Error Reduction in Structural Stiffness Identification

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Abstract (in Czech)

Článek se zabývá vlivem chyb v měření na metodu staticke identifikace. Prezentovaná metoda statické identifikace umožňuje určit tuhostní parametry rámových a přihradových konstrucki z měření posunutí a natočení styčníků dané konstrukce. Článek se zaměřuje na odstranění vlivu chyb měření na výsledky prezentované metody.

Keywords

redundant measurements; regularization; nonlinear least-squares; error reduction

1. Introduction

Stiffness identification [1] results from equation

$$^{g}K(k_i)\ u_j = f_j \tag{1}$$

where f_j is a column vector of j-th applied load. K is stiffness matrix of solved structure in the global coordinate system and it is function of unknown stiffness parameters k_i. According to [1] it is possible to rewrite (1) to

$$\begin{bmatrix} A_1(u) \\ \vdots \\ A_j(u) \end{bmatrix} k = \begin{bmatrix} f_1 \\ \vdots \\ f_j \end{bmatrix} \tag{2}$$

For the frame structure (Fig. 1) the dimensions of matrix A are 714x528 and the number of unknown stiffness parameters n = 528 [1].

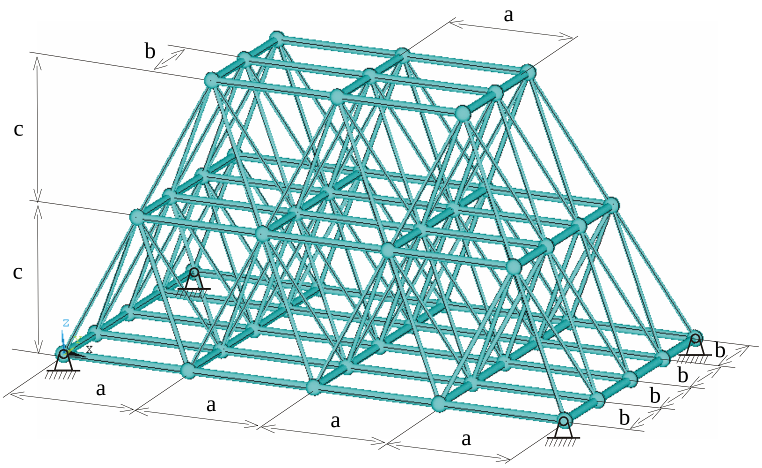

Fig. 1. *Solved frame structure.*

2. Construction of solution

If there exists no solution that exactly satisfies (2) we can only construct some pseudo-solutions [2]. They will satisfy (2) approximately in some sense, e.g. we require the minimization of vector norm of residui $\|Ak - f\|$. The solution that minimizes $\|Ak - f\|$ is

$$k = \left(A^T A\right)^{-1} A^T f \tag{3}$$

where $(A^TA)^{-1}A^T$ is Moore-Penrose generalized inverse of matrix A. The relation between solution of (10) and measurements errors was examined (Fig. 2). The correct solution would be horizontal line at level 0%.

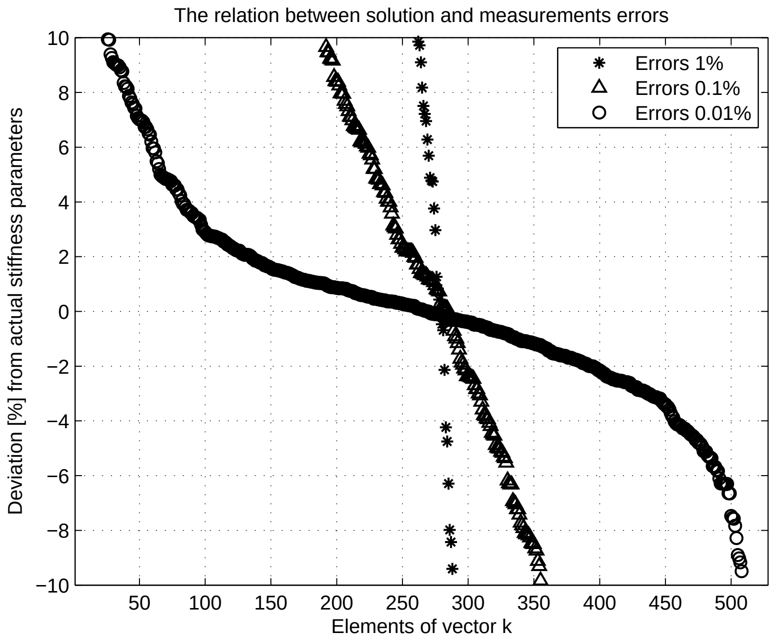

Fig. 2. *The relation between solution of (3) and measurements errors.*

3. Regularization of problem

Tikhonov regularization [3] is common method used for regularization of ill-posed problems. For overdetermined system (2) there is method called linear least squares. It seeks to minimize the residual

$$\left\|Ak - f\right\|^2 \tag{4}$$

where $\|.\|$ is the Euclidean norm. However matrix A can be ill-conditioned and the exact solution can't be found. In order to give preference to a particular solution with desirable properties, it is possible to add the regularization term

$$\left\|Ak - f\right\|^2 + \left\|\Gamma k\right\|^2 \tag{5}$$

where Γ is Tikhonov matrix. In many cases, this matrix is chosen as the identity matrix, giving the preference to solutions with smaller norms. The explicit solution can be found

$$k = \left(A^T A + \Gamma^T \Gamma\right)^{-1} A^T f \tag{6}$$

Effect of the regularization can be tuned via scale of a matrix Γ (tj. $\Gamma = \alpha E$). For $\Gamma = 0$ the problem reduces to unregularized least square method – provided that $(A^T A)^{-1}$.

There are many possibilities how to construct a Tikhonov matrix. Diagonal Tikhonov matrix

$$\Gamma_{diag} = \alpha E = \begin{bmatrix} \alpha & 0 & \cdots & 0 & 0 \\ 0 & \alpha & \ddots & & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & & \ddots & \alpha & 0 \\ 0 & 0 & \cdots & 0 & \alpha \end{bmatrix} \tag{7}$$

and tridiagonal Tikhonov matrix

$$\Gamma_{tridiag} = \begin{bmatrix} \alpha & \beta & 0 & & \cdots & 0 & 0 \\ \beta & \alpha & \beta & 0 & & & 0 \\ 0 & \beta & \alpha & \ddots & \ddots & & \vdots \\ & 0 & \ddots & \ddots & \ddots & 0 & \\ \vdots & & \ddots & \ddots & \alpha & \beta & 0 \\ 0 & & & 0 & \beta & \alpha & \beta \\ 0 & 0 & \cdots & & 0 & \beta & \alpha \end{bmatrix} \tag{8}$$

were used.

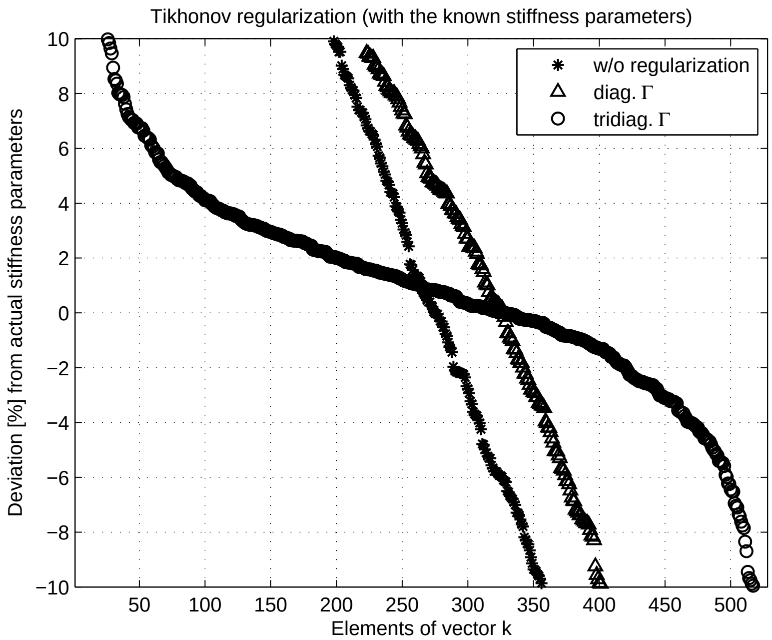

Fig. 3. *Possibility to find more accurate solution using Tikhonov regularization.*

Existence of suitable diagonal and tridiagonal Tikhonov matrix for solution (6) was examined [4]. Searching was carrying out on construction with known stiffness parameters and this information was used to set up the convergence criteria.

Fig. 3 shows promising results using simple Tikhonov matrix as a regularization term on measurements with errors.

3. Actual and old (known) parameters

There was a need of known actual stiffness parameters in the previous example of the Tikhonov regularization. It is possible to override this handicap. It is possible to assume that there exists some solution from a previous measurements or from assembly drawings which gives approximate values for convergence criteria. Relation between actual parameters and old parameters shows Fig. 4.

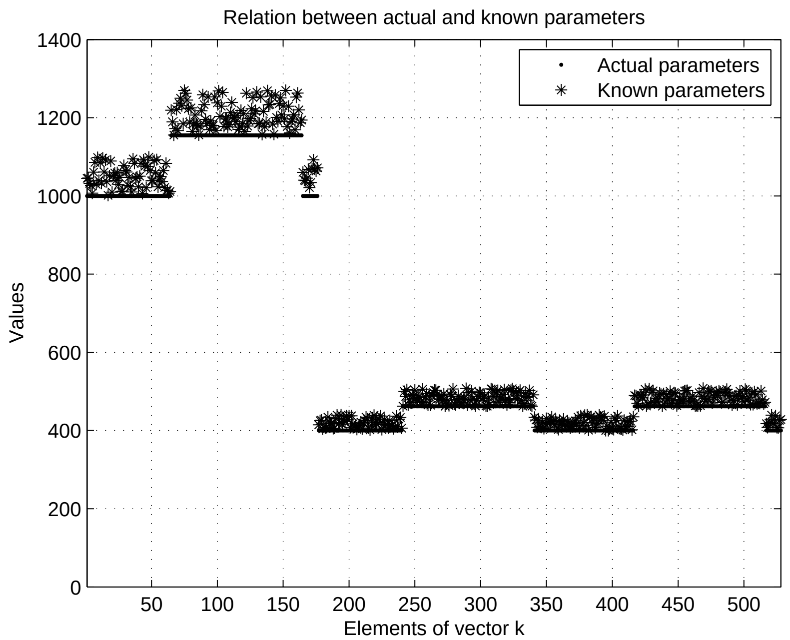

Fig. 4. *Relation between actual parameters and known (old) ones.*

4. Possibilities to improve results from Tikhonov regularization

It is possible to find unknown parameters α and β in tridiagonal Tikhonov matrix. The results are still not accurate enough. Fig. 5. shows, that it is possible to improve previous results. This is done by locking down the good stiffness parameters (compared to the known old parameters), e.g. stiffness parameters < 5%, and optimization of the corresponding parts of matrix A from eq. (2) to get better results. Convergence criteria is old (known) stiffness parameters.

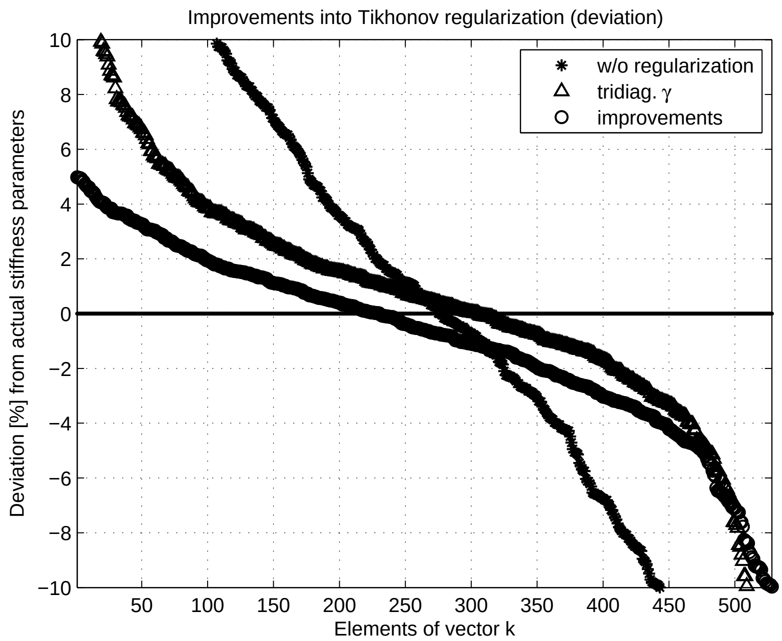

Fig. 5. *Improvements (deviation)*

It is possible to show results in way similar to Fig. 4. (Fig. 6.).

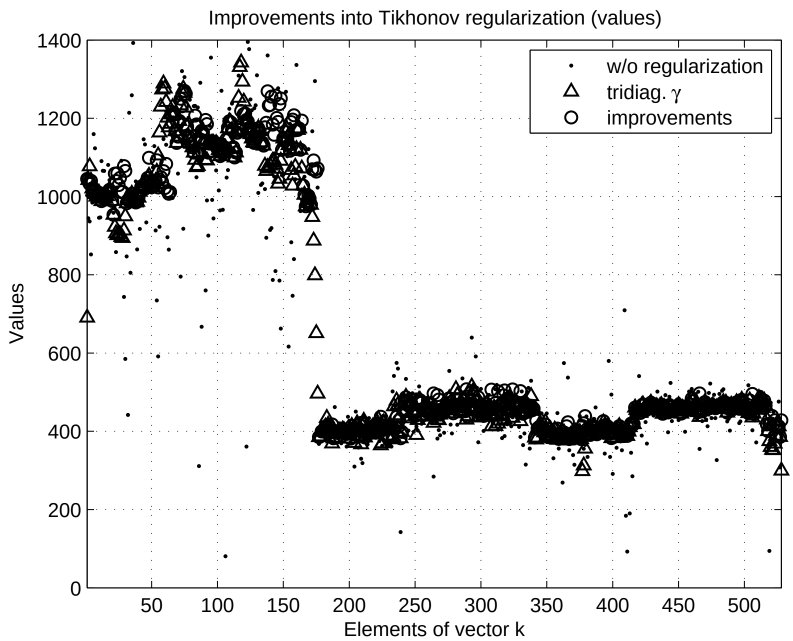

Fig. 6. *Improvements (values)*

Fig. 5. shows reduced stiffness parameters errors to interval < -10%, 5% > of actual values for error in the meassurements 0.1%. Therefore this method is still not acceptable for real applications.

Conclusions

Problem to minimize impact of measurements errors to stiffness reconstruction of 3D frame is very difficult as described above. There exist several techniques how can be this impact lowered. The possibility to use Tikhonov regularization was examined.

This topic need further inspect and it is necessary to find suitable solution that will provide realistic demands for the errors in meassurements and acceptable accuracy of the reconstructed parameters.

Acknowledgments

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References

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