Modelling of gas pipeline network systems

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1. Introduction

This paper presents the development of a mathematical model for gas pipeline network systems. This model can be used to analyze the actual operational state of a pipeline system and for the detection and location of potential leakages. It incorporates the natural gas properties as well as energy and mass balances and geometric data of the pipeline network. Different approaches for the calculation of the pressure drop inside the pipeline sections have been implemented in order to consider the compressibility of natural gas.

The operation of a gas pipeline network is evaluated and controlled on the basis of operating data such as

- Volume flow rate [m^3/sec, Nm^3/sec],
- Internal pressure [bar], and
- Temperature [K].

The measuring accuracy of the pipeline equipment used for this purpose is stated as ±0.5% in accordance with the current state of the art. This uncertainty margin is further increased by the inaccuracy of the measuring loops and, finally, of the metering line to the process computers in which the measured data is collected and stored.

These relatively high final uncertainties and tolerances often cause problems during the calculation of the quantities supplied by the pipeline network. To overcome these problems, the reconciliation of collected operational data using Gauss' Theorem is applied. This method closes all mass and energy balances by minimizing the overall uncertainty of all considered data.
In this way, not only a set of corrected measurement data is obtained, but also the accuracy of the single measurements is highly optimized, because singular measurements are transferred into a redundant network of information about the actual state of the pipeline system.

Where large quantities of gas are delivered over the time, reduced measurement tolerances may result in substantial financial benefits accumulating to millions of Euros or Dollars. Also, the detection of developing leakages within the pipeline length increases the availability and maintainability of the pipeline network. This yields also in protection against eventual environmental damages.

2. Theoretical background

2.1 Data reconciliation

All measured values are subject to distortions caused by avoidable, systematic or random uncertainties (DIN 1319). For more than 200 years the Gaussian correction principle that is complemented by taking boundary conditions into account has been available as an estimation method in the statistical-mathematical sense that allows these measurement uncertainties to be detected.

The basic idea of this method is to use not only the minimum quantity of measured variables required to obtain a solution but to record all accessible measured variables along with the respective variances and covariances. Additionally the true values of the measured variables must meet the boundary conditions:

- mass balances
- energy balances
- material balances (stoichiometric laws)

This method is described in VDI 2048 [1, 2] and is the best possible quality control method available to detect serious measurement errors. This methodology allows consistent estimations of the true values of the measured variables to be derived from conflicting measured values. The consistent estimated values thus obtained correspond to the true values with a 95% probability.

2.1.1 Gaussian correction principle

Corrections ν are made to the measured values x in accordance with Equation (1) in order to obtain estimated values (reconciled values) $\bar{x}$.

$$\bar{x} = x + \nu \tag{1}$$

The corrections ν must be determined in such a manner that the quadratic form

$$\xi_0 = \nu^T \cdot S_X^{-1} \cdot \nu \Rightarrow Min \tag{2}$$

ξ_0 ... square form of errors

S_X^{-1} ... inverse empirical covariance matrix

becomes a minimum. The empirical covariance matrix S_x is the estimated value for the uncertainty of the measured variables X. This general formulation also covers the existence of covariances, i. e. the interdependencies of the measuring points. The improved covariance matrix $S_{\bar{x}}$ is calculated from the empirical covariance matrix S_x and the covariance matrix of the improvements S_v as follows:

$$S_{\bar{x}} = S_x - S_v \tag{3}$$

2.1.2 Quality control

As a quality-control measure, two criteria must be fulfilled for process data reconciliation based on VDI 2048. For one thing, the square form of errors ξ_0 contained in Equation (2) must be smaller than $\chi^2_{95\%}$ (95% quantile of CHI square).

$$\text{VDI 2048 criterion 1: } \xi_0 < \chi^2_{95\%} \tag{4}$$

The 95% quantile of CHI square is a statistical measure for the number of model redundancies and is included e.g. in [3] as a table. The number of model redundancies is dependent both on the number of equations contained in the model and on the number of embedded measured values and reflects the over determined character of the system. The relationship between the square form of errors and CHI square is referred to as reconciliation quality, see Equation (5). Generally the following applies: The smaller the reconciliation quality, the better is the quality of the model/measured values.

$$\text{Reconciliation quality} = \xi_0 \;/\; \chi^2_{95\%} \quad . \tag{5}$$

Additionally the value of the individual penalty must be smaller than the statistical coefficient of 1.96. The value of the individual penalty of a measuring point is the ratio of the square of the improvement v_i to the difference between the estimated uncertainty of the measured value $s_{x,ii}$ and the calculated standard deviation of the reconciled value $s_{\bar{x},ii}$, see Equation (6).

$$\text{VDI 2048 criterion 2:} \quad \frac{|v_i|}{\sqrt{s_{x,ii} - s_{\bar{x},ii}}} = \frac{|v_i|}{\sqrt{s_{v,ii}}} \leq 1.96 \quad . \tag{6}$$

This criterion must be complied with for all measured values i. If it is not complied with, a serious fault exists for the corresponding measuring point i or for the estimated value of the associated uncertainty.

In this case the reconciled value as well as the measured value itself is questionable. If both of the above criteria are complied with, the reconciled measured values $\bar{x}$ correspond to a 95% probability of the true physical state variables.

2.1.3 Consequences for an industrial process

When process data reconciliation based on VDI 2048 is applied to an industrial process, measured temperatures, mass flows and pressures lose their singular character. The physical relationships between the measured parameters generated via secondary conditions such as mass balance and energy balance result in a process image that corresponds to the physical basis of the process as closely as possible. The relationships thus generated can be represented via the correlation coefficients that result from the improved covariance matrix; see [1].

2.2 Calculations of the pressure gradient

One of the key quantities in the calculation of pipeline systems and the evaluation of the fluid flow is the pressure gradient along the length of the pipe. There are several reasons for this pressure gradient. One is the loss of energy inside the fluid due to inertia and inner friction. This effect is included in the following calculation by considering density and viscosity. Another reason for the decrease of the pressure along the pipeline is the friction between the fluid and the surface inside the pipe. To take the surface friction into account, the surface roughness is considered as a parameter in order to calculate a so-called friction factor. Different approaches to calculate the friction forces between the surface and the fluid can be found in the literature.

2.2.1 Colebrook

The formula of Colebrook, Equation (7), relates to pipes with technical surface roughness, where

$$65\frac{d}{k} < \mathrm{Re} < 1300\frac{d}{k}$$

with d as pipe diameter, k as surface roughness and Re as the Reynolds number. It provides great accordance with experimental data and is used as a standard approach for calculations of this type in many cases found in literature [3].
The friction factor λ is given by

$$\lambda = \frac{1}{\left[2\lg\left(\frac{2{,}51}{\mathrm{Re}\sqrt{\lambda}} + \frac{0{,}27}{d/\varepsilon}\right)\right]^2} \tag{7}$$

The pressure drop between two position 1 and 2 over the pipeline length can now be calculated for the isothermal case ($T = const.$) with:

$$\Delta p = p_1\left[1 - \sqrt{1 - \frac{\lambda v^2 l \rho}{p_1 \cdot d}}\right] \tag{8}$$

where l is the length of the pipeline section or the distance between position 1 and 2, v is the velocity of the fluid, ρ is the density of the fluid and p_1 is the pressure at position 1.

2.2.2 Shapiro

Another approach to calculate the pressure drop between position 1 and 2 for isothermal flows with friction is given by Shapiro [4]. Although the Mach numbers for such flows are usually quite low, there are significant changes in the pressure over the length of the pipeline section on which friction is acting, so the flow is highly incompressible. It can be solved as

$$\Delta p = p_1\left[\frac{1-kM^2}{1+kM^2} - \sqrt{\left(\frac{1-kM^2}{1+kM^2}\right) - \frac{kM^2}{1+kM^2}\left(4\lambda\frac{l}{d}\right)}\right] \tag{9}$$

where k is the ratio of specific heats (k = 1.31 for natural gas), d is pipe diameter, l is the length of the pipeline section or the distance between position 1 and 2, p_1 is the pressure at position 1, λ is the friction factor according to Equation (7) and M is the Mach Number which is expressed by

$$kM^2 = \left(\frac{m}{\pi d^2/4}\right)^2 \frac{RT}{p_1^{\,2}} \tag{10}$$

with m as the mass flow, T as the temperature (t = 14,5°C) and R as the gas constant of natural Gas (R = 518,3 J/kg K^{-1} for methane, main component).

2.3 Calculation of the leakage volume

The detection of the loss of natural gas over the pipeline length is one basic requirement to enhance the efficiency in a economical and environmental point of view.

The approach which is used in this work is based on the ideal gas law. The leakage flow rate between two sections or positions 1 and 2 is expressed as

$$p_1 v_1 = p_2 (v_2 + v_x) \tag{11}$$

where $p_{1/2}$ is the pressure at position 1 respectively 2, $v_{1/2}$ is the velocity of fluid flow at position 1 respectively 2 and , v_x is the velocity of unknown leakage flow rate as shown in Figure 1. The temperature is assumed to be constant (isothermal case).

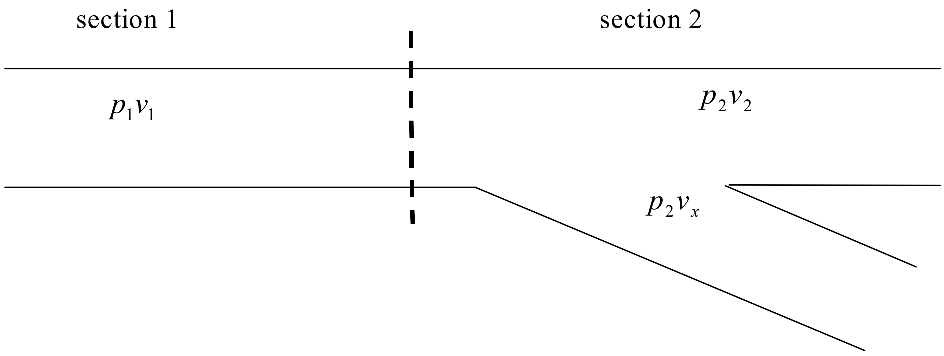

Figure 1: Calculation of the leakage flow rate

Due to the principle of mass conservation, the velocities at section 1 and 2 must be equal:

$$v_1 = v_2 \ , \tag{12}$$

and therefore:

$$v_x = \frac{v_1 \cdot (p_1 - p_2)}{p_2} \tag{13}$$

The working pressure in today's pipeline systems ranges between 55 and 75 bar, depending on the pipeline diameter [5]. For pressure values in this range, the ideal gas law is only valid when an additional compressibility factor is applied. In the investigated case only difference values between two section are considered, see Equation 13). Therefore, the errors coming from the high pressure values at both sections cancel each other out. and the ideal gas law provides sufficient accurate results for this calculation.

3 Pipeline model

3.1 Geometry and Layout

The pipeline system investigated in this paper is a virtual one. The design parameters as well as the measured values used for this study are typical values for this type of application and can be replaced by real values at any time. The length of the pipeline is approximately 360 km and is divided into 12 sectors with a length of about 30 km each. The diameter is specified with 1.42 m. Attached to the main pipeline there are 5 consumers, each divided into 2 sectors with a length of 25 km each and a diameter of 0.3 m. Compressors to establish the working pressure of 75 bars are installed every 120 km, respectively every 4 sectors. The transport volume of the main pipeline is 460 Nm^3/s (there are 4 pipelines installed in parallel, so the overall transport volume is 1840 Nm^3/s) at a constant temperature of 14.5 °C (isothermal case, insulated pipeline). The surface roughness of the used steel inside the pipeline is defined as 0.15 mm. A detailed sketch of the system is given in Figure 2.

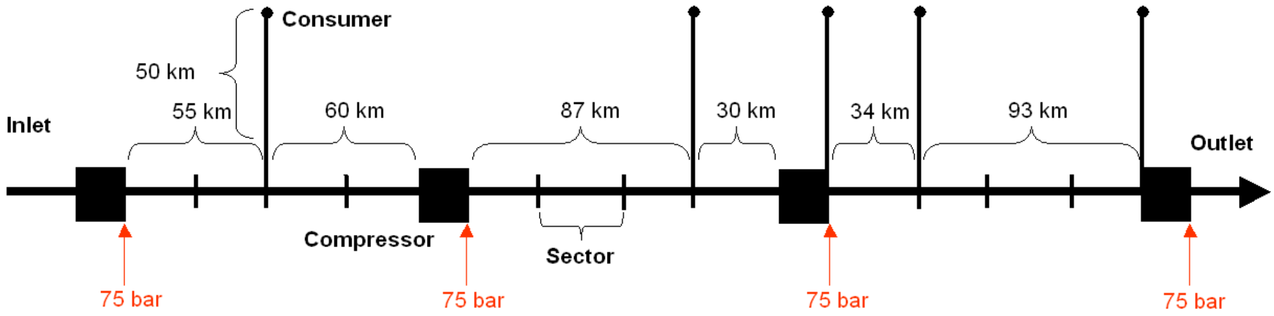

Figure 2: Investigated pipeline system

The composition of the natural gas taken in this case is given in the following Table 1. The data is obtained from [6].

Component	Volume Ratio [%]
Methane CH4	97.25
Ethane CH6	1.25
Propane CH8	0.48
Butane CH10	0.09
Carbon Dioxide CO2	0.06
Nitrogen N2	0.85

Table 1: Components of the natural gas

3.1 Data reconciliation model

The mathematical model of the pipeline system has been created by using the commercial software package VALI 4.2 from the Belgian software company BELSIM. This software provides a graphical user interface (GUI) to input all the necessary boundary conditions and automatically generates the resulting mathematical equations as described in Chapter 2.1.
An overview of the model setup in the GUI is given in Figure 3.
The main components used for this model are:

- Material streams to model a section of a pipeline in which natural gas is flowing.
- Valves (DPVAL) to model pressure drop through streams and valves.
- Black Boxes (BBXVAL): The BBXVAL unit allows to consider mass balances, energy balance and pressure drops for any process unit where no chemical reaction occurs.

More detailed information is given in the manual of VALI 4.2 [7].

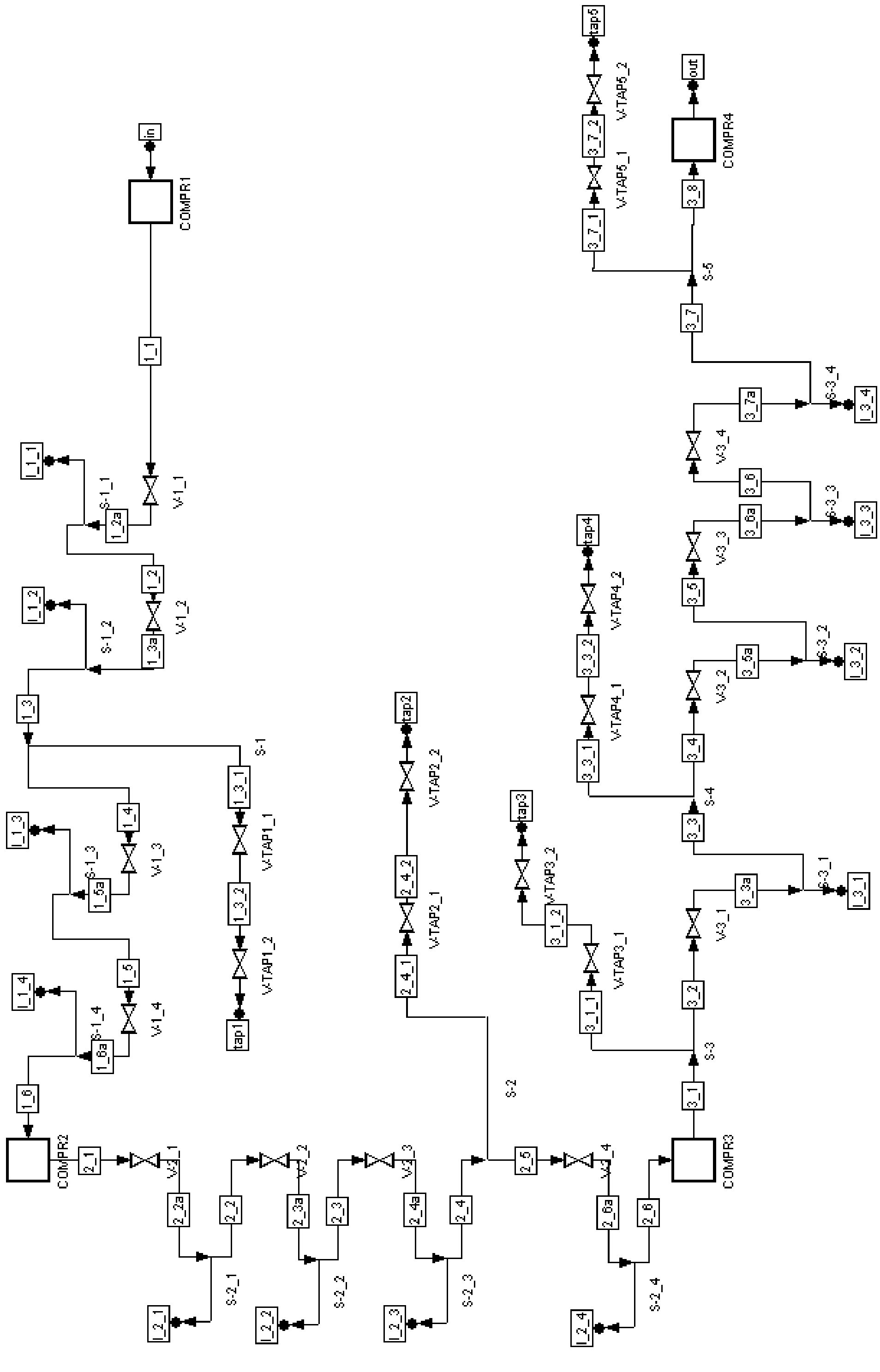

Figure 3: VALI 4.2 model of a pipeline system for data reconcilation

3.2 Location and uncertainties of the measurements

The gas pipeline system is equipped with different types of measurements devices. Pressure and volume flow are measured at different locations of the system. The measurements of the temperature are not taken into account, because the model is only valid for the isothermal case at this point of time. Therefore the temperature is set to a constant value of 14.5 °C, see Chapter 3.1. A detailed overview over the different measurements is given in Figure 4.

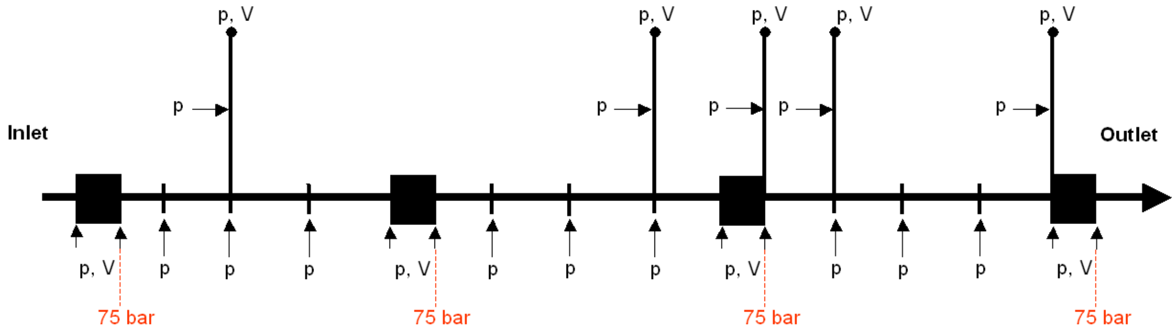

Figure 4: Location of measurements

As described in chapter 2.1, the measured values are provided with a specific uncertainty depending on the measured quantity. The uncertainties used in this model are

- 1 % for pressure values
- 1 % for volume flow rates at the inlet and the outlet of the main pipeline and at the consumers, 2 % at the compressors
- 10 % for the gas composition values, except methane with 3 %

3.3 Include calculated pressure gradients and leakage volume rates

The calculated values of the pressure gradients and the leakages volume flow rates are also considered in the data reconciliation model. This is done by assuming the calculated values to be equal to the measured values. In the case of the pressure gradient this is done be the following condition:

$$\frac{\Delta p_{measured}}{\Delta p_{calculated}} = 1 \quad \pm 10\% \ . \tag{14}$$

Including the calculation of the leakage volume is done by:

$$v_x - (\Delta v) = 0 \quad \pm 0.1 \ . \tag{15}$$

3.4 Results

The results of a reconciliation run are shown in Figure 5 with the following information displayed:

- Blue text fields: Measured values from the virtual process computer (*p* for pressure and *V* for volume flow).
- Yellow text fields: Reconciled values corresponding to the measured values or calculated leakage volume.
- White text fields:
  - Uns. MEA: Uncertainty of the measured value
  - Uns. REC: Uncertainty of the reconciled value
  - dp: Reconciled pressure difference due to mass and energy balances
  - dp_FLEX: Calculated pressure difference (see Chapter 2.2)
- χ^2: Number of redundancies, as a quantification for the amount of information used in this calculation run (see Chapter 2.1)
- L: Length of the pipeline section

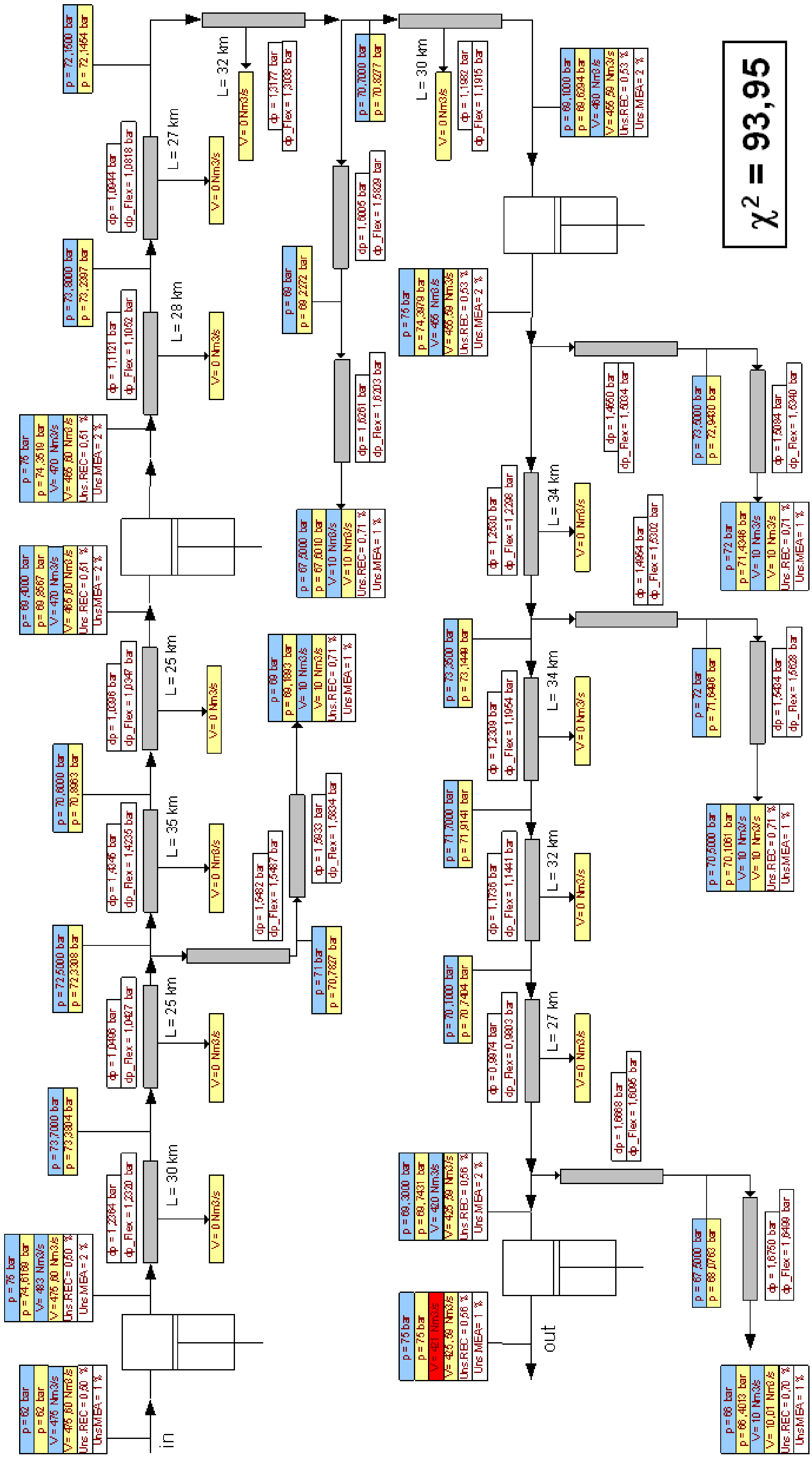

Figure 5:Reconciliation run, considering leakage flow

The first dataset in Figure 5 shows the obtained results for a reconciliation run with the pressure difference calculated according to the Colebrook approach. There is a small deviation between the calculated pressure value and the reconciled value due to the uncertainty of 10 %, defined at Equation (14). Obviously, the uncertainty of the measured volume flow at the inlet of the pipeline system is reduced significantly from 1% to 0.61 %, respectively 0.66 % at the outlet.
Figure 6 illustrates the consequences of the reduced uncertainty more clearly.

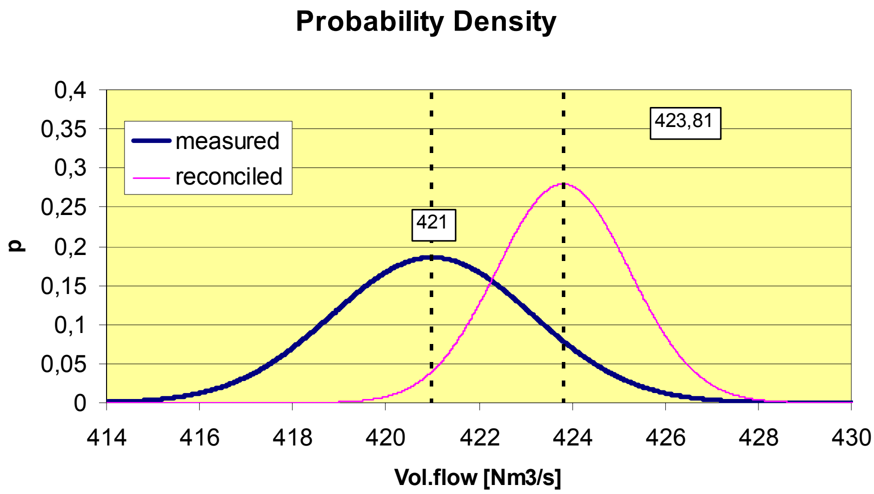

Figure 6: Corrected measurement and reduced uncertainty after reconciliation run

If the calculation of the pressure gradient over a pipeline sector is done with the Shapiro approach, a much higher value for the pressure drop is calculated. Taking the first sector of the pipeline, the *dp* increases from 1.2465 bar to a value of 4.23 bar. The results for *dp* obtained by the two different approaches will be compared to real measured data in the near future to investigate more detailed which one of the approaches is more suitable to obtain better results.

5 Literature

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[3] W. Beitz, K.-H. Küttner: Dubbel, Taschenbuch für den Maschinenbau, 17th Ed., Springer Verlag, 1990

[4] Ascher H. Shapiro, The Dynamics and Thermodynamics of Compressible Fluid Flow, Volume I, The Ronald Press Company, New York, 1953

[5] Wuppertal Institut für Klima, Umwelt und Energie / Max-Planck-Institut für Chemie Mainz, Endbericht: Treibhausgasemissionen des russischen Erdgas-Exportpipeline-Systems, February 2005

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[7] Belsim, VALI 4.2 User’s guide, Belsim S.A., B-4032 Liege, Belgium