Kinematical Solution of Parallel Mechanisms by Structural Approximation: A Hexapod 3R3R Example

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Abstract (in Czech)

Příspěvek se zabývá řešením dopředné kinematiky hexapodu pomocí metody strukturní aproximace. Tento koncept nahrazuje řešenou strukturu její zjednodušenou podobou, která je analyticky řešitelná. Toto řešení je základem iteračního cyklu. Metoda byla vyzkoušena na řešení inverzní kinematiky neanalytických seriových mechanismů. Tento příspěvek ukazuje použití metody pro řešení dopředné kinematiky paralelních mechanismů – metoda je demonstrována na případě hexapodu v 3R3R konfiguraci. Je ukázáno sestavení rovnic pro iterační cyklus. Dále je porovnána vypočetní náročnost metody s klasickým řešení Newtonovou iterační metodou.

Key words líčová slova

Kinematical solution; structural approximation; hexapod.

1. Introduction

The investigated problem is the solution of positional kinematical problem for analytically non-solvable (so called non-simple) mechanical systems (mechanisms) [2]. The traditional solution method is the Newton method. However, this paper deals with a new method for positional kinematical solution of mechanisms with loops. The method is based on the concept of structural approximation, i.e. the structure of the mechanism being solved is simplified in such a way that the mechanism with simplified structure is analytically solvable. The analytical solution is the basis of the iteration. This method has been successfully applied for the inverse kinematical solution of non-simple serial robots [1]. This paper extends this method for mechanisms with loops and specifically for forward kinematical solution of parallel kinematical structures. The method of structural approximation is demonstrated on Hexapod.

2. Method of Structural Approximation

If a kinematical structure is not analytically solvable then it includes usually some structural (topological) feature that is responsible for this non-solvability. If this feature is removed the resulting kinematical structure becomes solvable (Fig. 1). Such feature is for example the distance of rotational axes in non-spherical robot wrist [1]. If this distance is set to zero, the serial robot becomes simple and the inverse kinematical problem is solvable. This analytical solution is then computed for the perturbed right-hand side of kinematics constraints evaluated from the pervious values of coordinates. This is the basis for iterations.

The kinematical structure is described by the coordinates ***s***. These coordinates are constrained by the kinematical constraints:

$$\mathbf{f}(\mathbf{s}) = \mathbf{0} \tag{1}$$

These equations are not analytically solvable. But they can be split into the simple part $\boldsymbol{f}_S$ that is analytically solvable and the non-simple part $\boldsymbol{f}_{NS}$ that is causing the non-solvability. This corresponds to the Fig. 1.

$$\mathbf{f}(\mathbf{s}) = \mathbf{f}_S(\mathbf{s}) + \mathbf{f}_{NS}(\mathbf{s}) = \mathbf{0} \tag{2}$$

Because the part $\boldsymbol{f}_S$ that is analytically solvable it can be developed an iteration scheme

$$\mathbf{f}_S(\mathbf{s}) = -\mathbf{f}_{NS}(\mathbf{s}) \tag{3}$$

$$\mathbf{s}_{i+1} = \mathbf{f}_S^{-1}(\mathbf{f}_{NS}(\mathbf{s}_i)) \tag{4}$$

This iteration scheme converges and this can be checked by traditional means [2] of comparison of magnitudes of partial derivatives on left and right hand sides of (3).

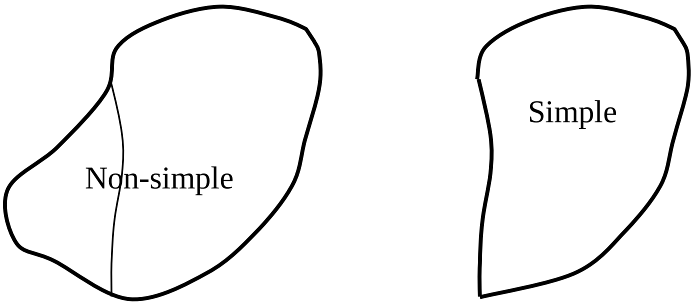

Fig. 1.*Non-simple kinematical structure and its structural approximation*

3. Hexapod

The forward kinematical problem of Hexapod, that is not analytically solvable, has been solved by the method of structural approximation.

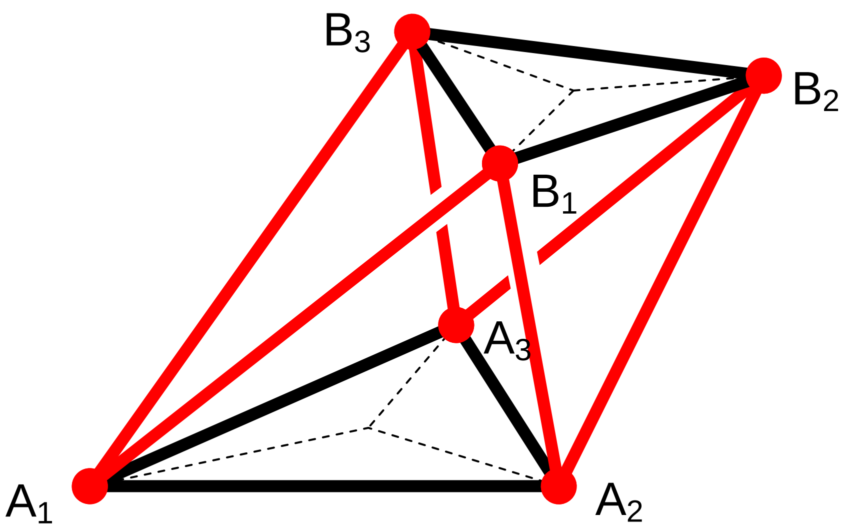

Fig. 2. *Traditional hexapod in 3R3R configuration*

The traditional Hexapod is in Fig. 2. The structural approximation is introduced by setting the distance B1B2 to zero. However, the particular structural approximation is carried out by supposing the knowledge of the vector B1B2 from the previous solution (iteration). The point A2 is moved by B1B2 to the point A2' and the resulting mechanism is analytically solvable (Fig. 3). The position of the point B1 is determined as the intersection of three spherical surfaces from points A1, A2, A2'.

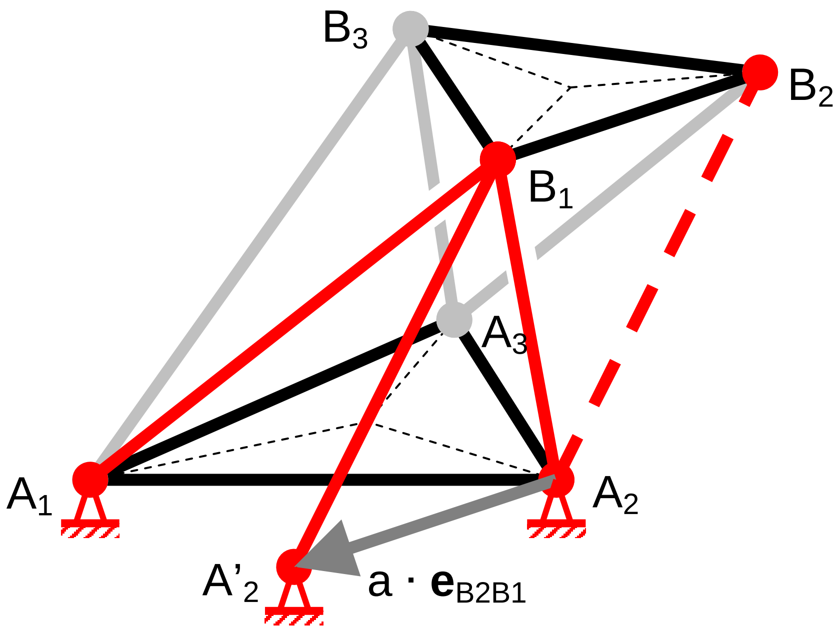

Fig. 2. *Traditional hexapod in 3R3R configuration*

This can be written as:

$$\begin{aligned} \left|\mathbf{r}_{B1}-\mathbf{r}_{A1}\right| &= L_{11} \\ \left|\mathbf{r}_{B1}-\mathbf{r}_{A2}\right| &= L_{12} \\ \left|\mathbf{r}_{B2}-\mathbf{r}_{A2}\right| &= L_{22} \end{aligned} \tag{5}$$

Structural approximation is expressed as:

$$\begin{aligned} \left|\mathbf{r}_{B1}-\mathbf{r}_{A2}+\mathbf{r}_{B1B2}\right| &= L_{22} \\ \left|\mathbf{r}_{B1}-\left(\mathbf{r}_{A2}-\mathbf{r}_{B1B2}\right)\right| &= L_{22} \\ \left|\mathbf{r}_{B1}-\left(\mathbf{r}_{A2}-b\cdot\mathbf{e}_{B1B2}\right)\right| &= L_{22} \\ \left|\mathbf{r}_{B1}-\mathbf{r}_{SAP1}\right| &= L_{22} \end{aligned} \tag{6}$$

where $\mathbf{r}_{SAP1}$ is a vector of the structural approximation, i.e. the radiusvector of the point A'2.These equations give the intersection of three spherical surfaces:

$$\begin{aligned} \left(x_{B1}-x_{A1}\right)^2+\left(y_{B1}-y_{A1}\right)^2+\left(z_{B1}-z_{A1}\right)^2 &= L_{11}^2 \\ \left(x_{B1}-x_{A2}\right)^2+\left(y_{B1}-y_{A2}\right)^2+\left(z_{B1}-z_{A2}\right)^2 &= L_{12}^2 \end{aligned}$$

$$\left(x_{B1}-x_{SAP1}\right)^2+\left(y_{B1}-y_{SAP1}\right)^2+\left(z_{B1}-z_{SAP1}\right)^2=L_{22}^2 \tag{7}$$

where

$$\mathbf{r}_{A1}=\begin{bmatrix} x_{A1} \\ y_{A1} \\ z_{A1} \end{bmatrix},\ \mathbf{r}_{B1}=\begin{bmatrix} x_{B1} \\ y_{B1} \\ z_{B1} \end{bmatrix},\ \text{etc.} \tag{8}$$

After multiplication, the second equation can be subtract for the first one and the third form the first one:

$$\begin{gathered} 2\left(x_{A2}-x_{A1}\right)x_{B1}+2\left(y_{A2}-y_{A1}\right)y_{B1}+2\left(z_{A2}-z_{A1}\right)z_{B1}=\dots \\ \dots=L_{11}^2-x_{A1}^2-y_{A1}^2-z_{A1}^2-L_{12}^2+x_{A2}^2+y_{A2}^2+z_{A2}^2 \\ 2\left(x_{SAP1}-x_{A1}\right)x_{B1}+2\left(y_{SAP1}-y_{A1}\right)y_{B1}+2\left(z_{SAP1}-z_{A1}\right)z_{B1}=\dots \\ \dots=L_{11}^2-x_{A1}^2-y_{A1}^2-z_{A1}^2-L_{22}^2+x_{SAP1}^2+y_{SAP1}^2+z_{SAP1}^2 \end{gathered} \tag{9}$$

Equation (9) can be solved as a set of linear equation in unknowns xB1, yB1. Using the Cramer's Rule the solution is linear in a variable zB1:

$$\begin{aligned} x_{B1}&=k_{1,1}\cdot z_{B1}+q_{1,1} \\ y_{B1}&=k_{2,1}\cdot z_{B1}+q_{2,1} \end{aligned} \tag{10}$$

By insertion of the solution (10) in the equation (7c) we get a quadratic equation for zB1:

$$\begin{gathered} \left(k_{1,1}z_{B1}+q_{1,1}\right)^2-2\left(k_{1,1}z_{B1}+q_{1,1}\right)x_{SAP1}+\dots \\ \left(k_{2,1}z_{B1}+q_{2,1}\right)^2-2\left(k_{2,1}z_{B1}+q_{2,1}\right)y_{SAP1}+\dots \\ z_{B1}^2-2z_{B1}z_{SAP1}=L_{22}^2-x_{SAP1}^2-y_{SAP1}^2-z_{SAP1}^2 \end{gathered} \tag{11}$$

The correct solution is the maximum.

$$z_{B1}=\max\left[\left(z_{B1}\right)_1,\left(z_{B1}\right)_2\right]=\frac{-b_{z,1}+\sqrt{b_{z,1}^2-4a_{z,1}c_{z,1}}}{2a_{z,1}} \tag{12}$$

This is repeated for each point B1, B2, B3 of the platform. Then the knowledge of these points is used for the computation of the orientation of the platform and hence again the position of the vector B1B2. It is very essential that the orientation is determined only as the elements of the matrix of direction cosines and not as the Euler or Cardan angles. The Euler or Cardan angles are computed only at the end of the iterations. Or the matrix of direction cosines can be computed stepwise using the last iteration of Bi points. The iteration progress is shown in Fig. 4 and 5. The convergence of the method is very robust – the number of the iteration steps depends on the initial iteration only imperceptibly.

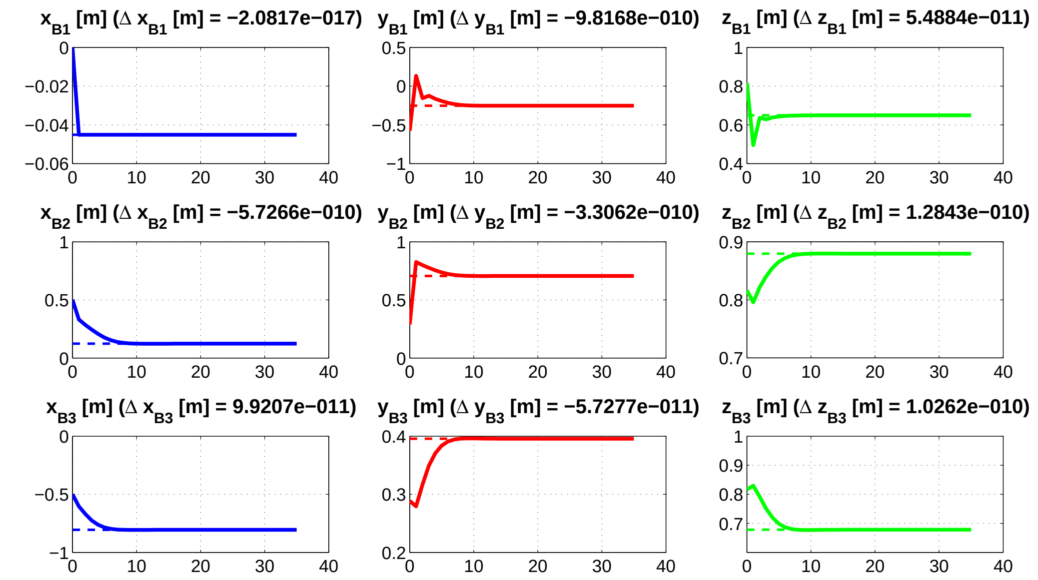

Fig. 4. *Iteration process – Fixed initial iteration*

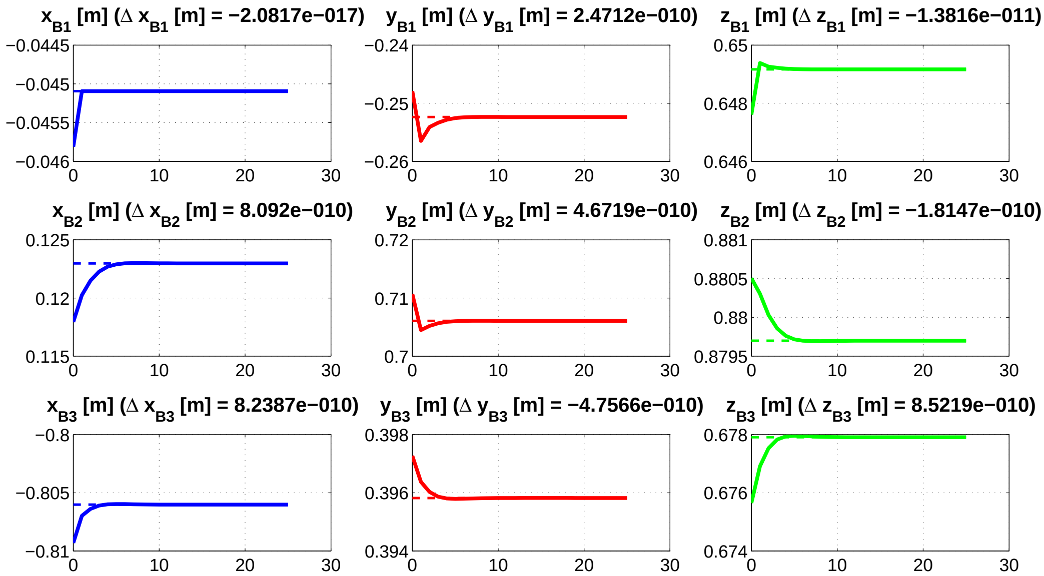

Fig. 5. *Iteration process – Initial iteration equals previous position on trajectory*

This procedure has been compared with traditional solution by Newton iterations. The comparison is shown on a computation of the forward kinematics problem on a sample trajectory. The trajectory was chosen from a general position near an edge of working space (Fig. 6) back to a basic position (**s** = **0**). Tests were carried out on several computers with various computational performance and the results were compared. The comparison of the computational complexity (elapsed time – actual time depends on computer performance but the ratio remains the same) is in Fig. 7. The method of structural approximation is favourable.

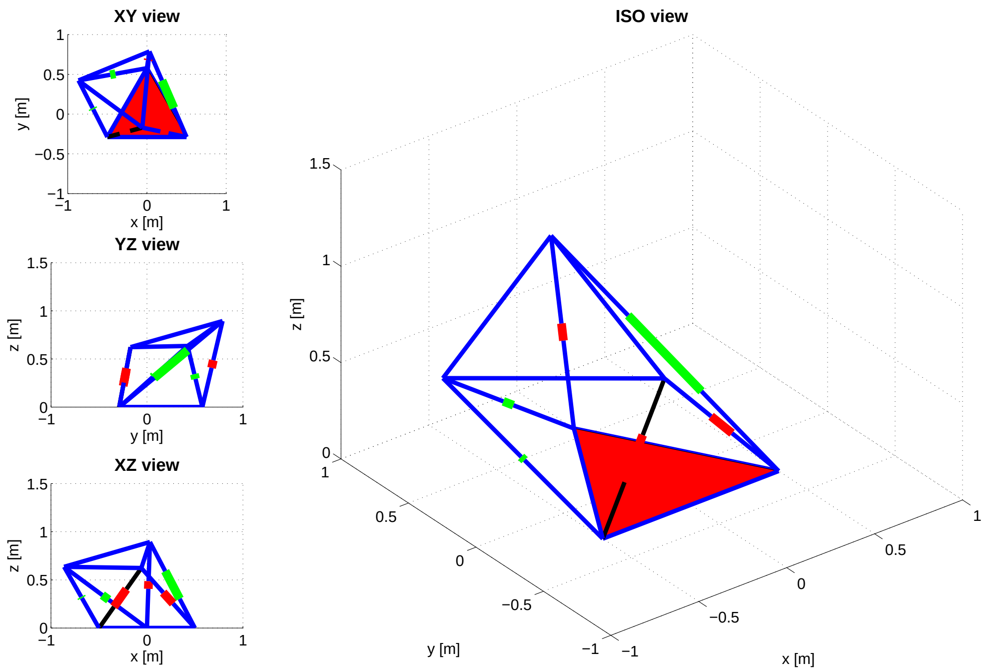

Fig. 6. *Hexapod in position near edge of working space*

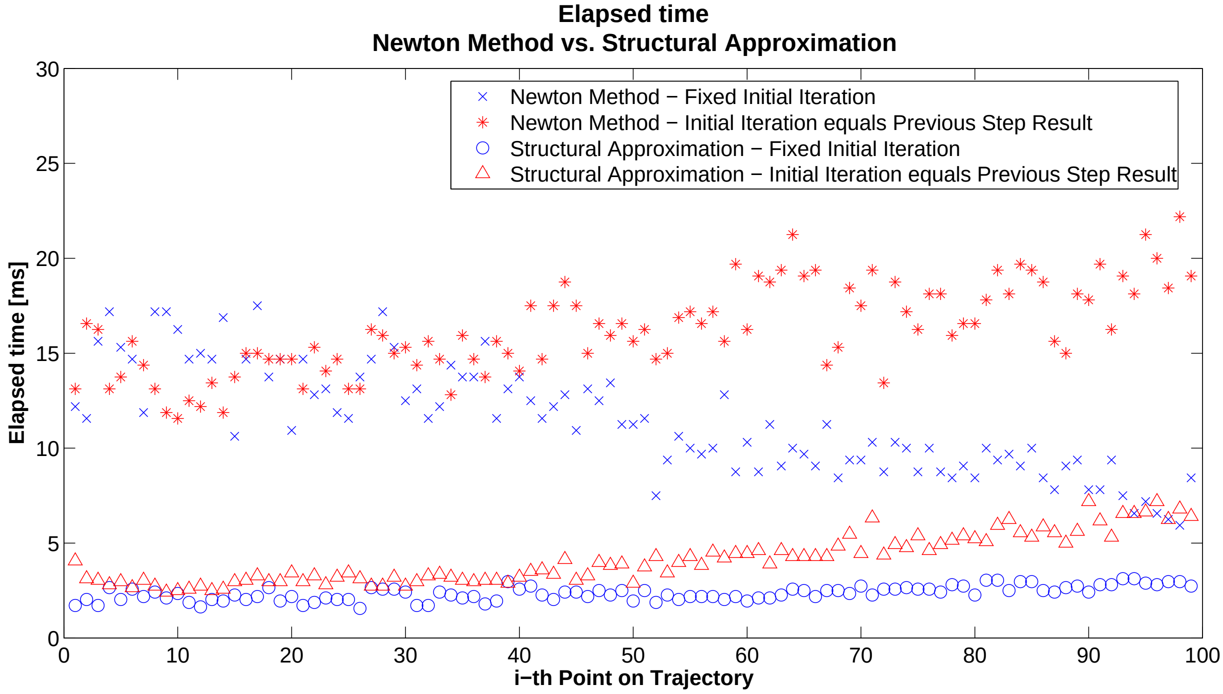

Fig. 7. *Computational complexity of kinematical solution*

Furthermore, the influence of the initial iteration is shown in Fig. 7. The robustness of the structural approximation is much more bigger in some regions of working space.

Because the hexapod in the presented 3R3R configuration is difficulty to design the future interest will be given to the extension of this method for solving the kinematical problem of the more general 6R6R hexapod.

Conclusions

The paper describes a new procedure of structural approximation for the solution of positional kinematical solution of parallel kinematical structures that are not analytically solvable. This procedure achieves better computational complexity than the traditional Newton method. The procedure has been used for the solution of forward kinematical problems of the hexapod in the 3R3R configuration.

Acknowledgements

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List of symbols

$\mathbf{s}$	coordinates vector	[m]
$\mathbf{r}_{Ai}$	radius-vector of point A_i	[m]
$\mathbf{r}_{Bi}$	radius-vector of point A_i	[m]
x_{Ai}, y_{Ai}, z_{Ai}	x, y, z – component of point A_i radius-vector	[m]
x_{Bi}, y_{Bi}, z_{Bi}	x, y, z – component of point A_i radius-vector	[m]
L_{ij}	length of the leg between points A_i and B_j	[m]
$\mathbf{r}_{SAP}$	structural approximation vector	[m]
$x_{SAP}, y_{SAP}, z_{SAP}$	x, y, z – component of structural approximation vector	[m]

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