Hilbert-Huang Transform, its features and application to the audio signal

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Abstrakt:
Hilbert-Huangova transformace (HHT) je nová metoda vhodná pro zpracování a analýzu signálů; zejména nelineárních a nestacionárních. Skládá se ze dvou základních částí, a to z EMD (Empirical Mode Decomposition - empirický rozklad na vnitřní modální funkce) a Hilbertovy transformace (HT). Příspěvek se zabývá volbou parametrů a modifikací algoritmu EMD a jejich vlivem na zpracování signálu metodou HHT. Praktické experimenty byly provedeny zejména na digitálním záznamu zvuku v prostředí Matlab.

Abstract:
Hilbert-Huang Transform (HHT) is new method suitable for signal analysis; especially nonlinear and non-stationary. It consists of two basic parts – EMD (Empirical Mode Decomposition) and Hilbert Transform. This paper deals with options and modifications of EMD algorithm and their influence to HHT signal analysis. Practical experiments were made with digital sound recording in Matlab environment.

1. Introduction – what is HHT

HHT is the method which allows us to deal with non-linear and non-stationary data and make no assumptions of linearity and/or stationarity as it is necessary with other traditional signal analysis methods like *Fourier spectral analysis*, *wavelet analysis*, *Wigner-Ville distribution* and many others. Many of them are more or less based on Fourier Transform and that's why they suffer similar problems [1].

If there is a talk about signal analysis there must be frequency talked too. And in frequency definition is also one of main differences between traditional approaches and *HHT* approach. In traditional (Fourier based) approach is frequency defined through integral. But if we want to analyze non-stationary data we cannot rely on global view. And that is the difference. *HHT* deals with "instantaneous frequency" defined in the same way as velocity – through time derivative [1]. Table 1.1 shows these analogy relations.

Tab.1.1

Velocity		**Frequency**	
Mean	**Instantaneous**	**Mean**	**Instantaneous**
$v = \frac{s}{t}$ (1.1)	$v = \frac{ds}{dt}$ (1.2)	$f = \frac{1}{T}$ (1.3)	$f = \frac{d\Phi}{dt}$ (1.4)

IF – instantaneous frequency

Instantaneous frequency is defined through analytical signal and the derivative of its phase angle. That means, if we want to compute *IF* we need to obtain appropriate analytical signal from our real signal. This is made through Hilbert Transform because analytical signal is such a signal whose imaginary part is the Hilbert transformation of its real part.

IMF – intrinsic mode functions

The problem with instantaneous frequency computed as mentioned above is that it functions only for signals that satisfy some condition which are not (especially in real data) usually fulfilled. In order to this problem there was in [1] proposed an *EMD* (Empirical Mode Decomposition) algorithm which is able to decompose input signal into set of *IMF*s (Intrinsic Mode Functions) which fulfill conditions. The result is that it is possible to compute instantaneous frequency from any signal without losing physical meaning as happen when we compute IF directly from the original signal.

Conditions those are necessary to successful computation of *IF*:

1) The number of zero crossing must be equal to or differ at most by one to number of local maximums and minimums
2) Mean value in the whole signal must be zero.

Example:

Let's consider simple sinusoidal signal (its Hilbert transformation is cosine)

$$y = \sin(t) + a \qquad \textbf{(1.5)}$$

and look what happens with instantaneous frequency when a grows up.

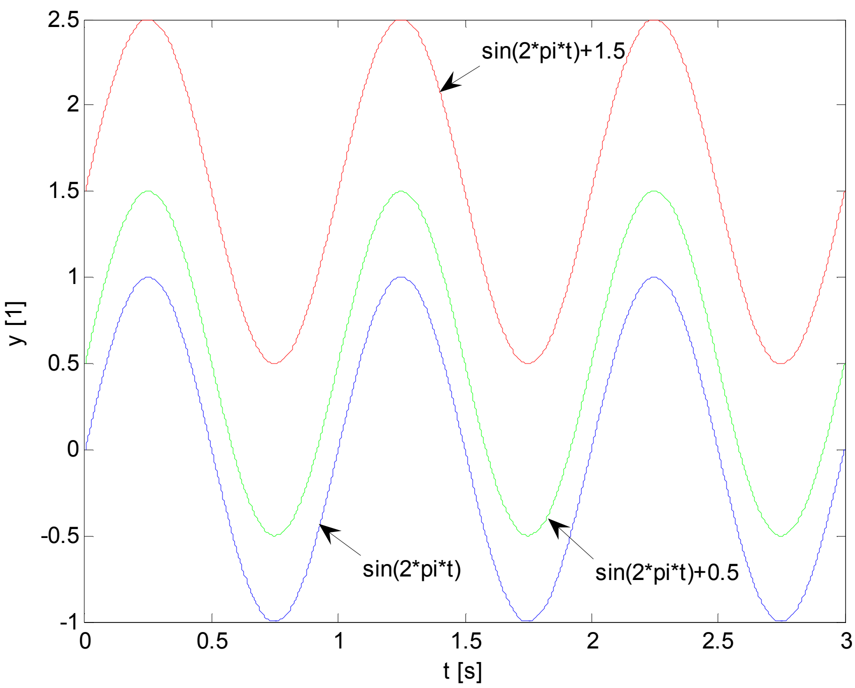

Fig.:1.1

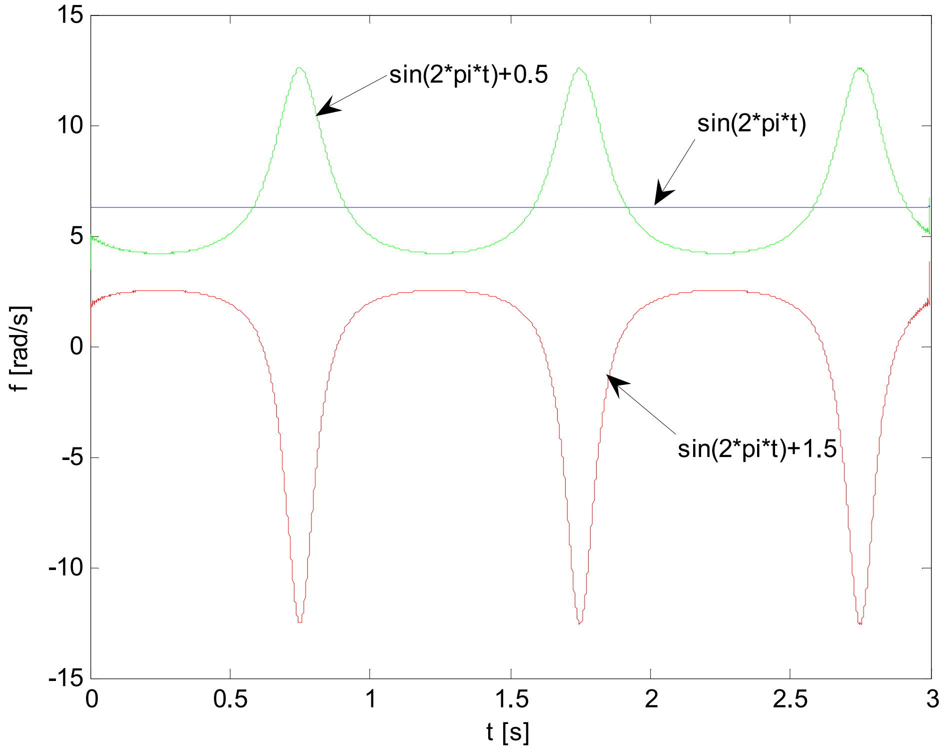

Fig.1.2: The influence of non-zero mean value of signal on instantaneous frequency

The negative frequencies have appeared. From Fig.1.3 it is clear why. It is also clear that those frequencies have no physical meaning and their occurrence is in order to the way of definition and computation.

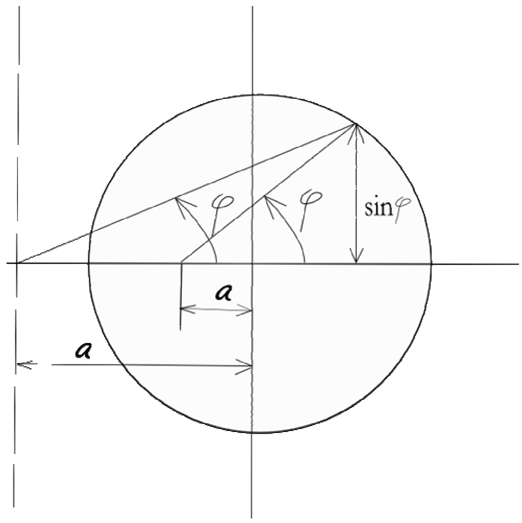

Fig.1.3: The mechanism of negative frequency generation

EMD – empirical mode decomposition

As mentioned above, EMD is an algorithm which pre-processes data to enable computation of instantaneous frequency from wide family of signals. But EMD can serve for signal analysis or for signal classification itself. The decomposition into IMFs can show us a useful view on the data as shown later in the chapter 3. The EMD algorithm has many parameters and variation which affects the results and it also depends on the character of analyzed data which features come out.

EMD is an iterative process when IMFs are by the sifting process which consists of [1]:

a) Finding local extremes

b) Computing spline envelopes through these extremes – one for maximums and one for minimums.

c) Computing instantaneous mean value as average value of both envelopes.

$$m_1 = \frac{1}{2}\left(e_{1\max} + e_{1\min}\right) \tag{1.6}$$

d) Subtraction data and mean. The result is the first iteration of finding IMF.

$$h_1 = X(t) - m_1 \tag{1.7}$$

These steps go around while conditions for IMF aren't fulfilled. After obtaining first IMF this is subtracted from original data and the result is the starting point for next round of iterations. At the end of this process constant function, trend or single wave function remains [1].

2. Implementation

EMD algorithm and computation of instantaneous frequency have been implemented in Matlab as m-functions and the development is still in progress. During testing we have encountered some limitations mostly concerning performance implied by amount of data and number of operations. There are a number of possibilities of result visualization in Matlab. Its example is in Fig.2.1 which shows processed blood glucose profile as smoothed colored time-frequency-amplitude Hilbert spectrogram where amplitude is expressed as color. This is much harder to do with audio data because of the amount of data. Development of appropriate optimization computation algorithm and results visualization are two of interests for future work.

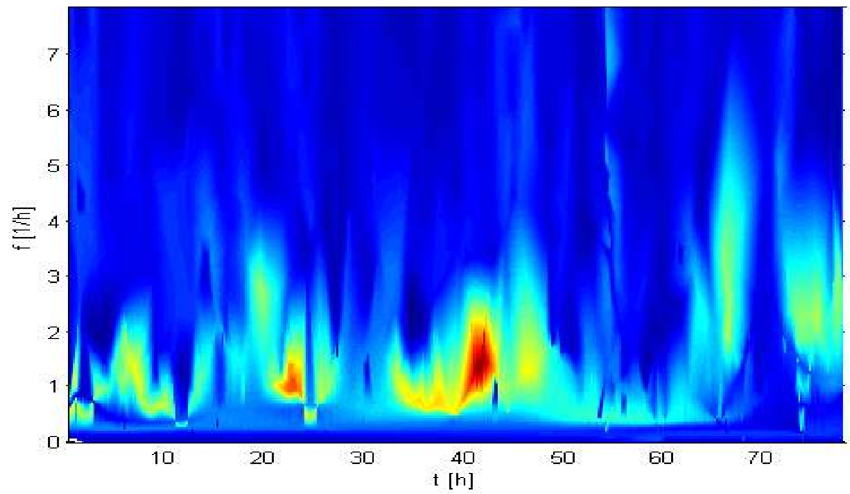

Fig.2.1:Smoothed Hilbert spectrum of blood glucose profile

3. Experiments

Testing and experiments are made on various data sets. For this paper I have chosen audio signal as an example. In Fig.3.1 we can see the EMD decomposition computed from about 0.5 second long recording of chord played on guitar and recorded with common PC microphone. As mentioned earlier this decomposition significantly clarifies the view on the data. We can see high frequency components, low frequency components and we have the amplitude information on y-axis. What is not clear and is the subject of further research is the exact interpretation of components because although they differ by frequency it seems not matching the traditional notion of harmonics.

In Fig.3.2 there is Hilbert spectrum computed from the components (IMFs) obtained by EMD algorithm. We can see that frequency is time variable but again the character of components is clear. The few singular points have appeared in order to non-perfectly tuned EMD algorithm but the quality of results are sufficient for application on this type of signal.

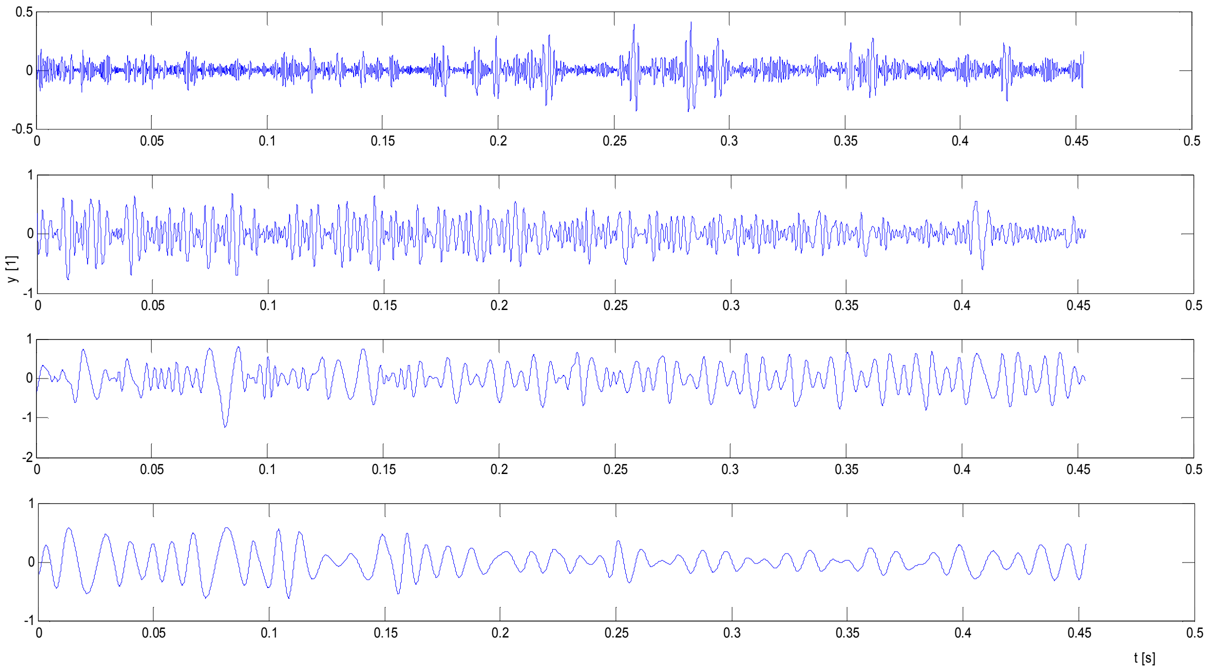

Fig.3.1a: First four IMF of analyzed audio signal (the A-minor chord played on guitar)

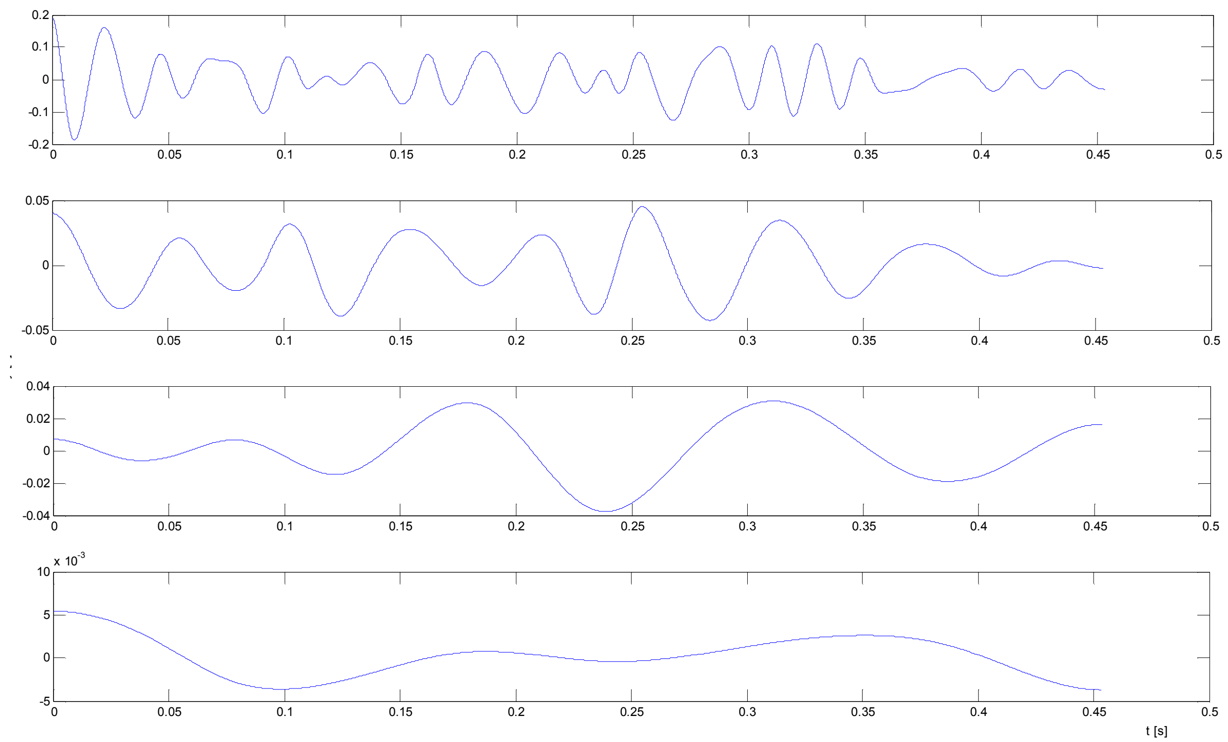

Fig.3.1b.: Next four IMFs of analyzed audio signal (the A-minor chord played on guitar)

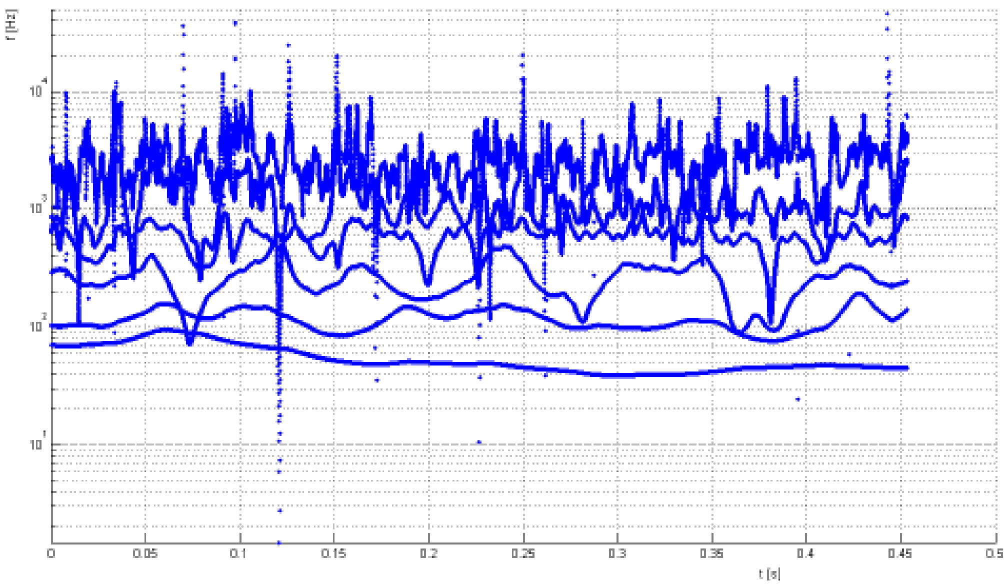

Fig.3.2: Hilbert spectrum of audio signal (the A-minor chord played on guitar) in logarithmic axis. There can be clearly seen various frequencies of various signal components.

4. Conclusion

This paper shows some of rich possibilities of HHT-based signal analysis. Short overview of this method is introduced at the beginning and its advantages are discussed. Main problem with computing instantaneous frequency is explained and showed on an example.

Applicability of implemented algorithm is verified by experiments with audio recording analysis. The results indicate possibilities of the method in this branch and the need of proper interpretation of components.

References:

[1] N.E. Huang, Z. Shen, S.R. Long, M.L. Wu, H.H. Shih, Q. Zheng, N.C. Yen, C.C. Tung and H.H. Liu: **The empirical mode decomposition and Hilbert spectrum for nonlinear and nonstationary time series analysis**, Proc. Roy. Soc. London A, Vol. 454, pp. 903–995, 1998.