencies (some mode shapes are listed in section 7.3 in attachment), so in order to fully determine a resonance condition, three parameters are needed: resonance frequency, damping and a mode shape.

Also, extensive hands-on experience of working with instrumentation and software in the lab was acquired, as well as theoretical background in mechanical resonance and Modal Testing.

Overall, this project has greatly contributed to its participants' engineering education, their professional and personal skills, and encouraged them to continue staying involved in research activities of the Mechanical Engineering Department of CTU in Prague.

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ATTACHMENT

7.3 Mode shapes

A mode shape is a way the wing oscillates, which is determined by a number of nodes of oscillation and their location. In other words, a mode shape describes the curvature of vibration. As part of the results analysis (section 7 of the paper), mode shapes for each modal frequency were determined manually because the software was not efficient in making quantitative comparison of different shapes, even though it could visualize the individual shapes (fig. 13).

Several modal shapes are displayed below (each graph corresponds to one resonance (modal) frequency). On the *x*-axis there is a dimensionless wing span, and the *y*-axis represents so called Normalized deflection amplitude, which is a way to measure the wings response and is a ratio of the wing's response in terms of acceleration, and the applied excitation force. Since the sensors (accelerometers) were placed on the edges of the wing, the value of Normalized deflection was recalculated with respect to the so called ¼ point, which is a point that lies ¼ chord back from the leading edge. This normalized deflection magnitude is proportional to the wing's deflection. On each graph below, the wing is shown in the position of the maximum value of the deflection magnitude which allows comparing quantitatively the mode shapes with each other. Beginning of the coordinates corresponds to the wing root.

It is notable that certain measurements (fig. 2 and 4) showed contrastly different mode shapes at certain frequencies. For example, Measurement 3 from Experiment 1, whose mode shape looks distinctly different from the other shapes in fig. 2, did not differ incase of other frequencies. The same can be concluded about both E1 M1 and E2 M1, whose shapes in fig. 4 (i.e. for 83.74 Hz) look odd, but for other frequencies they do not deviate greatly from the mean. This is why we classify such occasional deviations as resulting from random measurement imperfections. Nevertheless, deviations in resonance frequencies, which are of much higher importance, do not exceed 1.2%, characterizing achieved results as solid and reliable.

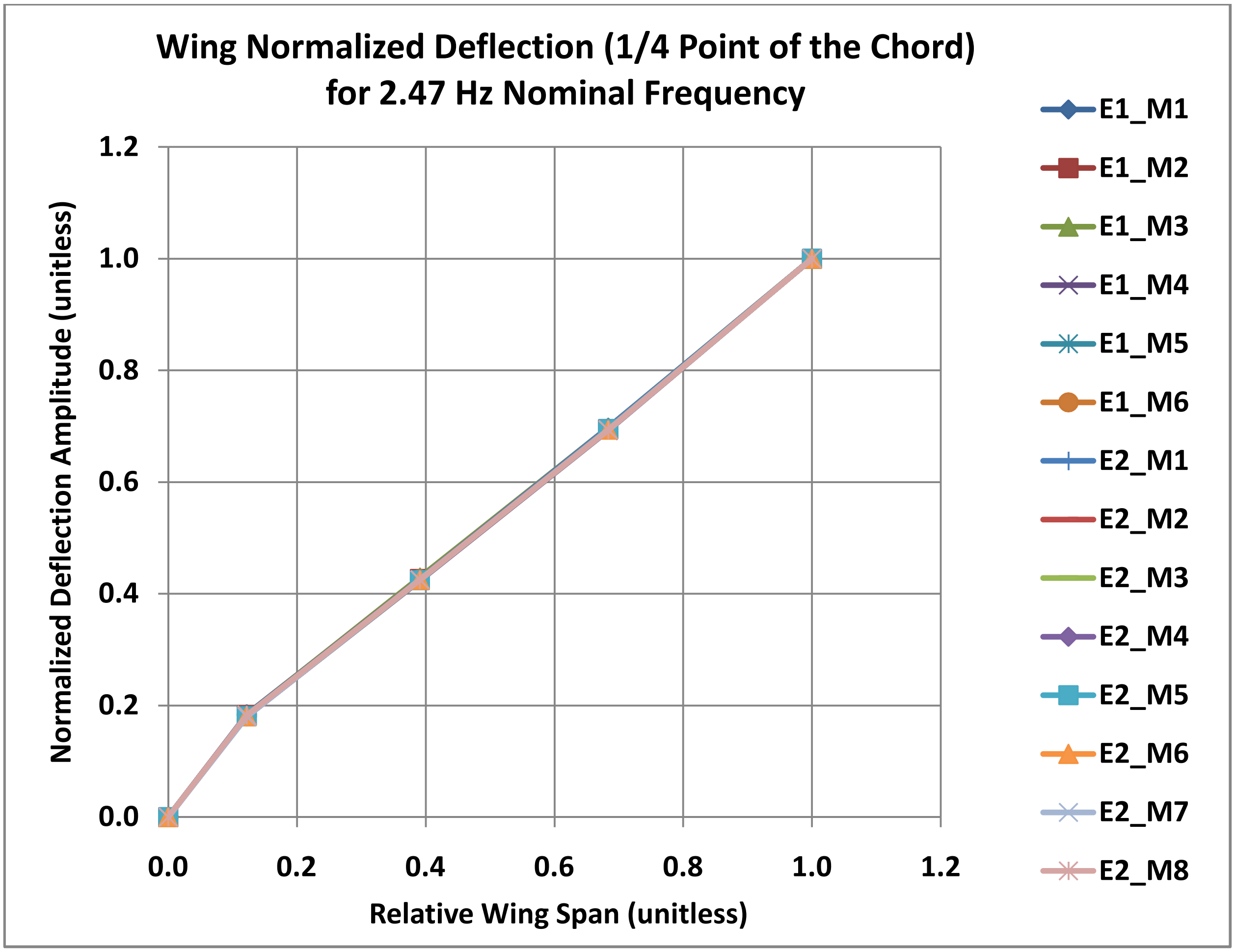

Fig. 1 Normalized deflection for 2.47 Hz.

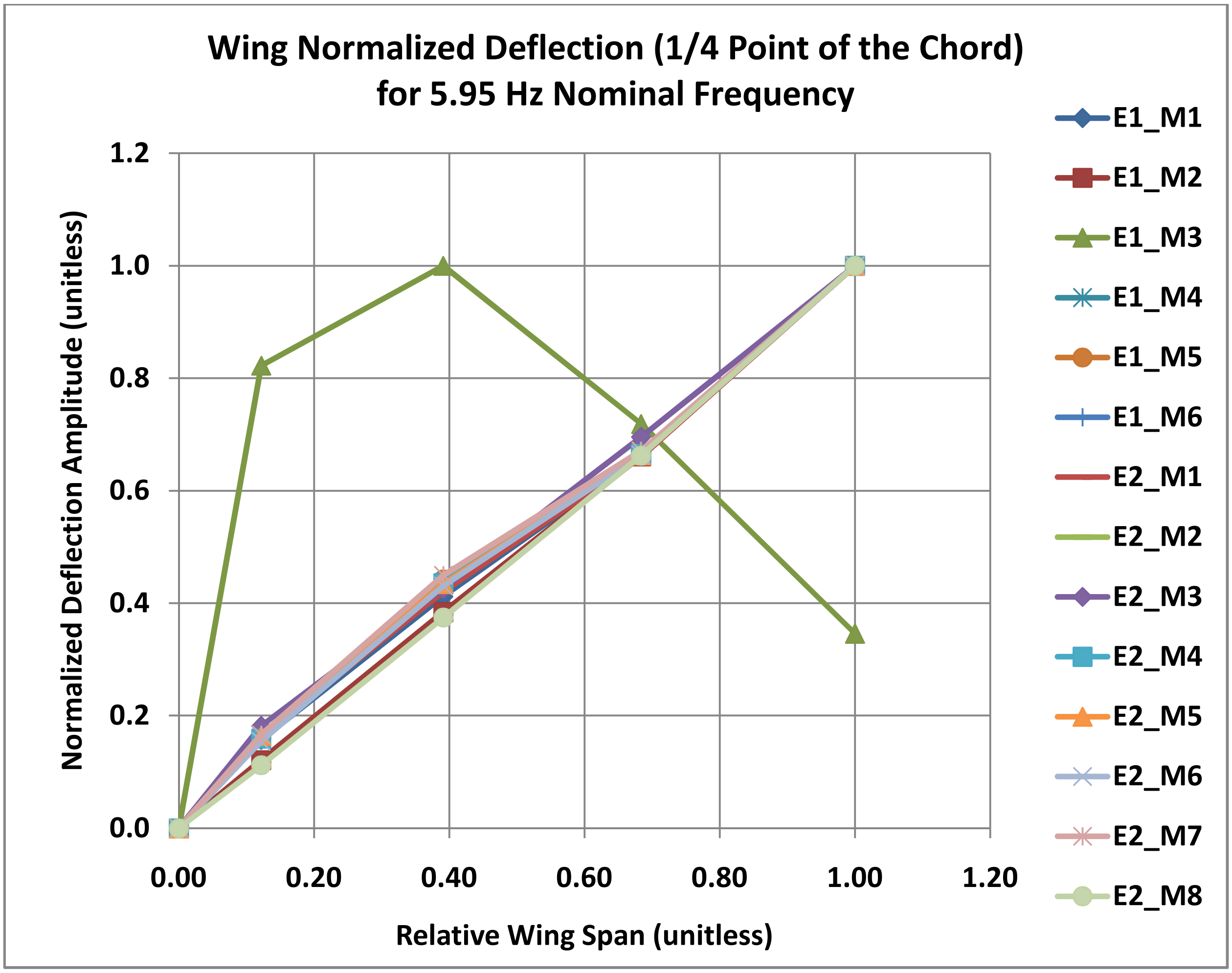

Fig. 2. Normalized deflection for 5.95 Hz.

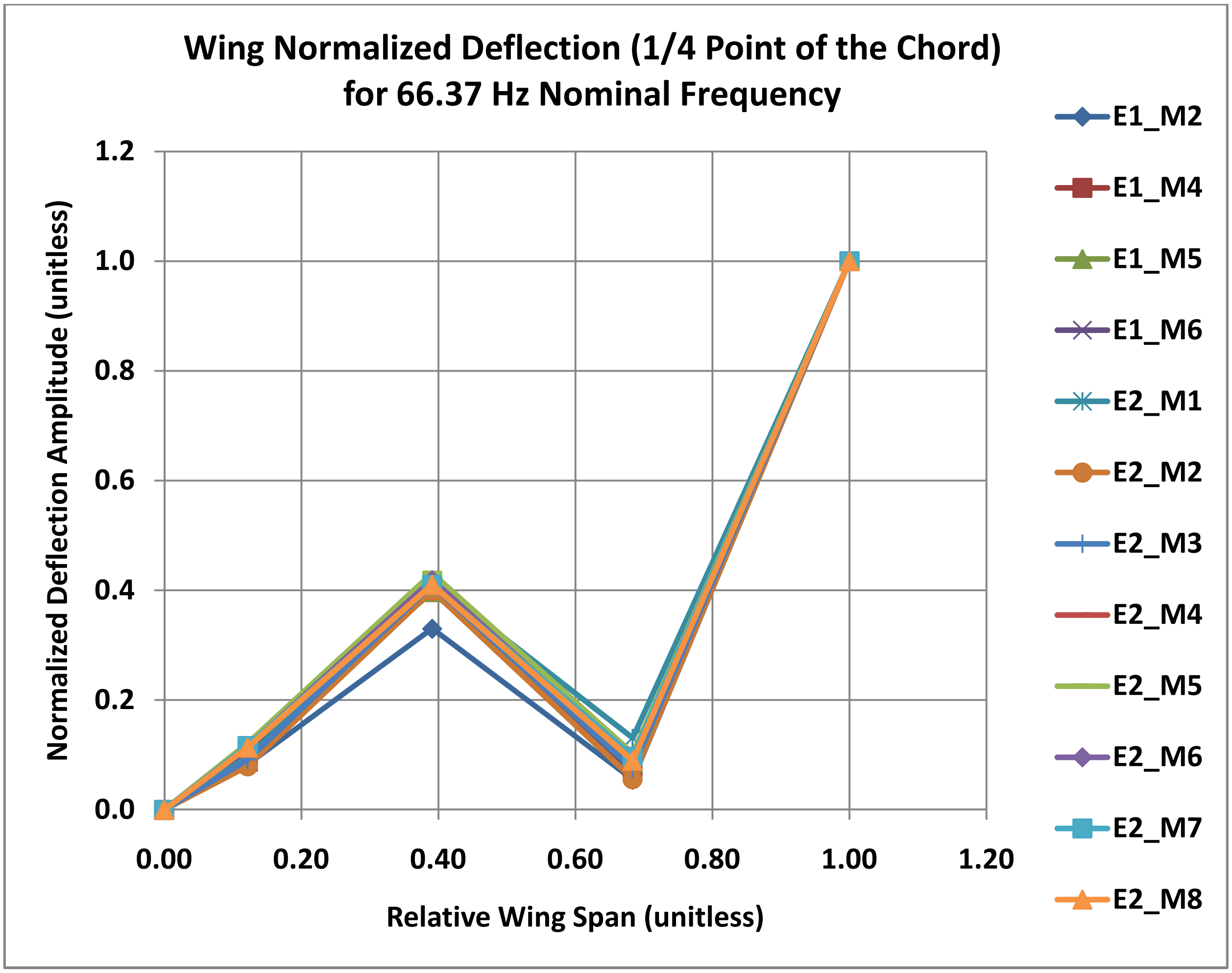

Fig. 3. Normalized deflection for 66.37 Hz.

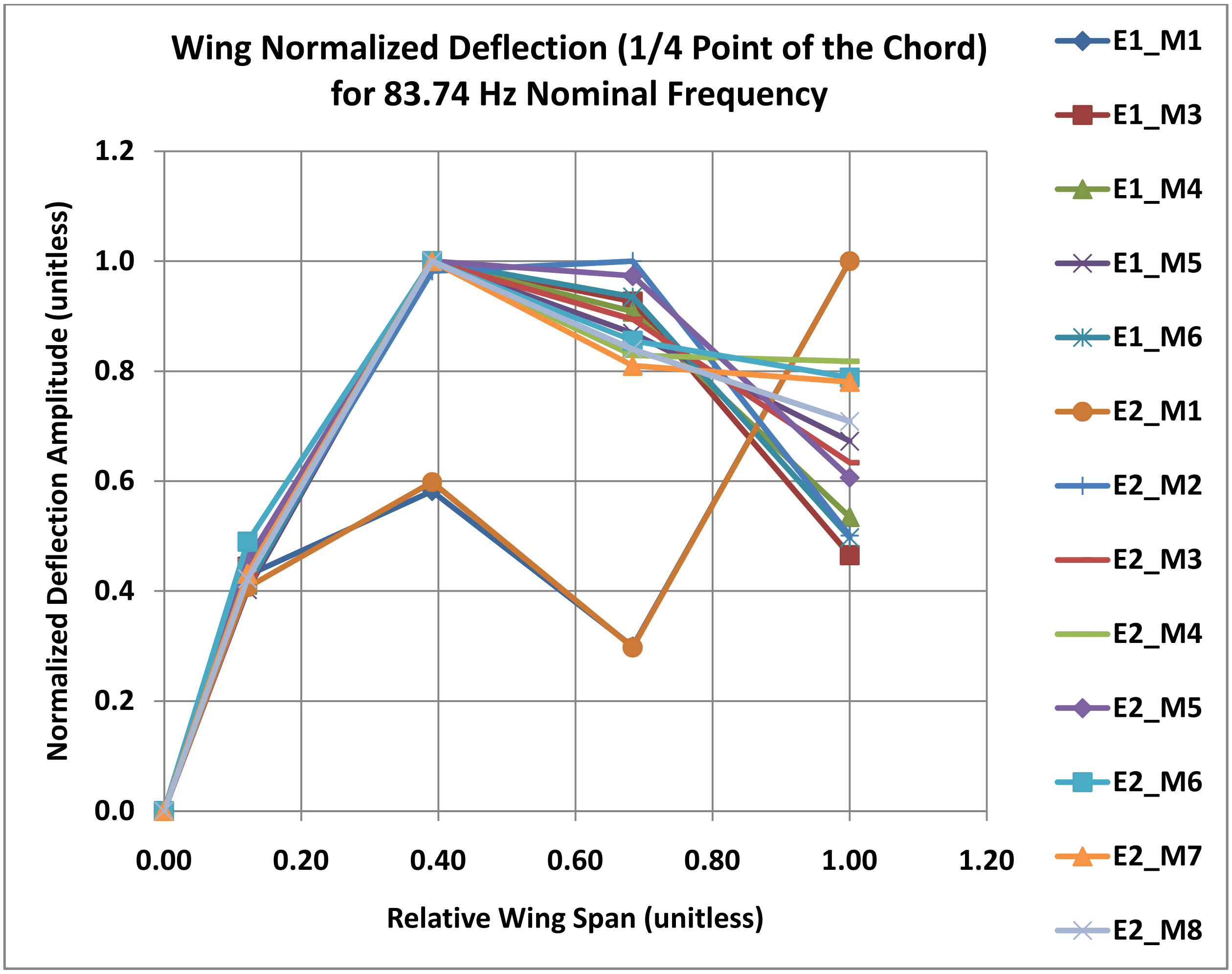

Fig. 4. Normalized deflection for 83.74 Hz.