# Internal Model Control Based on the Static Characteristic Knowledge

Ing. Nebojša Jovičić, Czech Technical University in Prague, Faculty of Mechanical Engineering, Department of Instrumentation and Control Engineering, nebojsa.jovicic@fs.cvut.cz

*Abstract* **—This lecture is focused on the usage of the processes described by second - order models with time – delay using internal model control based on the static characteristic knowledge. The reason why we use such approach is that in practice is very difficult to determine the dynamical behavior of the real process. The objective of this approach is the determination of the parameters, e.g. time constants, for which the control deviation is minimal. All derived results are validated by computer simulation Matlab – Simulink.**

*Keywords* **— Internal Model Control, Time Delay Systems, Frequency Domain.**

## I. INTRODUCTION

The internal model control (IMC) structure is first reviewed in the context of the YJBK parameterization in 1976 and soon after was introduced in 1980s by Manfred Morari and his coworkers as a general method combining the advantages of different model predictive control schemes. One of the features of IMC is that it requires an explicit model of the plant to be used as part of the controller. The concept of IMC is widely used due to its simple control structure, straightforward design procedure and strong robustness properties. We will consider any processes that include delays not only in the control but also in internal feedback loops, i.e., state delays. There are many examples of such industrial and chemical processes and some of them are: steam boilers, chemical reactors, distillation columns, heat exchangers, the model of mass flows in a sugar factory, wind tunnel control. Determining of the dynamic model for such processes is difficult to get so we will try to approximate it with its static model. The fact that we will use only open – loop stable systems significantly simplifies the question of stability. Necessary and sufficient conditions for stability are reduced to the simple requirement that controlled process and IMC controller must be stable.

## II. GENERAL CONTROLLER DESIGN

The IMC structure with IMC controller Q(s), plant $G_P(s)$ and plant model $G_M(s)$ is shown in Fig. 1. If we want to determine the transfer functions between the inputs and the process output, one of the easiest ways is to first redraw Fig. 1. as a simple feedback system, as shown in Fig. 2., applying the following rule:

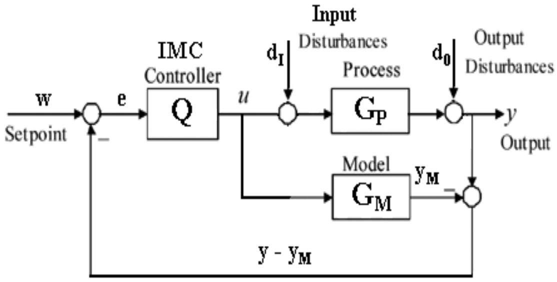


Fig. 1. Internal model control structure

The transfer function between any input and the output of a single – loop feedback system is the forward path transmission from the input to the output divided by one plus the loop transmission for negative feedback. For the feedback controller R(s) of Fig. 3., the rule gives the following equation:

$$R(s)=\frac{u(s)}{e(s)}=\frac{Q(s)}{1-Q(s)G_M(s)} \qquad (1)$$

The negative term in the denominator arises from the positive feedback around the Q(s). The design of IMC controller is based on the factorization of the plant model into the invertible and noninvertible components. The assumption of a perfect model $G_M(s) = G_P(s)$ and the absence of disturbances $d_I = d_0 = 0$ yields $y - y_M = 0$. Hence, the feedback signal disappears and Q(s) acts as a feedforward controller. Thus, for perfect control, we need a perfect model and the controller must perfectly invert that perfect model. An ideal control system would force the process output to track its setpoint instantaneously and perfectly suppress all disturbances so that they do not affect output.

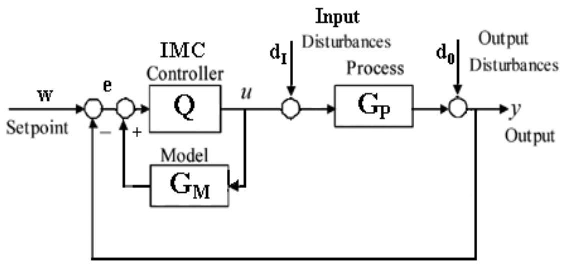


Fig. 2. Equivalent feedback system

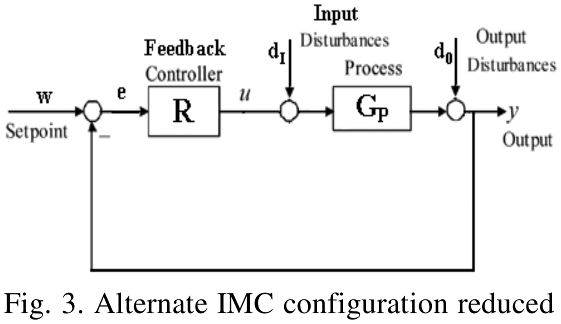


Fig. 3. Alternate IMC configuration reduced to a conventional closed – loop structure

The proposed plant model transfer function has following expression

$$G_p(s)=\frac{Ke^{-\tau s}}{(T_1 s+1)(Ts+e^{-\theta s})} \qquad (2)$$

The following two equations represent the real and the imaginary part of frequency response of the plant model, respectively.

$$R_eG_p(j\omega)= \frac{\left(\frac{K\cos(\tau\omega)}{1+T_1^2\omega^2}-\frac{K\sin(\tau\omega)T_1\omega}{1+T_1^2\omega^2}\right)\cos(\theta\omega)}{\cos^2(\theta\omega)+(T\omega-\sin(\theta\omega))^2}+ \frac{\left(-\frac{K\sin(\tau\omega)}{1+T_1^2\omega^2}-\frac{K\cos(\tau\omega)T_1\omega}{1+T_1^2\omega^2}\right)(T\omega-\sin(\theta\omega))}{\cos^2(\theta\omega)+(T\omega-\sin(\theta\omega))^2} \qquad (3)$$

$$I_mG_p(j\omega)= \frac{\left(-\frac{K\sin(\tau\omega)}{1+T_1^2\omega^2}-\frac{K\cos(\tau\omega)T_1\omega}{1+T_1^2\omega^2}\right)\cos(\theta\omega)}{\cos^2(\theta\omega)+(T\omega-\sin(\theta\omega))^2}- \frac{\left(\frac{K\cos(\tau\omega)}{1+T_1^2\omega^2}-\frac{K\sin(\tau\omega)T_1\omega}{1+T_1^2\omega^2}\right)(T\omega-\sin(\theta\omega))}{\cos^2(\theta\omega)+(T\omega-\sin(\theta\omega))^2} \qquad (4)$$

The invertible part of the plant model is described by equation

$$G_{p0}(s)=\frac{K}{(T_1 s+1)(Ts+e^{-\theta s})} \qquad (5)$$

A low – pass filter $F_2(s)$ is usually added to attenuate the effects of process – model mismatch. The filter is the only part of the controller that can be selected to meet control requirements.

$$F_2(s)=\frac{1}{(T_{F_2}s+1)^2} \qquad (6)$$

Thus, the internal model controller is usually designed as the inverse of the process model in series with a low – pass filter

$$Q_2(s) = F_2(s) G_{p0}^{-1}(s) \quad (7)$$

By combining equations 1,2,5,6 and 7 we obtain the transfer function of the controller

$$R_2(s) = \frac{(T_1 s + 1)(Ts + e^{-\theta s})}{K\left((T_{F_2} s + 1)^2 - e^{-\tau s}\right)} \quad (8)$$

The following two equations represent the real and the imaginary part of frequency response of the controller, respectively.

$$R_e R_2(j\omega) = \frac{\left[\cos(\theta\omega) - T_1\omega(T\omega - \sin(\theta\omega))\right]\left[-T_{F_2}^2\omega^2 + 1 - \cos(\tau\omega)\right]}{K\left[\left(-T_{F_2}^2\omega^2 + 1 - \cos(\tau\omega)\right)^2 + \left(2T_{F_2}\omega + \sin(\tau\omega)\right)^2\right]} + \frac{\left[T_1\omega\cos(\theta\omega) + T\omega - \sin(\theta\omega)\right]\left(2T_{F_2}\omega + \sin(\tau\omega)\right)}{K\left[\left(-T_{F_2}^2\omega^2 + 1 - \cos(\tau\omega)\right)^2 + \left(2T_{F_2}\omega + \sin(\tau\omega)\right)^2\right]} \quad (9)$$

$$I_m R_2(j\omega) = \frac{\left[T_1\omega\cos(\theta\omega) + T\omega - \sin(\theta\omega)\right]\left[-T_{F_2}^2\omega^2 + 1 - \cos(\tau\omega)\right]}{K\left[\left(-T_{F_2}^2\omega^2 + 1 - \cos(\tau\omega)\right)^2 + \left(2T_{F_2}\omega + \sin(\tau\omega)\right)^2\right]} - \frac{\left[\cos(\theta\omega) - T_1\omega(T\omega - \sin(\theta\omega))\right]\left(2T_{F_2}\omega + \sin(\tau\omega)\right)}{K\left[\left(-T_{F_2}^2\omega^2 + 1 - \cos(\tau\omega)\right)^2 + \left(2T_{F_2}\omega + \sin(\tau\omega)\right)^2\right]} \quad (10)$$

## III. STATIC MODEL BASED CONTROLLER DESIGN

The design procedure in this case is almost the same as in previous section. The only difference here is that we propose using the approximation of dynamic model by its static model. The transfer function of the proposed model is presented as

$$G_{M_1}(s) = \frac{K}{\Theta s + 1} \quad (11)$$

The IMC filter must be of first order since the model's relative degree equals one

$$F_1(s) = \frac{1}{T_{F_1} s + 1} \quad (12)$$

The internal model controller is again designed as the inverse of the process model in series with a low – pass filter

$$Q_1(s) = F_1(s) G_{M_1}^{-1}(s) \quad (13)$$

$$Q_1(s) = \frac{\Theta s + 1}{K(T_{F_1} s + 1)} \quad (14)$$

By combining and substituting proper equations we can find the transfer function of the controller

$$R_1(s) = \frac{Q_1(s)}{1 - Q_1(s) G_{M_1}(s)} \quad (15)$$

$$R_1(s) = \frac{\Theta s + 1}{K T_{F_1} s} \quad (16)$$

In this case, equations of frequency response are not so complicated and they have following form

$$R_e R_1(j\omega) = \frac{\Theta}{K T_{F_1}} \quad (17)$$

$$I_m R_1(j\omega) = -\frac{1}{K T_{F_1}\omega} \quad (18)$$

## IV. APPLICATION EXAMPLE

In this example is used model of the process (eq. 2) with the following parameters:

$T = 8.8,\ T_1 = 1.22,\ \tau = 1.73,\ \theta = 3.6, K = 1$

It is shown that for chosen parameter e.g. $T_{F_2} = 8.5$ it is possible to find out the parameters of PI controller (eq. 16) with the following values $\Theta = 3,\ T_{F_1} = 18.5$ for which is achieved best curve fitting of frequency responses (Fig.7) of open-loop systems with the PI controller (Fig. 6) and the second order controller with time delay (Fig.5). Best curve fitting is achieved by using mean square error in frequency domain. Error function was defined as:

$$E = \sqrt{[R_e R_1(j\omega) - R_e R_2(j\omega)]^2 + [I_m R_1(j\omega) - I_m R_2(j\omega)]^2}$$
$$= E(\omega, \Theta, T_{F_1})$$

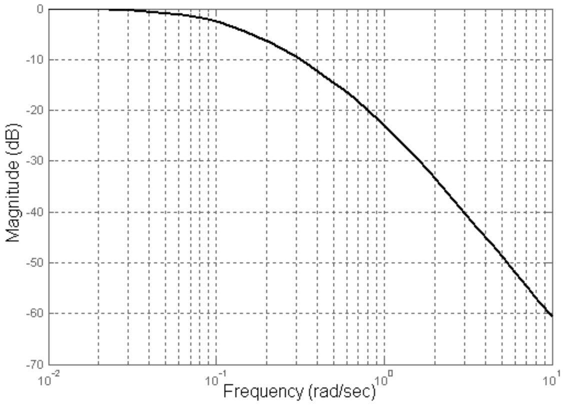


Fig 4. Logarithmic Magnitude Frequency Response of Process Model

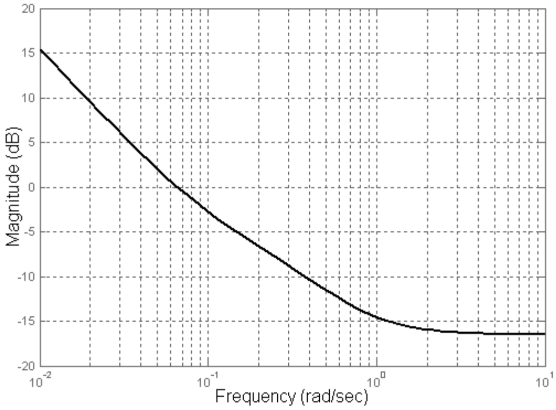


Fig 5. Logarithmic Magnitude Frequency Response of Second Order Controller with Time Delay

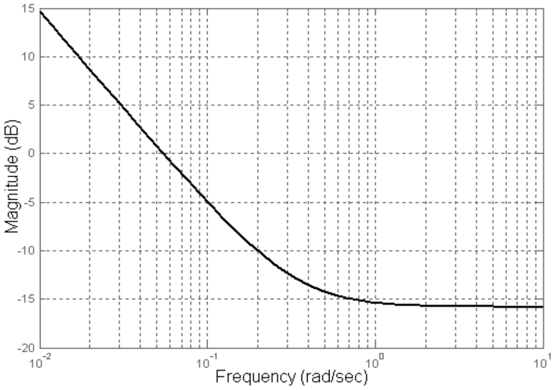


Fig. 6. Logarithmic Magnitude Frequency Response of PI Controller

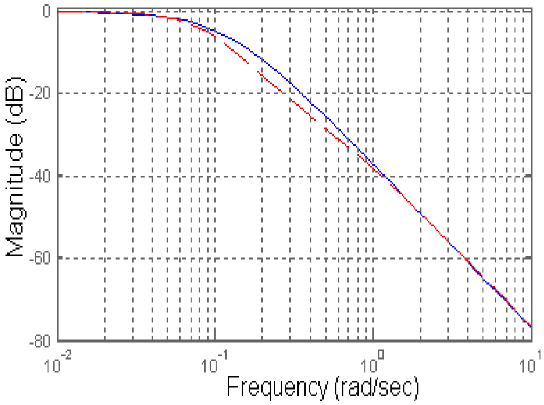


Fig. 7. Frequency responses of both open loop systems, with second (solid) and first (deshed) order controller.

## V. CONCLUSION

We have proposed the extension of IMC schemes for open – loop stable plants with lumped delays in both state and control using the approximation of dynamic model by its static model. Both the ideal case and static case designs were considered. A comparison between these two control structures is done on the basis of their frequency responses. It is shown that these control structures have the same behavior. Ann illustrative example has demonstrated the usefulness of the proposed idea. The presented controller should find its particular application in practice, thanks to its simple realization.

## REFERENCES


[1] Zítek, P. and Víteček, A. (1999). Control of Non – linear Time – delay Systems. CTU in Prague.

[2] Zítek, P. and Vyhlídal, T. (2006). Static – Model – Based Residue Generation for Hereditary Process Fault Detection. Institute of Instrumentation and Control Engineering , Center for Applied Cybernetics, Czech Technical University in Prague

[3] Zítek, P. and Hlava, J. (2001). Anisochronic internal model control of time – delay systems. Control Eng. Practice No. 9, pp. 501 – 516

[4] Morari, M. and Zafiriou, E. (1989). Robust Process Control. Prentice – Hall

[5] Goodwin, G.C. and Graebe, S.F. and Salgado, M.E. (2000). Control System Design. Prentice – Hall

[6] Brosilow, C. and Joseph, B. (2002). Techniques of Model – Based Control. Prentice - Hall