Input Shaping for Residual Vibration Suppression

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Abstrakt: *Jedním z problémů při řízení poddajných mechanismů jsou nežádoucí zbytkové vibrace. Tvarování vstupu je metoda dopředného řízení založená na takové modifikaci vstupního signálu, aby výstup měl požadované vlastnosti. V tomto příspěvku je popsán obecný princip metody tvarování vstupu s využitím Laplaceovy transformace v konečném čase, včetně podmínek nutných pro bodové řízení bez zbytkových vibrací. V druhé části příspěvku je pak provedena ukázka řízení bez zbytkových vibrací na systému se třením.*

Abstract: *In the control of flexible mechanical structures many methods are used to reduce unwanted residual vibration. Input shaping is a feed-forward control method based on modification of the input signal so that the output performs the demanded behaviour. In this paper the general basic principle of input shaping using finite-time Laplace transform is described including the derivation of necessary conditions for point-to-point control with zero residual vibration. In the second part the vibration-less control of system with Coulomb friction is shown.*

1. Introduction

Precise point-to-point motion is a common operation for many flexible mechanical systems but it is frequently corrupted by residual vibration. To avoid this unwanted performance engineers usually use strong motors and mechanical components of a very high stiffness. But despite these design properties machines are still limited by their own dynamics and control actions cause vibration of the overall system.

Basically two approaches can be applied to control flexible structures: feedback or feedforward. In this paper one particular method of the latter group – input shaping is described. Input shaping is based on modification of input signal so that it leads to zero residual vibration. This principle proved to be efficient in many applications such as robot manipulator [Chang, 2005], telescopic handler [Park, 2004], antisway crane [Valášek, 1995] etc. Elimination of vibration using modified input is shown in Fig. 1.1.

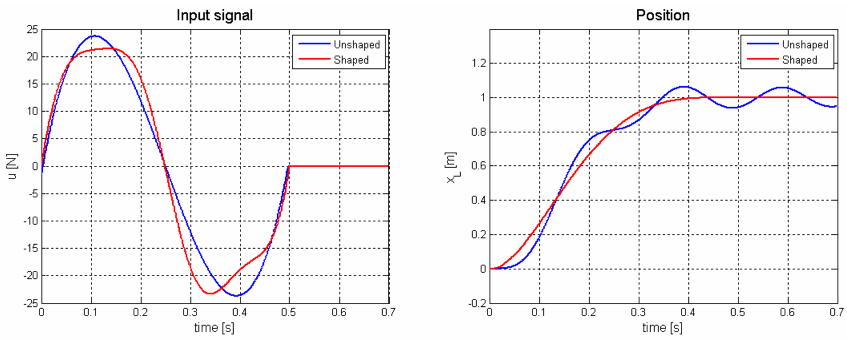

Fig. 1.1 – Using input shaping to eliminate vibration

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There are several approaches to vibration suppression based on modification of the input signal. Singer and Seering (1990) proposed a pre-shaping technique which consists of convolving a control input with a sequence of impulses. Their method is still in progress but it leads to time delays, especially in the case of more complex systems, that could be unacceptable. Miu (1993) published a method that explains many other theories using formulation of the point-to-point control problem in Laplace s-domain. This paper is based on the same approach but the description is extended to systems with multiple inputs. The necessary conditions for zero residual vibration are then derived using simple mechanical model and the results are demonstrated by simulation experiment. In the last chapter it is shown that the idea of input shaping can be applied to the flexible systems with Coulomb friction that adds nonlinear dynamics as well.

2. Problem description

The behaviour of under-actuated linear and time-invariant dynamical system can be characterised by the matrix equation

$$\dot{\mathbf{y}}(t) = \mathbf{A}\mathbf{y}(t) + \mathbf{B}\mathbf{u}(t) \tag{1}$$

where **A** and **B** are constant system matrixes, vector **y**[m×1] denotes system states and **u**[n×1] is control input, where at least m/2>n, i.e. the system includes at least m/2 degrees of freedom (DOF) the number of which is larger than the number n of actuators.

The aim of the solution of point-to-point control problem is to evaluate control input **u**(t) required to transformation of the system from initial state $\mathbf{y}(t_1)$ to the final state $\mathbf{y}(t_2)$ that are precisely defined.
The solution of (1) is:

$$\mathbf{y}(t_2) = e^{\mathbf{A}(t_2 - t_1)}\mathbf{y}(t_1) + \int_{t_1}^{t_2} e^{\mathbf{A}(t_2 - \tau)}\mathbf{B}\mathbf{u}(\tau)d\tau \tag{2}$$

Assuming full controllability, Eq. (2) can be transformed by a unique transformation to the Jordan canonical form and with some rearrangement it can be re-written in the following form [Bhat et al, 1991].

$$e^{-\mathbf{J}t_2}\mathbf{z}(t_2) - e^{-\mathbf{J}t_1}\mathbf{z}(t_1) = \int_{t_1}^{t_2} e^{-\mathbf{J}\tau}\mathbf{C}\mathbf{u}(\tau)d\tau \tag{3}$$

where

$$e^{\mathbf{J}t} = diag\left\{e^{\mathbf{J}_i t}\right\} \tag{4}$$

$$e^{\mathbf{J}_i t} = e^{p_i t}\begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \\ t & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \dfrac{t^{r_i - 1}}{(r_i - 1)!} & \dfrac{t^{r_i - 2}}{(r_i - 2)!} & \cdots & 1 & 0 \\ \dfrac{t^{r_i}}{r_i!} & \dfrac{t^{r_i - 1}}{(r_i - 1)!} & \cdots & t & 1 \end{bmatrix} \tag{5}$$

p_i is the pole of the i-th Jordan block $\mathbf{J}_i$ with multiplicity of $r_i + 1$. If k is the order of the system then $k = \sum_{i=1}^{r}(r_i + 1)$, where r is the total number of Jordan blocks.

The right hand side of (3) can be rewritten as a sum of contributions from individual inputs u_l

$$e^{-\mathbf{J}t_2}\mathbf{z}(t_2)-e^{-\mathbf{J}t_1}\mathbf{z}(t_1)=\sum_{l=1}^{n}\int_{t_1}^{t_2}e^{-\mathbf{J}\tau}u_l(\tau)d\tau\cdot c_l \tag{6}$$

where c_l is a column of the matrix $\mathbf{C}$ corresponding to u_l.

The expressions on the right-hand side of (6) resembles the finite time Laplace transform of the control input as defined in [Miu, 1993]

$$U(s)=\int_{t_1}^{t_2}e^{-s\tau}u(\tau)d\tau \tag{7}$$

therefore

$$\sum_{l=1}^{n}U_l(s)\big|_{s=\mathbf{J}}\cdot c_l=e^{-\mathbf{J}t_2}\mathbf{z}(t_2)-e^{-\mathbf{J}t_1}\mathbf{z}(t_1) \tag{8}$$

This equation is equivalent with (1) and connects Laplace transform of the control input with the state of the system at the beginning and at the end of the point-to-point maneuver. Interesting and important fact is that (8) is algebraic whereas (1) is differential equation.

3. Necessary conditions for zero residual vibration

The Laplace transform of the control input has to satisfy equation (8). In this chapter additional constrains will be derived in order to ensure zero residual vibration. Exact formulation of these conditions depends on the description of the system so the simple mechanical model will be used.

The left hand side of (8) is just the algebraic sum of contributions from individual inputs so without lost of generality the system with single input can be used for derivation of necessary conditions, e.g. two-mass spring damper model in Figure 3.1.

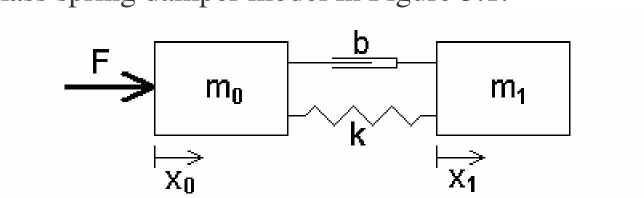

Fig. 3.1 - Spring mass system

This system with one input F and two DOFs x_0, x_1 is described by equation

$$\mathbf{M}\ddot{\mathbf{x}}(t)+\mathbf{B}\dot{\mathbf{x}}(t)+\mathbf{K}\mathbf{x}(t)=\mathbf{F}(t) \tag{9}$$

where

$$\mathbf{x}=\begin{bmatrix}x_0\\x_1\end{bmatrix},\ \mathbf{M}=\begin{bmatrix}m_0&0\\0&m_1\end{bmatrix},\ \mathbf{B}=\begin{bmatrix}b&-b\\-b&b\end{bmatrix},\mathbf{K}=\begin{bmatrix}k&-k\\-k&k\end{bmatrix},\ \mathbf{F}=\begin{bmatrix}f\\0\end{bmatrix} \tag{10}$$

Demand for point-to-point maneuver without residual vibrations leads to the following conditions

$$\mathbf{x}(t)\big|_{t=t_2}=\begin{bmatrix}X_0\\X_0\end{bmatrix},\ \dot{\mathbf{x}}(t)\big|_{t=t_2}=\begin{bmatrix}0\\0\end{bmatrix} \tag{11}$$

Using modal coordinates and transformation to the Jordan canonical form [Miu, 1993] equation (9) can be rewritten as

$$\underbrace{\begin{bmatrix}\dot z_1\\ \dot z_2\\ \dot z_3\\ \dot z_4\end{bmatrix}}_{\dot{\mathbf{z}}}=\underbrace{\begin{bmatrix}0&0&0&0\\1&0&0&0\\0&0&p&0\\0&0&0&p^*\end{bmatrix}}_{\mathbf{J}}\underbrace{\begin{bmatrix}z_1\\ z_2\\ z_3\\ z_4\end{bmatrix}}_{\mathbf{z}}+\underbrace{\begin{bmatrix}1\\0\\1\\1\end{bmatrix}}_{\mathbf{c}}u \tag{12}$$

where $p=-\zeta\omega+j\sqrt{1-\zeta^2}\,\omega$, $p^*=-\zeta\omega-j\sqrt{1-\zeta^2}\,\omega$ are complex conjugated poles of the flexible mode and $\omega=\sqrt{\dfrac{k(m_0+m_1)}{m_0m_1}}$, $\zeta=\dfrac{b(m_0+m_1)}{2\omega\, m_0m_1}$. Now the system is described as by (1) and (3). Conditions (11) for zero residual vibration are transformed by above mentioned transformation to

$$\mathbf{z}(t_2)=[0;X_0;0;0] \tag{13}$$

and necessary conditions for $U(s)$ can be derived. It is obvious that the system has two poles at zero and two conjugated poles p and p^* (see $\mathbf{J}$ in (12)). Therefore according to (8) the conditions must be formulated for $U(s)\big|_{s=0}$, $\dfrac{dU(s)}{ds}\Big|_{s=0}$, $U(s)\big|_{s=p}$ and $U(s)\big|_{s=p^*}$.

Assuming that $t_1=0$, $t_2=T$ and zero initial conditions it can be derived

$$U(s)\big|_{s=0}=\lim_{s\to 0}\int_0^T e^{-s\tau}u(\tau)d\tau=\int_0^T u(\tau)d\tau=z_1(T)=0 \tag{14}$$

$$\frac{dU(s)}{ds}\bigg|_{s=0}=\lim_{s\to 0}\frac{d}{ds}\int_0^T e^{-s\tau}u(\tau)d\tau=\lim_{s\to 0}\int_0^T -\tau e^{-s\tau}u(\tau)d\tau=-\int_0^T \tau u(\tau)d\tau \tag{15a}$$

and integration of (15a) per-partes results in

$$\frac{dU(s)}{ds}\bigg|_{s=0}=-[\tau\cdot z_1(\tau)]_0^T+\int_0^T z_1(\tau)d\tau=-T\cdot z_1(T)+z_2(T)=z_2(T)=X_0 \tag{15b}$$

The Laplace transform of the third row in the Eq. (12) is

$$sZ_3(s)=p\cdot Z_3(s)+U(s) \tag{16}$$

therefore

$$sZ_3(s)\big|_{s=p}=pZ_3(s)\big|_{s=p}+U(s)\big|_{s=p}\quad\Rightarrow\quad U(s)\big|_{s=p}=0 \tag{17}$$

and analogically

$$sZ_4(s)\big|_{s=p^*}=p^*\,Z_4(s)\big|_{s=p^*}+U(s)\big|_{s=p^*}\quad\Rightarrow\quad U(s)\big|_{s=p^*}=0 \tag{18}$$

The whole set of necessary conditions for zero residual vibration is then (compare with (13))

$$U(s)\big|_{s=0}=0,\ \ \frac{dU(s)}{ds}\Big|_{s=0}=X_0,\ \ U(s)\big|_{s=p}=0,\ \ U(s)\big|_{s=p^*}=0 \tag{19}$$

A physical interpretation of these conditions is that for point-to-point control without residual vibrations the time-bound input signal has to contain zero resultant energy at the poles of the flexible modes. If the system is un-damped and so has poles along the imaginary axis this condition means that the Fourier transform of the input signal has zero amplitude at the system resonant frequency.

Note: Conditions (19) are formulated in [Miu, 1993] as well but without derivation.

4. Simulation experiment

The following simulation experiment was performed with two mass model of a machine tool drive with transfer function between load and motor that can be written as [Souček, 2004] (assuming zero external force on load)

$$G(s)=\frac{Y(s)}{U(s)}=\left.\frac{x_L(s)}{a_M(s)}\right|_{F_L=0}=\frac{1}{p}\cdot\frac{1}{s^2}\cdot\frac{\frac{2\zeta_M}{\omega_M^*}s+1}{\frac{s^2}{\omega_M^{*2}}+\frac{2\zeta_M}{\omega_M^*}s+1} \tag{20}$$

where p is gearing ratio, ω_M^* is the anti-resonant locked motor frequency and ζ_M is the damping coefficient. This system has two poles at zero and two complex conjugated poles $q=-\zeta_M\omega_M^*+j\sqrt{1-\zeta_M{}^2}\,\omega_M^*$, $q^*=-\zeta_M\omega_M^*-j\sqrt{1-\zeta_M{}^2}\,\omega_M^*$. It is the same situation as (12) so the necessary conditions for $U(s)$ zero residual vibration are (19). The control input was considered in the following form

$$u(t)=\lambda_0+\lambda_1 t+\lambda_2 e^{at}\sin bt+\lambda_3 e^{at}\cos bt+\lambda_4 t^2+\lambda_5 t^3 \tag{21}$$

where $a=\zeta_M\omega_M^*$, $b=\sqrt{1-\zeta_M{}^2}\,\omega_M^*$ and λ_i are weighting coefficients that were computed using s-domain synthesis technique [Miu, 1993]. This form of control input was derived as a minimum energy solution and it is modified by polynomials of the second and third the order to fulfill time domain constrains as well. Time domain conditions were formulated to ensure time domain continuity of the input signal

$$u(t_1)=0,\ u(t_2)=0 \tag{22}$$

The results of simulation experiment are in figures bellow.

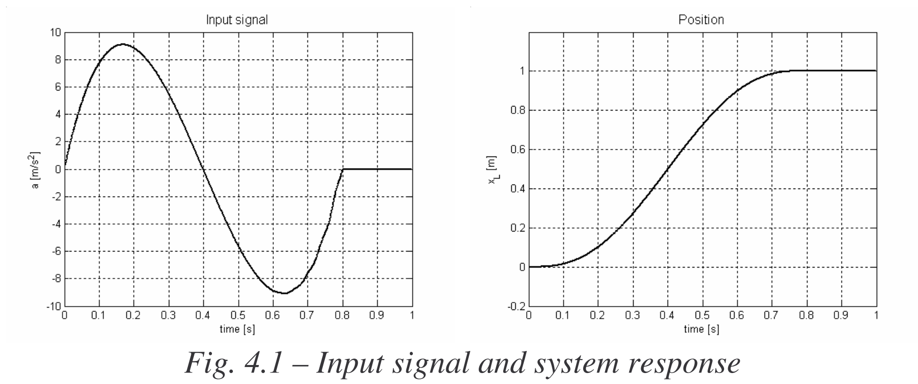

Fig. 4.1 – Input signal and system response

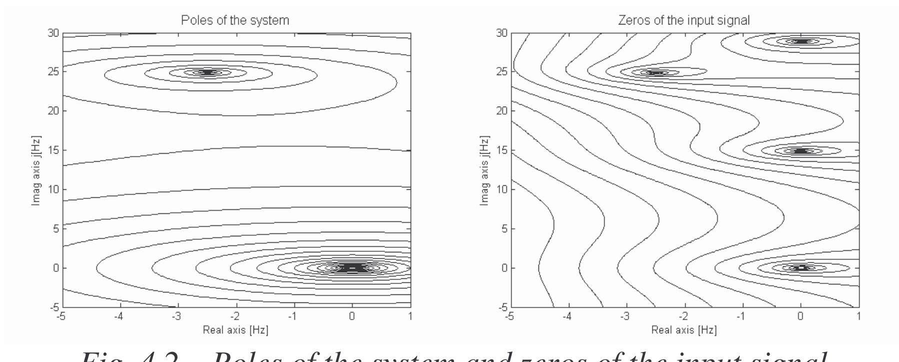

Fig. 4.2 – Poles of the system and zeros of the input signal

The figure of the system response shows that there are no residual vibrations. The contour maps of the system and the input signal illustrate that derived conditions (19) are fulfilled and zeros of the input signal are precisely at the poles of the system. By multiplication in the Laplace domain the zeros cancel the poles and thus the system has zero residual vibration.

5. System with Coulomb friction

Precise positioning of real-world systems with Coulomb friction is very difficult and it is further complicated if the system has flexible dynamics. To ensure that the input shaping is able to deal with this situation the simulation model in Fig. 5.1 was used.

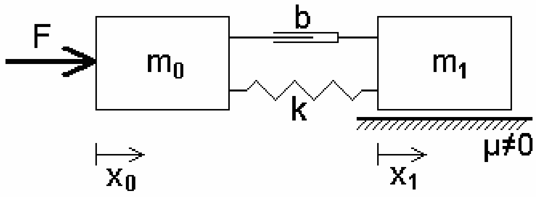

Fig. 5.1 – System with Coulomb friction

Coulomb friction forces were simulated by very simple model

$$F_C = N \cdot \mu \cdot sign(\dot{x}_1) \tag{23}$$

where N is the normal force and μ is the friction coefficient.

The response of this system to the step input is in Fig. 5.2. It shows the problem of precise positioning. Moreover after the final stop of the m_1 mass (due to the friction force) there still remains some amount of energy trapped in the flexible mode of the system. This energy causes vibration of the m_0 mass.

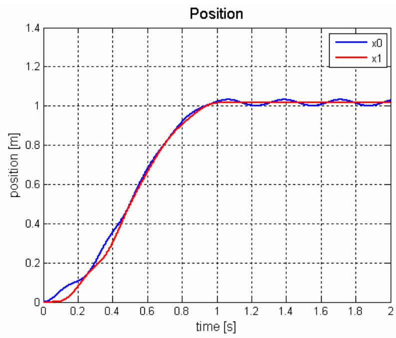

Fig. 5.2 – System with Coulomb friction - Response to unshaped input

The nature of the signum function leads to the model that switches between three analytical equations. It would be difficult to perform transformation to Jordan canonical form and consequently find the solution in s-domain as mentioned in chapter three. Therefore the problem was solved in time domain.

Input was assumed in the form of polynomial function that has to full fill conditions (11). The results are in Fig. 5.3.

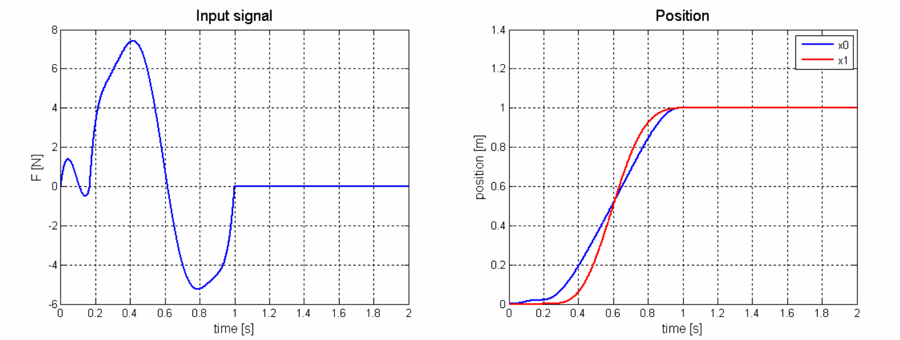

Fig. 5.3 – System with Coulomb friction - shaped input

6. Conclusions

The basic principle of the input shaping control using formulation in Laplace domain was described considering the system with multiple inputs. The derivation of necessary conditions for zero residual vibration, that has been in [Miu, 1993] omitted, was described and the results were demonstrated on the simulation model of machine tool drive.
The way of precise and vibration-less control of the system with Coulomb friction using shaped signal was shown. The aim of further studies is implementation of this solution to the Laplace domain formulation of the input shaping.

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